PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

PROCEEDINGS

OF

THE ROYAL SOCIETY

OF

EDINBURGH

VOL. LII.

1931-1932.

EDINBURGH:
PRINTED BY NEILL AND COMPANY, LIMITED.
MDCCCCXXXIII.

CONTENTS.

CALENDAR OF HUME MSS.

IN THE POSSESSION OF
THE ROYAL SOCIETY OF EDINBURGH.

BY

J. Y. T. GREIG AND HAROLD BEYNON.

(FORMING VOLUME LII, PART I, OF
THE PROCEEDINGS OF THE ROYAL SOCIETY OF EDINBURGH.)

EDINBURGH:
PRINTED BY NEILL AND COMPANY, LIMITED.
MDCCCCXXXII.

CONTENTS.

PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

VOL. LII. 1931–32.

CALENDAR OF HUME MSS.

INTRODUCTION.

The MSS. here calendared were presented to the Royal Society of Edinburgh by David Hume the Younger, nephew of the philosopher and Baron of the Scottish Exchequer.

* * *

The editors desire gratefully to acknowledge a grant made by the Research Committee of Armstrong College, Newcastle-upon-Tyne, in the University of Durham, to assist them in the work of compiling the *Calendar*.

* * *

The numerals in heavy type indicate the volume and folio where the document is to be found.

When the document contains the addressee's name and address we have set these down in a shortened form. Thus, " Francis Hutcheson, Glasgow," stands for " To|Mr Francis Hutcheson|Professor of Philosophy|at Glasgow," or a similar mode of address. When the document contains no address we have enclosed the addressee's name in square brackets. An entry like " Colonel [A.] " indicates that the letter, known to be written to Colonel Abercromby, begins " Dear Colonel " but contains no address.

Immediately after the addressee's name and address are printed the place from which the letter was written, and its date. If these are enclosed in square brackets, they have been derived from external evidence alone.

" [1½ pp. + 2½ pp., 11·5 × 23 cm.] " means that the document consists of 1½ pages of text and 2½ pages that are either completely blank, or blank except for the address; and that the page measures approximately 11·5 cm. in breadth by 23 cm. in length.

After the measurements follow references to books where the document has already been, or is about to be, printed, in full or in part. We have not attempted to give all these references, but only the more important of them.

All letters are in the mother tongue of the writer, unless it is otherwise stated. The following abbreviations have been used :—

Burton = " Life and Correspondence of David Hume," by John Hill Burton, 2 vols., Edinburgh, 1846.

Greig = " The Letters of David Hume," edited by J. Y. T. Greig, 2 vols., Oxford (forthcoming). (Hume's own letters are referred to by their numbers in this edition.)

Greig, *Life* = " David Hume," by J. Y. T. Greig, London, 1931.

Cald. Papers = " Caldwell Papers, being the Papers of the Family of Mure of Caldwell," edited by William Mure, 2 vols. (Vol. II in 2 Parts), Glasgow, 1854.

E.P. = " Letters of Eminent Persons addressed to David Hume," edited by John Hill Burton, Edinburgh, 1849. (It should be noted that the text of this book is extremely inaccurate.)

Exposé succinct = " Exposé succinct de la contestation . . . entre M. Hume et M. Rousseau," Paris, 1766.

Hill = " Letters of David Hume to William Strahan," edited by G. Birkbeck Hill, Oxford, 1888.

Oswald = " Memorials of James Oswald of Dunnikier," Edinburgh, 1825.

Smith to Strahan = Adam Smith's letter of 9 Nov. 1776, to William Strahan, usually printed along with Hume's " My Own Life."

H. = Hume.

Inc. = incomplete.

LETTERS BY HUME.

To Colonel James Abercromby of Glassauch.

I, 1. Colonel [A.]. Ninewells, 7 Aug. 1747. *Signed.* [4 pp., 19 × 24 cm.; Burton, I, 222–3, inc.; Greig, No. 57.]

On H.'s claim for half-pay. Necessity of winning over Mr Pelham. H.'s voyage to Berwick.

I, 2. Colonel [A.]. Ninewells, 12 Nov. 1747. *Signed.* [1 p. + 1 p., 18·5 × 22 cm.; Greig, No. 59.]

On H.'s claim for half-pay.

I, 3–5. Hon. Col. James Abercromby, Westminster. Ninewells, 16 Feb. 1751. [*Three documents as under :—*]

A. *Signed.* [1½ pp. + 2½ pp., 11 × 19 cm.; Burton, I, 311–12; Greig, No. 69.]

Revealing to A. a practical joke to be played on a common friend, Fraser, in connection with recent Westminster Election.

B. *Signed.* [1 p. + 3 pp., 19 × 23 cm.; Burton, I, 313; Greig, No. 69A.]

Ostensible letter to A., to be shown to Fraser, declaring H.'s disgust at the latter's proceedings.

C. [5 pp. + 1 p., 19 × 31 cm., 2 pp., 18 × 23 cm.; Burton, I, 308–11; Greig, App. A.]

Enclosure to *B.*, being supposed Petition to " Lord Chief Justice Reason," complaining of Fraser's recent doings.

To the Rev. Hugh Blair.

I, 6. Dr Blair, Edinburgh. [Paris, Dec. 1763.] *Signed.* [4 pp. + 2 pp., 17 × 19·5 cm.; Burton, II, 180–1, inc.; Greig, No. 227.]

Advice and encouragement to B. on undertaking the investigation of the authenticity of Ossian. H.'s life at Paris: at present it gives him "more pain than pleasure." The agreeableness of the men of letters; the attraction of the ladies. Lord Hertford's kindness.—Advice on domestic matters, B. having become H.'s tenant in Edinburgh. Unlikelihood of H.'s early return.

I, 7. Rev. Dr Hugh Blair, Edinburgh. Paris, 26 Apr. 1764. *Signed.* [5½ pp. + 2½ pp., 18·5 × 23 cm.; Burton, II, 193–8, inc.; Greig, No. 237.]

H.'s inability to introduce Col. Lesly to French Society.—Rejoices that John Adam has not been left destitute by recent financial failure.—Voltaire has written a very witty reply to Lord Kames. State of literature in France: Voltaire's edit. of Corneille, etc. H.'s happy life: the civilities he receives from the nobility.—Ferguson and the Chair of Moral Philosophy at Edinburgh.

I, 8. Dr Hugh Blair, Edinburgh. Paris, 6 Apr. 1765. *Signed.* [3¼ pp. + 2¾ pp., 20 × 30 cm.; Burton, II, 266–70, inc.; Greig, No. 272.]

A common letter to all H.'s "Protestant Pastors."—*To Robertson*: Success of the French translation of his *Hist. of Scot.*—*To Jardine*: A short story to illustrate H.'s success with the French ladies.—*To Carlyle*: No use writing to Lord Eglintoune about a living for Wilson.—*To Ferguson*: F.'s recent change of Chair, which only half-pleases H.—*To Blair*: Admiration in France of B.'s *Dissertation on Fingal*; though some still doubt authenticity of poems.—Differences between England and France, especially in attention shown to men of letters. H.'s personal success, and plans for the future.

I, 9. Dr Blair. Compiègne, 20 July 1765. *In third person.* [1 p. + 3 pp., 11·5 × 18 cm.; Burton, II, 286–7; Greig, No. 284.]

Greetings to Edinburgh friends. Dauphin has asked about Robertson. Possibility of electing Dauphin to Poker Club.

I, 10. Dr [B.]. Paris, 23 Aug. 1765. *Unsigned.* [3 pp. + 1 p., 15·5 × 20 cm.; Burton, II, 288–90, inc.; Greig, No. 287.]

B.'s Dissertation much admired. H. somewhat sceptical about extreme antiquity of poems.—Lord Hertford must leave France. H. receives a pension; will probably settle in Edinburgh.—*P.S.* Sir James Macdonald's success in Paris.

I, 11. Dr Hugh Blair, Edinburgh. Paris, 28 Dec. 1765, and P.S. 1 Jan. 1766. *Signed.* [8 pp., 18·5 × 23 cm.; Burton, II, 293 and 297–303, inc.; Greig, No. 295.]

H.'s intention of remaining some time longer in France; but must cross to

London for business reasons and to settle Rousseau.—R.'s past life; persecution at Neufchâtel, etc. His great popularity in Paris. Description of R. Comparison with Socrates.—*P.S.* H. still hesitating where to settle, but asks B. to vacate house in Edinburgh.

I, 12. Dr Hugh Blair, Edinburgh. Lisle Street [London], 11 Feb. 1766. *Signed.* [1½ pp. + 2½ pp., 18 × 23 cm.; Burton, II, 310, inc.; Greig, No. 303.]

H.'s low opinion of Ferguson's papers [MS. of *Essay on Civil Society*]. Begs B. and Robertson to dissuade F. from publishing.—Excellent state of H.'s friendship with Rousseau. R., the most devout philosopher, has been persecuted the most.

I, 13. Dr Hugh Blair, Edinburgh. Lisle Street, Leicester Fields [London], 25 Mar. 1766. *Signed.* [5½ pp. + 2½ pp., 18·5 × 23 cm.; Burton, II, 312–17; Greig, No. 314.]

Rousseau says details of *Nouvelle Héloïse* fictitious. Has an absurd preference for *Contrat social* over his other works. About to settle on the Peak with Mr Davenport. His acute sensibility: story of Davenport's hiring of the chaise, and R.'s sulks and reconciliation with H. the night before his departure. Reason for R.'s dislike of company.—R.'s modesty about his own works. R. is meditating a continuation of the *Emile*.

I, 14. Dr Hugh Blair, Edinburgh. Lisle Street, Leicester Fields [London], 5 June 1766. *Signed.* [1 p. + 3 pp., 18·5 × 23 cm.; Burton, II, 318, inc.; Greig, No. 326.]

Regrets Dr Jardine's death. Begs B. to try to persuade Dr Drysdale to take Mr Dodwell's son as a boarder.

I, 15. Dr Hugh Blair, Edinburgh. Lisle Street, Leicester Fields [London], 1 July 1766. *Signed.* [1½ pp. + 2½ pp., 18·5 × 23 cm.; Burton, II, 344–5, inc.; Greig, No. 334.]

Competition among Scottish divines for two small offices.—Rousseau is the blackest villain in existence. H. begs B. to conceal all H.'s former letters in praise of R., and to recover any copies that may have been made.

I, 16. Dr Hugh Blair, Edinburgh. [London] 15 July 1766. *Signed.* [1½ pp. + 2½ pp., 18·5 × 22·5 cm.; Burton, II, 345–6; Greig, No. 337.]

Outline of quarrel between H. and Rousseau, with summary of latter's absurd charges.

I, 17. [H. B.]. London, 24 Feb. 1767. *Signed.* [2 pp. + 2 pp., 18·5 × 23 cm.; Burton, II, 365 and 386; Greig, No. 371.]

Favourable opinion of Ferguson's book [*Essay on Civil Society*] expressed by

ates * having $(\bar{x}^{\mu})$ as origin. These are defined as functions of the original coordinates (x^{μ}) by the equation †

$$(3.12) \qquad y^{\mu} = -\bar{g}^{\mu\alpha}\frac{\partial\Omega}{\partial\bar{x}^{\alpha}}, \qquad (\mu = 0, 1, 2, 3)$$

where Ω, as above, represents one-half of the square of the geodesic distance between $(\bar{x}^{\mu})$ and (x^{μ}). For the moment Ω is not zero, since the x^{μ} are being employed as current coordinates instead of as the world-coordinates of the star.

In the fundamental galilean case,

$$\Omega = \tfrac{1}{2}\{(t-\bar{t})^2 - (x-\bar{x})^2 - (y-\bar{y})^2 - (z-\bar{z})^2\},$$
$$\bar{g}^{00} = 1,\ \bar{g}^{11} = \bar{g}^{22} = \bar{g}^{33} = -1, \quad \bar{g}^{\mu\nu} = 0 \text{ if } \mu \neq \nu, \quad \text{so}$$
$$y^0 = -\partial\Omega/\partial\bar{t} = t - \bar{t}$$

and similarly $y^1 = x - \bar{x}$, $y^2 = y - \bar{y}$, $y^3 = z - \bar{z}$. Hence in this case the transference to normal coordinates reduces to a mere change of origin without rotation.

Expressed in terms of the new variables, the equations of any geodesic through $(\bar{x}^{\mu})$ are of the form

$$(3.13) \qquad y^{\mu} = a^{\mu}\, s, \qquad (\mu = 0, 1, 2, 3)$$

where s is the length of its arc from the origin $(y^{\mu}) = 0$ to the point (y^{μ}), and the constants a^{μ} determine the direction of the geodesic. In fact

$$(3.14) \qquad a^{\mu} = \frac{dy^{\mu}}{ds} = \left(\frac{dx^{\mu}}{ds}\right)_0$$

where $\left(\frac{dx^{\mu}}{ds}\right)_0$ is the value at $(\bar{x}^{\mu})$ of $\frac{dx^{\mu}}{ds}$ for the geodesic in question. In particular, the observer's world-line has the equations

$$(3.15) \qquad y^{\mu} = k^{\mu}\, s. \qquad (\mu = 0, 1, 2, 3)$$

It should be noticed that, in terms of the new coordinates, the equations (3.13) of the geodesics through $(\bar{x}^{\mu})$ are linear.

By (3.14),

$$\bar{g}_{\mu\nu}\, a^{\mu}\, a^{\nu} = \bar{g}_{\mu\nu}\left(\frac{dx^{\mu}}{ds}\right)_0\left(\frac{dx^{\nu}}{ds}\right)_0$$
$$= 1 \qquad \text{by (3.1).}$$

So, multiplying by s^2 and using (3.13), we deduce that the square of the four-dimensional distance from the observer of a star whose normal world-coordinates are (y^{μ}) is given by

$$(3.16) \qquad s^2 = \bar{g}_{\mu\nu}\, y^{\mu}\, y^{\nu}.$$

* Veblen, *Invariants of Quadratic Differential Forms* (Camb. Math. Tract No. 24, 1927), ch. vi.

† Ruse, *Proc. London Math. Soc.*, 32 (1931), 90.

I, 24. Dr Blair, Links of Leith. Doncaster, 27 June [1776]. *Unsigned.* [1 p. + 3 pp., 10 × 14 cm.; Burton, II, 506–7; Greig, No. 529.]

Inviting B. to dinner on H.'s return home.

To [Thomas Bradshaw].

I, 25. [T. B.]. *Undated, unsigned draft letter.* [1¼ pp. + ¾ p., 16 × 18 cm.; Greig, No. 542.]

About H.'s pension.

To the Rev. Alexander Carlyle.

I, 26, 27. [A. C.]. Edinburgh, 3 Feb. 1761. *Signed.* [1 p. + 1 p., 18·5 × 23 cm., *and copy in C.'s hand*, 2 pp., 20 × 25 cm.; Burton, II, 88–9; Greig, No. 182.]

H. jestingly claims to be the author of [Ferguson's] pamphlet, *Sister Peg.*

I, 28. [A. C.]. Lisle Street, Leicester Fields [London], 15 Sept. 1763. *Signed.* [2 pp. + 2 pp., 19 × 23 cm.; Burton, II, 164–5; Greig, No. 214.]

On the distress of Thomas Blacklock. H. intends to go to Paris; the prospect of happiness there.

To George Carre, afterwards Lord Nisbet.

I, 29. George Carre of Nisbet, Advocate. Ninewells, 12 Nov. 1739. *Signed.* [1½ pp. + 2¾ pp., 18·5 × 23 cm.; Greig, No. 14.]

Desiring post of travelling governor to Lord Haddington and his brother.

To [Dr George Cheyne, London].

I, 30. [G. C.]. [London, Feb. or Mar. 1734.] *Unsigned.* [10½ pp. + 3½ pp., 18·5 × 30 cm.; Burton, I, 30–39; Greig, No. 3.]

Detailed draft letter about H.'s physical and mental ailments ("desertion of the spirit," scurvy spots, *e.g.*) during 1729–34. The progress of his studies during this period. Request for medical advice.

To James and/or Thomas Coutts, Bankers.

I, 31. [J. or T. C.]. Edinburgh, 14 Feb. 1775. *Signed.* [1 p. + 3 pp., 19 × 23 cm.; Greig, No. 502.]

Desiring payment of £250 to nephew, Cornet [Joseph] Home, to buy a lieutenancy.

I, 32. James and Thomas Coutts in Coy. Edinburgh, 5 Apr. 1775. *Signed.* [1 p. + 3 pp., 18·5 × 23 cm.; Greig, No. 504.]

Desiring payment of money necessary for nephew's journey to France.

I, 33. [James or Thomas Coutts]. [Edinburgh, Spring 1775.] *Signed fragment.* [½ p. + ½ p., 19 × 11·5 cm.; Greig, No. 506.]

Concerning the money advanced for the purchase of Joseph Home's lieutenancy.

To [John Crawford of Auchenaimes].

I, 34. [J. C.]. [Edinburgh, Nov. or Dec. 1773.] *Unsigned, draft letter much corrected and altered.* [2½ pp. + 1½ pp., 18·5 × 23 cm.; Burton, II, 149, inc.; Greig, No. 491.]

On H.'s reasons for neglecting to continue his demands for a pension due to him.

To Mrs Dysart of Eccles.

I, 35. [Mrs D.]. Ninewells, 19 Mar. 1751. *Copy only.* [3 pp. + 1 p., 18·5 × 23 cm.; Burton, I, 337–40; Greig, No. 74.]

H. proposes to move with his sister to Berwick. Jokes on his obesity.

I, 36. Mrs Dysart, Eccles. [Edinburgh] 9 Oct. [1754]. *Copy only.* [1 p. + 3 pp., 18·5 × 23 cm.; Burton, I, 410–11; Greig, No. 96.]

Sending Mrs D. a copy of *Hist. of Stuarts*, Vol. I.

To [Patrick, Lord Elibank].

I, 37. [Ld. E.]. [Edinburgh, end of 1759 or beginning of 1760.] *Unsigned, draft letter.* [3 pp. + 1 p., 18·5 × 23 cm.; Burton, II, 252–6; Greig, No. 172.]

Defending his account of Mary, Queen of Scots, and complaining of the attacks of "the Answerer" [William Tytler].

I, 38. Lord [E.]. Fontainebleau, 3 Nov. 1764. *Signed.* [3 pp. + 1 p., 19 × 23 cm.; Burton, II, 257–60; Greig, No. 259.]

Regretting the violent letter E. has written to him. Defending his own previous letter and his conduct towards [Alexander] Murray [E.'s brother], in Paris.

To Alexander Home of Eccles.

I, 39. Alexander Home, Edinburgh. [Ninewells] 11 Dec. [1743]. *Signed.* [1½ pp. + 2½ pp., 19 × 23·5 cm.; Greig, No. 23.]

Dullness of the country. Recent meeting with A. H.'s uncle in Berwick. Weddings of Jenny Kinloch and Betty Dalrymple.

I, 40. Alexander Home, Solicitor-General, Edinburgh. Portsmouth, 23 May 1746. *Signed.* [1 p. + 3 pp., 15·5 × 18·5 cm.; Burton, I, 208–9; Greig, No. 50.]

H.'s intended voyage to America. Warning against party violence towards defeated rebels.

To John Home of Ninewells.

I, 41. [J. H.]. Quiberon Bay in Brittany, 4 Oct. 1746, with P.S. 17 Oct. *Unsigned.* [5½ pp. + 2½ pp., 18·5 × 23 cm.; Burton, I, 213–7; Greig, No. 53.]

An account of Gen. St Clair's military expedition to the Coast of France.

I, 42. [J. H.]. *Unsigned Journal.* [24 pp.: 1–4, 19 × 23 cm.; 5–8, 17 × 21 cm.; 9–12, 20 × 31 cm.; 13–16, 21 × 32·5 cm.; 17–20, 22 × 35 cm.; 21–24, 19 × 24 cm.; Burton, I, 240–66, inc.; Greig, No. 64.]

Journal of H.'s tour through Europe as A.D.C. to Gen. St Clair: The Hague, 3 Mar. 1748; The Hague, 10 Mar.; Breda, 16 Mar.; Nimeguen, 20 Mar.; Cologne, 23 Mar.; Bonne, 24 Mar.; Coblentz, 26 Mar.; Frankfurt, 28 Mar.; Würtzburg, 30 Mar.; Ratisbon, 2 Apr.; the Danube, 7 Apr.; Vienna, 15 Apr.; Vienna, 25 Apr.; Knittelfeldt in Styria, 28 Apr.; Clagenfurt in Carinthia, 4 May; Trent, 8 May; Mantua, 11 May; Cremona, 12 May; Turin, 16 June.—Descriptions of country; the prosperous state of Germany; the battlefield of Dettingen; the Austrian Court, etc.

I, 43. John Home of Ninewells. Paris, 24 June 1765. *Signed.* [2 pp. + 2 pp., 17 × 21·5 cm.; Burton, II, 278–9, inc.; Greig, No. 278.]

H.'s appointment as Secretary to the British Embassy.

I, 44. John Home of Ninewells. Compiègne, 14 July 1765. *Unsigned.* [2½ pp. + 1½ pp., 19 × 23 cm.; Burton, II, 284–6; Greig, No. 282.]

Possibility of Lord Hertford's becoming Lord Lieutenant of Ireland, with H. as Secretary. Risk of fall of Ministry in London meantime.

I, 45. John Home of Ninewells. Compiègne, 4 Aug. 1765. *Unsigned.* [1½ pp. + 2½ pp., 18·5 × 23 cm.; Burton, II, 287, inc.; Greig, No. 285.]

Lord Hertford has been appointed Lord Lieutenant of Ireland, and promises to appoint H. as Secretary. H.'s prospects and plans.

I, 46. [J. H.]. [Paris, Aug. 1765.] *Signed.* [2 pp. + 2 pp., 19 × 23 cm.; Burton, II, 290–2; Greig, No. 289.]

H. gives up idea of becoming Secretary for Ireland. Receives a pension instead.

I, 47. John Home of Ninewells. London, 2 Feb. 1766. *Signed.* [3 pp. + 1 p., 18·5 × 23 cm.; Burton, II, 308–10, inc.; Greig, No. 301.]

H.'s regrets at leaving Paris. His financial affairs.—His opinion of Rousseau, whom he has brought to England with him.

I, 48. John Home of Ninewells. Lisle Street, Leicester Fields [London], 22 Mar. 1766. *Signed.* [1¾ pp. + 2¼ pp., 19 × 23 cm.; Hill, 79, inc.; Greig, No. 312.]

Financial matters. Rousseau and London Society.

I, 49. [J. H.]. London, 28 July 1767. *Signed.* [3 pp. + 1 p., 18·5 × 24 cm.; Burton, II, 396–7 and 403–4; Greig, No. 397.]

Ministerial changes, with special reference to Gen. Conway. Suggestions for the education of nephew "Josey" at Eton.

I, 50. [J. H.]. London, 6 Oct. 1767. *Signed.* [2 pp. + 2 pp., 18·5 × 23 cm.; Burton, II, 397–8, inc.; Greig, No. 405.]

H.'s preparations to write more history. Domestic matters.

I, 51. [J. H.]. London, 13 Oct. 1767. *Signed.* [2 pp. + 2 pp., 18·5 × 23 cm.; Burton, II, 404–5, inc.; Greig, No. 408.]

On the educational accomplishments and progress of nephew "Josey."—H.'s investments.

To David Home the Younger, afterwards Baron Hume of the Scottish Exchequer.

I, 52. David Home at Ninewells. St Andrews Square [Edinburgh], 30 Aug. 1775. *Signed.* [1 p. + 3 pp., 18·5 × 22 cm.; Burton, II, 474–5; Greig, No. 507.]

On his nephew's literary style. Domestic matters.

I, 53. David Home. Edinburgh, 8 Dec. 1775. *Signed.* [2¼ pp. + 1¾ pp., 18·5 × 23 cm., *slightly torn*; Burton, II, 480–2; Greig, No. 512.]

The necessity of exercise to preserve one's health. The advantages of a republican government. Harrington's *Oceana* impracticable.

I, 54. David Home at Ninewells. Edinburgh, 15 Aug. 1776. *Signed.* [1 p. + 3 pp., 18·5 × 23 cm.; Burton, II, 509; Greig, No. 537.]

His nephew's presence not necessary. He will be warned in case H. relapses.

To Professor Francis Hutcheson.

I, 55. Francis Hutcheson, Glasgow. Ninewells, 17 Sept. 1739. *Signed.* [3 pp. + 1 p., 19 × 30 cm.; Burton, I, 112–5; Greig, No. 13.]

Discussion of Hutcheson's remarks on H.'s papers [MS. of *Treatise*, Bk. III]. H.'s alleged lukewarmness in cause of virtue. The meaning of *natural* and of *virtue*. Defence of free-thought.—*P.S.* Actions to be considered as the reflection of qualities. There must be other motives to action besides the desire for virtue.

I, 56. Francis Hutcheson, Glasgow. Ninewells, 4 Mar. 1740. *Signed.* [2 pp. + 2 pp., 18·5 × 23 cm.; Burton, I, 116, inc.; Greig, No. 15.]

Requesting introduction to an honest bookseller for the publication of *Treatise*, Bk. III.

I, 57. Francis Hutcheson, Glasgow. Edinburgh, 16 Mar. 1740. *Signed.* [3 pp. + 1 p., 18·5 × 23 cm.; Burton, I, 117–20; Greig, No. 16.]

Requesting promised introduction to Thomas Longman. H.'s hasty bargain made for *Treatise*, Bks. I and II. Further discussion on ethical points.

I, 58. Francis Hutcheson, Glasgow. Edinburgh, 10 Jan. 1743. *Unsigned.* [2 pp. + 2 pp., 18·5 × 23 cm.; Burton, I, 146–50; Greig, No. 19.]

Thanking Hutcheson for his present [the *Philosophiae Moralis Institutio Compendiaria*]. Detailed comments thereon.

To [Matthew Maty, Chief Librarian, British Museum].

I, 59. [M. M.]. Edinburgh, 23 Jan. 1767. *Signed.* [1 p. + 3 pp., 18·5 × 23 cm.; Burton, II, 359–60; Greig, No. 368.]

Requesting that originals of Rousseau's letters to H. be deposited in and retained by the Museum.

To Andrew Millar, Bookseller, London.

I, 60. [A. M.]. Edinburgh, 11 Apr. 1755. *Signed.* [3 pp. + 1 p., 18·5 × 23 cm.; Burton, I, 415–7; Greig, No. 108.]

Requesting M. to take over the publication of the *Hist.*, which has not gone well in the hands of Bailie [Gavin] Hamilton.—The second vol. will give less offence than the first.—Dr Birch's story about the order for Lord Loudon's execution probable.—H. has made an attempt at [translating] Plutarch.

I, 61. [A. M.]. Edinburgh, 12 June 1755. *Signed.* [1½ pp. + 2½ pp., 18·5 × 22·5 cm.; Burton, I, 421–2; Greig, No. 111.]

Approves M.'s project for a weekly paper, but cannot enter into an undertaking about this till the *Hist.* completed.—Offers four dissertations for 50

guineas.—Hopes M.'s prophecy, that the *Hist.* will not sell till publication of the second vol., will not prove true.

I, 62. Andrew Millar, Strand, London. Edinburgh, 27 May 1756. *Signed.* [1 p. + 3 pp., 19 × 14·5 cm.; Greig, No. 117.]

H.'s satisfaction at M.'s letter. A note on the *Hist.*, Vol. I. The number of copies bound by Bailie Hamilton is embarrassing.

I, 63. Andrew Millar, Bookseller. Edinburgh, 22 Sept. 1756. *Signed.* [½ p. + 2½ pp., *torn*, 18·5 × 13·5 cm.; Burton, II, 2–3; Greig, No. 118.]

Strahan will soon have completed the printing of the *Hist.*, Vol. II. H. asks for 15 copies.—Begs M. to ask Mallet to check volume for Scotticisms.

I, 64. [A. M.]. Edinburgh, 4 Dec. 1756. *Signed.* [1½ pp. + 2½ pp., 18·5 × 22·5 cm.; Burton, II, 3–5; Greig, No. 121.]

Pleased at the sale of the *Hist.* Lord Lyttelton's objection is ill-grounded. Would like a list of the Scotticisms from Mallet.—Kincaid and Donaldson have offered to take 200 sets of H.'s "philosophical pieces," to be paid for by a proportionate number of shares in the new quarto edit.

I, 65. Andrew Millar, Strand, London. Edinburgh, 11 Jan. 1757. *Signed, initials only.* [1 p. + 1 p., 18 × 21 cm.; Greig, No. 124.]

Returning Dedication [for *Four Dissertations*].—John Home is revising last act of *Douglas.* Then he will set out for London.

I, 66. Andrew Millar, Strand, London. Edinburgh, 18 Jan. 1757. *Signed.* [1 p. + 1 p., 20 × 23·5 cm.; Greig, No. 125.]

Sending a corrected copy of his own works to be used by printer when making a new edit. Prepared to make an index to 4to edit.—Has received a book from Count Algarotti, and asks M. to bind and send a copy of the *Dissertations* to A. in return.

I, 67. [A. M.]. Edinburgh, 20 Jan. 1757. *Signed.* [1 p. + 1 p., 18 × 23 cm.; Burton, II, 18; Greig, No. 126.]

It has been suggested that the Dedication of the *Dissertations* to John Home might injure the latter. H. therefore begs M. to suspend the printing till hearing again from him.

I, 68. [A. M.]. Edinburgh, 20 May 1757. *Signed.* [3 pp. + 1 p., 19 × 23·5 cm.; Burton, II, 22-5, inc.; Greig, No. 132.]

Has received duplicates of M.'s note for £600.—Sending back 8 L.P. copies

to M. Wishes account to be drawn up. Wishes to have a copy of "Answer" to the *Dissertations.*—Has begun third vol. of the *Hist.* with Henry VII.—Has heard of an attack on himself by a Dr Brown, and asks for a copy of it. Regrets that M. should have let a private letter be seen by Brown. H. will not answer Brown.—*P.S.* Begs M. to read this letter to Brown.

I, 69. [A. M.]. Edinburgh, 21 July 1757. *Signed.* [2½ pp. + 1½ pp., 19 × 24 cm.; *Englische Studien*, Bd. 63, Heft 3, 377–9; Greig, No. 136.]

Has received 4 letters from M., which he will deliver. Will not take the least notice of Dr Brown, but begs M. in future to burn all his (H.'s) letters.—Asks M. to send 3 copies of *Hist.* and *Dissertations* to France as presents; also a copy of *Douglas* —Desires a copy of *L'ami des hommes.*—Vol. 1 of French translation of the *Hist.* will appear next winter.

I, 70. Andrew Millar, Bookseller. Edinburgh, 3 Sept. 1757. *Signed.* [3 pp. + 1 p., 18 × 23 cm.; Burton, II, 34 8, inc.; Greig, No. 140.]

Sympathising with M. in recent loss.—H. will not positively defend his opinions, which are merely doubts.—Confident that Warburton wrote the letter to himself; but H. will not reply.—Has completed index.—New vol. of *Hist.* well advanced. Offers it for £700.—Well pleased with success of *Epigoniad*, and believes it would succeed better if M. would engage in it.—Invites M. to show the paragraph of his last letter to Warburton.

I, 71. [A. M.]. Edinburgh, 6 Apr. 1758. *Signed.* [2 pp. + 2 pp., 18 × 23 cm.; Burton, II, 42–3, inc.; Greig, No. 145.]

H. is glad that Robertson is coming to terms with M. The great merits of R.'s *Hist.* [*of Scotland*].—Little likelihood of this book interfering with H.'s new vol., which cannot be finished in time for Christmas. Hopes to be introduced to Dr Birch when he comes to London.—Regrets that the "extraordinary run" on Smollett's *Hist. of England* should be spoiling sales of H.'s *Hist.*, but this is only temporary.—*P.S.* Recommending Strahan as printer for Robertson's *Hist.*

I, 72. [A. M.]. Edinburgh, 20 June 1758. *Signed.* [2 pp. + 2 pp., 18·5 × 23 cm.; Burton, II, 43, inc.; Greig, No. 150.]

Surprised that M. has so many copies of Vol. I left.—Robertson would like more than one sheet per post.—Spelling of *honour* as *honor* has good authority, but H. does not mind which spelling is used.—Regrets M.'s illness.

I, 73. [A. M.]. Edinburgh, 18 Dec. 1759. *Signed.* [2 pp. + 2 pp., 19 × 23 cm.; Burton, II, 65, inc.; Greig, No. 171.]

Occupied in adding authorities to the *Stuarts*, and hopes to put these vols.

beyond controversy.—Intends to proceed with "antient English History."—A virulent attack on his account of Queen Mary has appeared; from which he hopes to benefit.—Considers an index unnecessary.—Compliments to friends, and congratulations on British victories and the prospect of a good peace.

I, 74. [A. M.]. Edinburgh, 22 Mar. 1760. *Signed.* [3½ pp. + ½ p., 18·5 × 22·5 cm.; Burton, II, 81–3, inc.; Greig, No. 173.]

Success of *Siege of Aquileia.*—List of books required [enclosed].—Expects early appearance of Lyttelton's and Mallet's *Histories.*—Barrenness of press during the winter.

I, 75. Andrew Millar, Strand, London. [Edinburgh] 27 Mar. [1760]. *In third person.* [¼ p. + 1¾ pp., 18·5 × 22 cm.; Greig, No. 174.]

Asking for another book.

I, 76. [A. M.]. Edinburgh, 27 Oct. 1760. *Signed.* [1 p. + 1 p., 19 × 23·5 cm.; Burton, II, 87–8, inc.; Greig, No. 177.]

Difficult to introduce someone whom he does not know at all to men whom he knows only slightly.—Hopes to publish the winter after next. Glad to hear that Clarendon papers have fallen into Dr Douglas's hands.—Pleased to hear of Mrs Millar's good health.

I, 77. [A. M.]. Edinburgh, 15 Mar. 1762. *Signed.* [2½ pp. + 1½ pp., 18·5 × 23 cm.; Burton, II, 130–2; Greig, No. 189.]

A new edit. probable. H. sends up corrected copies of the *Hist.* for printing it, when required.—Denies the rumour that he is going to write an ecclesiastical history.—Lord Kames's *Elements* [*of Criticism*], though good, not likely to prove popular.—Edit. of *Essays* exhausted. H. desires to make considerable corrections for next edit.

I, 78. [A. M.]. Edinburgh, 8 Apr. 1762. *Signed.* [4 pp., 19 × 23 cm.; Burton, II, 132–4, inc.; Greig, No. 191.]

[Charles] Townshend's accusation of inaccuracy in H. disproved by Dyson.—H.'s independence of great men.—Above 600 of both edits. of *Essays* sold.—Instructions to Strahan about printing new edit. of the *Hist.* H. would like to have the benefit of Lyttelton's *Hist.* in correcting his own.

I, 79. [A. M.]. Edinburgh, 17 May 1762. *Signed.* [2 pp. + 2 pp., 18 × 23 cm.; Burton, II, 135–6, inc.; Greig, No. 193.]

Prefers to print in vols. rather than in numbers. Thinks frontispiece and own portrait might be left out.—Still hopes to continue his *Hist.*, but fears party prejudice.—Information about Robertson's *Hist.* [*of Charles V*].

I, 80. Andrew Millar, Bookseller. Edinburgh, 22 Nov. 1762. *Signed.* [2 pp. + 2 pp., 19 × 23·5 cm.; Burton, II, 139–40; Greig, No. 199.]

Two new edits. well advanced. Still proposes to continue his *Hist.* Mallet's *Hist.* said to be ready for the press.—Lord Marischal's proposal for an English edit. of Rousseau's works. H. suggests that M. undertake an English translation to appear at the same time.—*P.S.* Requests 3 copies of Royal Paper edit. Thanks M. for making out his subscription. Does not think stocks yet at highest.

I, 81. [A. M.]. Edinburgh, 10 Mar. 1763. *Signed.* [2½ pp. + 1½ pp., 19 × 23 cm.; Burton, II, 147, inc.; Greig, No. 202.]

Advises immediate publication of 8vo. edit. of *Hist.*—Proposals from Kincaid and Donaldson to take 100 copies of *Essays* at reduced price.—Thanks to Strahan for careful printing. Hopes Mrs M. will recover from her illness.

I, 82. [A. M.]. Edinburgh, 28 Mar. 1763. *Signed.* [2 pp. + 2 pp., 19 × 23 cm.; Burton, II, 147–8, inc.; Greig, No. 205.]

M. should follow his own judgment about 8vo. edit.—H. still proposes to continue his *Hist.*, but will wait till prejudice against the Scots disappears.—Desires information about private papers of late [James] Ralph.—Kincaid and Donaldson accept M.'s terms.—Wilkes very much to blame in expressing so many national reflections.

I, 83. Andrew Millar, Bookseller. Edinburgh, 21 Apr. 1763. *Signed.* [2 pp. + 2 pp., 19 × 23 cm.; Burton, II, 143–4; Greig, No. 208.]

Requesting M. to send the vols. of the *Hist.* annotated by Mallet. Mallet fears that H., if he continues his *Hist.*, will interfere with Mallet's own sales.

I, 84. Andrew Millar, Bookseller. [London] 8 Oct. 1763. *Signed, initials only.* [1 p. + 3 pp., 15·5 × 19 cm.; Greig, No. 218.]

Enclosing letter to Algarotti. He has returned Burns's *Ecclesiastical Law* to M.—H. has received a present of [Robert] Strange's prints, and asks M. to send S. a copy of the *Hist.* Further copies to be sent to Paris.

I, 85. [A. M.]. Paris, 1 Dec. 1763. *Signed.* [1 p. + 1 p., 17 × 21·5 cm.; Burton, II, 179–80; Greig, No. 226.]

H. has discovered King James II's Memoirs in MS., and will make use of them to revise his *Hist.*—Requests additional copies of the *Hist.*

I, 86. Andrew Millar, Strand, London. Paris, 18 Mar. 1764. *Signed.* [2 pp. + 2 pp., 16·5 × 21·5 cm.; Burton, II, 200, inc.; Greig, No. 230.]

Request for some books. Desires to know how sales are proceeding. His idle and dissipated life in Paris. Little likelihood of a new edit. of the *Hist.*

I, 87. [A. M.]. Paris, 18 Apr. 1764. *Signed.* [2 pp. + 2 pp., 15·5 × 20 cm.; Burton, II, 200–1, inc.; Greig, No. 236.]

Memoirs of James II tell against the King himself and his brother. H., however, cannot use the information, being too much M.'s friend to increase the burden he has already laid on him by the *Hist.*—Vexed at M.'s having sent him an expensive edit. of Coke.—Mrs Mallet's whim of living alone in a hermitage.

I, 88. [A. M.]. Paris, 23 May 1764. *Signed.* [3 pp. + 1 p., 15·5 × 20 cm.; Burton, II, 201–2, inc.; Greig, No. 240.]

Surprised to learn of new 4to edit. of *Tudors.* He should have been warned, in order that he might provide a list of corrections. Sends the alterations for the *Stuarts.*—The *Commonwealth* and *Stuarts* have been perused by Father Gordon [of the Scots College], who has found little to alter.—The piety of Wilkes. W. claims to have a copy of H.'s two suppressed essays in his library in England, and prudently suggests that they be destroyed.

I, 89. [A. M.]. Paris, 3 Sept. 1764. *Signed.* [4 pp., 16 × 20 cm.; Burton, II, 231–3, inc.; Greig, No. 251.]

M.'s new life of ease. H. still desires to continue his *Hist.* More justice is being done to him. But he cannot at present leave Lord Hertford.—Satisfaction at the success of new edit. of *Essays.* Sending Coke back again.—Rouet expected in Paris. The Douglas Cause. Mrs Mallet offended at H.'s neglect of her.—The lowness of stocks.

I, 90. Andrew Millar, Strand, London. Paris, 14 Nov. 1764. *Signed.* [$1\frac{1}{4}$ pp. + $2\frac{3}{4}$ pp., 18·5 × 22·5 cm.; Greig, No. 260.]

Le Roy's Plans of Athens has been delivered in mistake to David Wilson instead of to M. H. has met Rouet.

I, 91. [A. M.]. Paris, 14 Jan. 1765. *Signed, initials only.* [2 pp. + 2 pp., 15·5 × 20·5 cm.; Burton, II, 264–5, inc.; Greig, No. 267.]

H. intends to continue his *Hist.* Value for this purpose of a visit to France. The unreasonable English prejudice against Scotsmen.—Desires the new edit. of *Essays* to be printed from the last in 8vo.—Encloses bill in settlement of his account.

I, 92. Andrew Millar, Strand, London. Paris, 4 May 1765. *Signed.* [1 p. + 3 pp., 16·5 × 22 cm.; Burton, II, 273, inc.; Greig, No. 274.]

Death of Mallet. H. begs M. to discover whether Mallet's *Hist.* was ever begun. Request for FitzOsborne's Letters.

I, 93. Andrew Millar, Bookseller. Edinburgh, 8 Oct. [Nov.] 1766. *Signed.* [2 pp. + 2 pp., 18·5 × 23·5 cm.; Hill, 100, inc.; Greig, No. 360.]

H.'s reasons for not having had his *Concise and Genuine Account* published by M.—New edit. of the *Hist.*, when required, will be as correct as possible. Quick sale of philosophical works and slow sale of the *Hist.* H. will sketch out the next vol.—*P.S.* Robertson expects to be ready [with *Charles V*] the winter after next.

I, 94. Andrew Millar, Bookseller, Strand. Edinburgh, 21 Oct. 1766. *Signed.* [1½ pp. + 2½ pp., 18 × 23 cm.; *Englische Studien*, Bd. 63, Heft 3, 380–1; Greig, No. 354.]

Macleane's questions will have been answered by the French edit. of the Rousseau pamphlet.—Allan Ramsay cannot find a suitable engraver. H. would much prefer to have an engraving of himself prefixed to the new edit. He is waiting to watch public opinion before starting on the new vol. Inquires about a mislaid, corrected copy of the *Hist.*—[*Note added by M. to Cadell*]: Does he know anything of this ?

I, 95. [A. M.]. London, 17 July 1767. *Signed.* [1¼ pp. + 2¾ pp., 18·5 × 22·5 cm.; Burton, II, 393–4 inc.; Greig, No. 394.]

A change of ministry imminent. His Majesty has promised to allow H. to examine the records in the Paper Office. H. will not continue his *Hist.*

I, 96. Andrew Millar, Kew Green. [London] 19 Oct. 1767. *Signed.* [1½ pp. + 2½ pp., 16 × 19 cm.; Burton, II, 408–9, inc.; Greig, No. 411.]

Concerning the engraving of H.'s head. An excellent drawing by Donaldson. H. is devoting his time to revising the *Hist.*

To William Mure of Caldwell.

II, 1. William Mure of C., at Caldwell. [Ninewells] 14 Nov. [? 1742]. *Unsigned.* [2 pp. + 2 pp., 18·5 × 22·5 cm.; Burton, I, 158–60; *Cald. Papers*, II, i, 38; Greig, No. 18.]

Defending his previous letter. Advice to M. for his parliamentary career. Compliments to friends.

II, 2. William Mure, M.P., London. [Ninewells] 26 Jan. [1743]. *Signed.* [2 pp. + 2 pp., 19 × 23 cm.; Burton, I, 155–6; *Cald. Papers*, II, i, 46–7; Greig, No. 20.]

Encouraging M. to cultivate the friendship of [James] Oswald [of Dunnikier]. The value of a friendship with honourable men. Advice to show steadiness and frugality.

II, 3. [W. M.]. [Ninewells] 30 June [1743]. *Unsigned.* [3½ pp. + ½ p., 19 × 30 cm.; Burton, I, 162–4, inc.; *Cald. Papers*, II, i, 50–2; Greig, No. 21.]

On [William] Leechman's Sermon on Prayer; with a number of suggested verbal alterations. Impossibility of arousing affections towards the Deity. Useless and blasphemous nature of prayer.—*P.S.* Asks for copy of *Dialogues on Devotion.* Ironically suggests they may be a means to his conversion.

II, 4. William Mure of C., at Caldwell. [Ninewells] 10 Sept. [? 1743]. *Unsigned.* [2 pp. + 2 pp., 18 × 22·5 cm.; Burton, I, 153–5; *Cald. Papers*, II, i, 30–2; Greig, No. 22.]

No subject to write on. Mock-heroic address to his pen. H.'s transformation from a gallant into a philosopher. Compliments to various friends.

II, 5. William [M.]. Edinburgh, 4 Aug. 1744. *Unsigned.* [4 pp., 18·5 × 23 cm.; Burton, I, 165–8; *Cald. Papers*, II, i, 63–5; Greig, No. 24.]

H.'s candidature for Edinburgh Chair. Success depends on Pringle's reply to Town Council's decision. H. has also a chance of being appointed tutor to son of Murray of Broughton. Surprised at the alleged opposition of Prof. Hutcheson to H.'s candidature in Edinburgh.—Oswald's business ability.

II, 6. William Mure of C., Glanderston, near Glasgow. [Edinburgh, Oct. 1754.] *Unsigned.* [1 p. + 3 pp., 18·5 × 17 cm., *torn*; Burton, I, 409–10; *Cald. Papers*, II, i, 111–2; Greig, No. 102.]

Sending copy of the *Hist.*, Vol. I. The aims of a historian. Sending also one to Abbé Le Blanc for a French edit., and will also make a Dublin edit.

II, 7. William Mure. [Edinburgh, Feb. 1757.] *Signed.* [3 pp. + 1 p., 18 × 23 cm.; Burton, II, 19–22; *Cald. Papers*, II, i, 112–4; Greig, No. 128.]

Feels Vol. I of the *Hist.* to be better than Vol. II. Annoyance at numbers of *Four Dissertations* having been issued without the Dedication to John Home. Thanks M. for critical remarks. Finds practice of "doubling the genitive" so common that he has adopted it.

II, 8. Baron Mure. Lisle Street [Leicester Fields, London], 1 Sept. 1763. *Copy only.* [2½ pp. + 1½ pp., 20 × 25·5 cm.; Burton, II, 158–60, inc.; *Cald. Papers*, II, i, 190–1; Greig, No. 212.]

Prospects of his journey to Paris and of his being appointed Secy. to Embassy. Opinions of his London friends on this change in his situation.—Account of recent politics.—Thomas Blacklock left as a legacy to M.

II, 9. Baron Mure. Paris, 22 June 1764. *Copy only.* [3 pp. + 1 p., 20 × 25·5 cm.; Burton, II, 203–5; *Cald. Papers*, II, i, 253–5; Greig, No. 242.]

Meuron is leaving for Scotland.—H. dined lately with Duchess of Perth.

Opinions on the Douglas Cause.—Pleased with his way of life. Secretaryship of Embassy expected daily.—Thanks Mrs M. for her letter.

II, 10. Baron Mure. London, 1 July 1767. *Copy only.* [2 pp., 20 × 25·5 cm.; Burton, II, 390–1; *Cald. Papers*, II, ii, 115–6; Greig, No. 392.]

H. is looking out for an usher for Graffigni.—Discovery of Graffigni's impostures.

II, 11. Baron Mure. London, 18 July 1767. *Copy only.* [2 pp., 20 × 25·5 cm.; *Cald. Papers*, II, ii, 118–9; Greig, No. 396.]

Account of Mure boys at Graffigni's school.—Confusion of political situation.—Satisfaction at Hamilton's victory in Douglas Cause [in Court of Session].

II, 12. Baron Mure. [London, summer, 1767.] *Copy only.* [2½ pp. + 1½ pp., 20 × 20·5 cm.; Burton, II, 391, inc.; *Cald. Papers*, II, ii, 120–1; Greig, No. 398.]

Report on the Mure boys at Norlands.—Graffigni a conceited and foolish fellow.

II, 13. Baron Mure. [London, summer, 1767.] *Copy only.* [2 pp., 20 × 25·5 cm.; Burton, II, 391–2, inc.; *Cald. Papers*, II, ii, 120–1; Greig, No. 399.]

Account of Mure boys at Norlands. The method of teaching languages there. H. is deliberating about his own nephews.

II, 14. Baron Mure. Edinburgh, 2 Oct. 1770. *Copy only.* [1¼ pp. + ¾ p., 20 × 25·5 cm.; Burton, II, 436; *Cald. Papers*, II, ii, 177–8; Greig, No. 451.]

H. is engaged in building a house.—Miss Kitty [M.]'s coat much admired.

II, 15. Baron Mure, Caldwal, near Glasgow. St David's Street [Edinburgh], 27 Oct. 1775. *Signed, initials only.* [1½ pp. + 2½ pp., 19 × 23 cm.; Burton, II, 478–9, inc.; *Cald. Papers*, II, ii, 259–60; Greig, No. 510.]

H. will not write a petition from the freeholders of Renfrewshire against America. He is an American in his sympathies. Begs M. to support Dr Wright's candidature for Divinity Chair at Glasgow.

To James Oswald of Dunnikier.

[All these letters are copies only.]

II, 16. J. O. Ninewells, 2 Oct. 1747. [Oswald, 54–8; Burton, I, 224, inc.; Greig, No. 58.]

Congratulating O. on his election. The present state of affairs. The misfortune of Bergen-op-Zoom. H.'s determination to print the *Philosophical Essays*. Little hope of success in soliciting his half-pay.

J. O. London, 29 Jan. 1748. [Oswald, 59–64; Burton, I, 236–8; Greig, No. 61.]

Has accepted Gen. St Clair's invitation to accompany him to Turin. Hopes the journey will be of value for his projected *Hist.*, but leaves home with regret.—Trouble in the stocks. A controverted election. Story of Lord Marchmont's falling suddenly in love in the playhouse.

J. O. Ninewells, 1 Nov. 1750. [Oswald, 65–71; Burton, I, 301–4; Greig, No. 67.]

Economic discussion: results of increase of money in a country; the comparison between a rich and a poor country; results of the distance of China on its exports; transferable bonds; influence of West Indies on Spain; paper money in the Colonies.—The aims of H.'s essay.

J. O. Jack's Land [Edinburgh], 28 June 1753. [Oswald, 72–5; Burton, I, 380–1; Greig, No. 86.]

Thanks O. for providing for H.'s cousin. Obscurity about old methods of raising subsidies. Need for a satisfactory Hist. of England. Impartiality of H.'s own.

J. O. Edinburgh, 3 Apr. 1763. [Oswald, 76–80; Burton, II, 149, inc.; Greig, No. 206.]

Difficulties in obtaining his half-pay.

J. O. Paris, 3 June 1765. [Oswald, 81–3; Burton, II, 275–7; Greig, No. 276.]

H. is entering on a correspondence with an Abbé [Morellet] on political and commercial matters. The likelihood of his being appointed Secretary for Ireland.

J. O. Ninewells, 1 Oct. 1766. [Greig, No. 352.]

On Mr Charteris's visit to Paris.

To Michael Ramsay.

II, 17. M[ichael R.]. [Ninewells] 4 July 1727. *Unsigned, mutilated.* [4 pp., 16 × 19 cm.; Burton, I, 12–6; Greig, No. 1.]

Acknowledging receipt of books. H.'s papers not yet sufficiently polished to be sent to R. His desultory reading and Saturnian happiness. Domestic affairs. The sublimity of Longinus.

II, 18. Michael Ramsay, Edinburgh. [Ninewells] Monday, March 1732. *Unsigned.* [1 p. + 3 pp., 15 × 19 cm.; *Englische Studien*, Bd. 63, Heft 3, 337–8; Greig, No. 2.]

Desiring to see R. again, and lamenting the shortness of their meetings.

II, 19. Michael [R.]. Rheims, 12 Sept. 1734. *Unsigned.* [4 pp., 17 × 21 cm.; Burton, I, 51–6; Greig, No. 4.]

Description of Rheims. Reflections on French politeness, compared with English. The advantages and disadvantages of set customs and phrases.

II, 20. Michael Ramsay, Marseilles. [London] 22 Feb. [1739]. *Signed, initials only.* [3 pp. + 1 p., 17 × 21 cm.; Burton, I, 107, inc.; Greig, No. 9.]

Interest in R.'s affairs. Expects little success immediately from his own "grand undertaking" [*Treatise*, I and II].—*P.S.* Attempting to gain a living for R. without bringing him into the feud between Lords Home and Marchmont.

II, 21. Michael Ramsay, Edinburgh. [Ninewells] 30 July [?]. *Unsigned.* [1½ pp. + 2½ pp., 16 × 19 cm.; *Englische Studien*, Bd. 63, Heft 3, 338–9; Greig, No. 541.]

Inviting R. to spend the vacation with him.

II, 22. M. R. Ninewells, 22 June 1751. *Copy only, enclosed in letter from Michael Ramsay the Younger to David Hume the Younger.* [Burton, I, 342 and 427, inc.; Greig, No. 75.]

H.'s present circumstances and worldly goods. Reasons for his determination to settle in Edinburgh. His sister Catherine will join him. [Omission in copy, probably relating to mother of David Hume the Younger.] Henry Home of Kames and his *Essays*.

To William Rouet.

II, 23. [W. R.]. [London] 15 May [1759]. *Signed.* [1 p. + 3 pp., 11·5 × 18·5 cm.; Burton, II, 62; Greig, No. 166.]

Asking for rapee and Scotch pebbles. [R.]'s expulsion from his university. Expected French invasion.

II, 24. William Ruat. [London] 6 July [1759]. *Signed.* [3 pp. + 1 p., 18·5 × 23 cm.; Burton, II, 62–5, inc.; Greig, No. 168.]

Bogus account of French invasion, Bishop Warburton's having turned Mahometan, etc.—*Last page by* "A. Armstrong Short" [? Dr John Armstrong].

To Adam Smith.

II, 25. [A. S.]. Riddal's Land, Lawnmarket [Edinburgh], 24 Sept. 1752. *Signed.* [1 p. + 3 pp., 20 × 32 cm.; Burton, I, 375–6; Greig, No. 78.]

H.'s *Hist.* The best period to begin at: 1603. Preparations for a new edit. of *Essays*.

II, 26. Adam Smith, Glasgow. Edinburgh, 17 Dec. 1754. *Signed.* [1½ pp. + 2½ pp., 18·5 × 23 cm.; Burton, I, 393–4 and 411–2; Greig, No. 104.]

Retains his post in Advocates' Lib., but hands the salary to Blacklock. The sale of the *Hist.*

II, 27. Adam Smith, Glasgow. Edinburgh, 9 Jan. 1755. *Signed.* [1 p. + 3 pp., 19 × 23 cm.; Burton, I, 417–8; Greig, No. 107.]

Apologising for not sending a paper to the [Glasgow Philosophical] Society. Remarks on various persecutions in history.

II, 28. Adam Smith. [Edinburgh, Feb. or Mar. 1757.] *Unsigned, mutilated.* [3 pp. + 1 p., 12 × 20 cm.; Burton, II, 16–8, inc.; Greig, No. 130.]

The *Four Dissertations.* The Dedication to John Home. The success of latter's play [*Douglas*].—H. hesitates whether to go backwards or forwards in his *Hist.*

II, 29. Adam Smith. [Edinburgh] 8 June 1758. *Signed.* [1½ pp. + 2½ pp., 18·5 × 23 cm.; Burton, II, 45–7; Greig, No. 148.]

Possibility of transferring S. from Glasgow to Edinburgh, and his prospects there.

II, 30. Adam Smith. London, 28 July 1759. *Signed.* [4 pp., 19 × 23 cm.; Burton, II, 59–61; Greig, No. 169.]

Has seen Mr Wilson. A new series of the classics proposed.—Praise of S.'s book [*Theory of Moral Sentiments*], with a criticism of one passage. The attitude of the Whigs to H.'s *Hist.* He intends to write the Hist. of England before Henry VII.

II, 31. Adam Smith, Glasgow. Ninewells, 29 June 1761. *Signed.* [1 p. + 3 pp., 18·5 × 23 cm.; Burton, II, 89–90; Greig, No. 185.]

Proposing Cumming for Chair of Hebrew in Glasgow.

II, 32. Adam Smith, Glasgow. Edinburgh, 28 Mar. 1763. *Signed.* [½ p. + 3½ pp., 19 × 23 cm.; Burton, II, 148–9, inc.; Greig, No. 204.]

Promising a visit.

II, 33. Adam Smith. Edinburgh, 21 July 1763. *Signed.* [½ p. + 3½ pp., 18·5 × 22·5 cm.; Burton, II, 150; Greig, No. 210.]

On the Douglas Cause. A paper thereon has been written by [William] Johnstone.

II, 34. Adam Smith, Glasgow. Edinburgh, 9 Aug. 1763. *Signed.* [1 p. + 3 pp., 18·5 × 23 cm.; Burton, II, 157–8; Greig, No. 211.]

H. has decided to accept Lord Hertford's invitation to accompany him to Paris.

II, 35. Adam Smith. Lisle Street, Leicester Fields [London], 13 Sept. 1763. *Signed.* [4 pp., 18·5 × 23 cm.; Burton, II, 160–3; Greig, No. 213.]

H. left Scotland with a certain regret. He is to aid Lord Beauchamp in his studies.—The political situation. Hostility to Lord Bute. Pitt's interview with the King.

II, 36. Adam Smith. Fontainebleau, 28 Oct. 1763. *Signed. Torn and partly illegible.* [4 pp. + 2 pp., 17 × 22 cm.; Burton, II, 168–72; Greig, No. 222.]

H.'s welcome in Paris, especially by the ladies. His distaste for such flatteries. S.'s *Theory of Moral Sentiments* is being translated into French.

II, 37. Adam Smith. Paris, 5 Sept. [wrongly dated Nov.] 1765. *Signed.* [4 pp., 16 × 20 cm.; Burton, II, 292–3, inc.; Greig, No. 290.]

Lord Hertford has been appointed Lord Lieutenant of Ireland, but has stated that he will accept only with H. as Secretary. H. receives a pension instead. Refuses post of Usher to Black Rod in Ireland. Hesitates where to settle, but prefers Paris.

II, 38. Adam Smith, Paris. [London, Jan. 1766.] *Signed.* [1 p. + 3 pp., 16 × 20 cm.; Burton, II, 392, inc.; Greig, No. 300.]

Recommending his French servant to S.—The encouragement he receives to continue his *Hist.*, but his disinclination through laziness.

II, 39. Adam Smith, Paris, readdressed to Compiègne. [London, Aug. 1766.] *Signed.* [2½ pp. + 1½ pp., 18·5 × 23 cm.; Burton, II, 348–9, inc.; Greig, No. 348.]

Complaining of a Paris bookseller's failure to send over books.—The quarrel with Rousseau. H.'s disinclination to publish the correspondence unless forced to do so by R.'s actions.—*P.S.* His hesitation whether to live in London or Paris.

II, 40. Adam Smith. London, 13 June 1767. *Signed, initials only.* [3 pp. + 1 p., 18·5 × 23 cm.; Burton, II, 388–90; Greig, No. 388.]

On the Comte de Sarsfield. Story of a heated outburst by the Bp. of Raphoe after a joke by H.

II, 41. Adam Smith. London, 14 July 1767. *Signed.* [½ p. + 3½ pp., 18 × 24 cm.; Burton, II, 395–6; Greig, No. 393.]

Note enclosed in a packet of the Comte de Sarsfield's papers.

II, 42. Adam Smith. [London, Oct. 1767.] *Signed, initials only.* [½ p. + 3½ pp., 18·5 × 23·5 cm.; Greig, No. 406.]

Thanking S. for his resentment against the Bp. of Raphoe.

II, 43. Adam Smith. London, 8 Oct. 1767. *Signed.* [4 pp., 19 × 30 cm.; Burton, II, 374–8, inc.; Greig, No. 407.]

Deeds of Rousseau: left Davenport's house for Spalding; then to Dover and crossed to France. His character. His subsequent neglect by French. Measures to regain his popularity.

II, 44. Adam Smith. London, 17 Oct. 1767. *Unsigned.* [2 pp. + 2 pp., 18·5 × 23 cm.; Burton, II, 378–80, inc.; Greig, No. 409.]

Rousseau is willing to return to Davenport. His Memoirs.

II, 45. Adam Smith. James's Court [Edinburgh], 20 Aug. 1769. *Signed.* [1 p. + 1 p., 18·5 × 22·5 cm.; Burton, II, 429–30; Greig, No. 432.]

Inviting S. to visit him so that they may discuss S.'s theories.

II, 46. Adam Smith, Kirkcaldy. Edinburgh, 6th Feb. 1770. *Signed.* [1½ pp. + 2½ pp., 18·5 × 22 cm.; Burton, II, 433, inc.; Greig, No. 438.]

Chiefly an extract from someone else's letter about the resignation of the Duke of Grafton and the succession of Lord North.

II, 47. Adam Smith, Kirkcaldy. Edinburgh, 28 Jan. 1772. *Signed.* [1 p. + 3 pp., 18·5 × 23 cm.; Greig, No. 467.]

Illness of H.'s sister, after whose recovery he hopes to see S.—*P.S.* Lack of good Italian prose.

II, 48. Adam Smith, Kirkcaldy. St Andrews Square [Edinburgh], 27 June 1772. *Unsigned. Torn.* [1½ pp. + 2½ pp., 19·5 × 24 cm.; Burton, II, 459–61, inc.; Greig, No. 476.]

The financial crisis in Scotland. Its value.

II, 49. [A. S.]. [Edinburgh, Oct. 1772.] *Signed, initials only.* [1¼ pp. + ¾ p., 18·5 × 23 cm.; Greig, No. 478.]

The financial crisis in the affairs of the Ayr Bank.

***II, 50.** Adam Smith, Kirkcaldy. [Edinburgh, Feb. 1770.] *Signed, initials only.* [½ p. + 3½ pp., 19 × 22 cm.; Greig, No. 440.]

Giving extract from a letter from London about the defeat of the Opposition when introducing a motion to disfranchise revenue officers.

* The editors first dated this letter October 1772; hence its position in Vol. II. of the MSS. Later investigations showed that they were wrong, and that the date ought to be as given above.

II, 51. Adam Smith, Kirkcaldy. St Andrews Square [Edinburgh], 23 Nov. 1772. *Signed.* [1 p. + 3 pp., 18·5 × 23 cm.; Burton, II, 461, inc.; Greig, No. 479.]

Suggesting a plan of work, and making a plea on behalf of Roby Arbuthnot's son.

II, 52. Adam Smith, Kirkcaldy. St Andrews Square [Edinburgh], 24 Feb. 1773. *Signed, initials only.* [½ p. + 2½ pp., 18 × 23 cm.; Burton, II, 466–7, inc.; Greig, No. 487.]

On Andrew Stuart's *Letters to Lord Mansfield* and Lord Monboddo's *Origin and Progress of Language.*

II, 53. Adam Smith, Kirkcaldy. St Andrews Square [Edinburgh], 10 Apr. 1773. *Signed, initials only.* [1 p. + 3 pp., 18·5 × 23 cm.; Burton, II, 467, inc.; Greig, No. 492.]

Further bankruptcies. Macpherson's *Homer.* Dalrymple's *Anecdotes.*

II, 54. Adam Smith. St Andrews Square [Edinburgh], 13 Feb. 1774. *Unsigned.* [1 p. + 3 pp., 18·5 × 23 cm.; Burton, II, 471, inc.; Greig, No. 495.]

Suggesting to S. that he should come to Edinburgh and act as Adam Ferguson's substitute during the latter's absence. On Benjamin Franklin's conduct, and Alexander Wedderburn's treatment of him.

II, 55. Adam Smith, British Coffee-house, London. Edinburgh, 8 Feb. 1776. *Signed.* [1 p. + 3 pp., 18·5 × 23 cm.; Burton, II, 483–4; Greig, No. 514.]

The printing of *Wealth of Nations* said to be completed.—H.'s bad state of health.—His attitude towards American Colonies.

II, 56. Adam Smith. Edinburgh, 1 Apr. 1776. *Signed, initials only.* [1½ pp. + 2½ pp., 19 × 23 cm.; Burton, II, 486–7; Greig, No. 517.]

Praising *Wealth of Nations,* with discussion on one or two points. Recommending Gibbon's *Decline and Fall.*

II, 57–8. Adam Smith, Kirkcaldy. London, 3 May 1776. [*Two documents as under*:—]

A. Signed. [1½ pp. + 2½ pp., 18·5 × 23 cm.; Burton, II, 491–2; Greig, No. 522.]

Referring to *B.* enclosed. His state of health and his proposed journey to Bath.

B. Signed. [1 p. + 3 pp., 18·5 × 23 cm.; Burton, II, 493; Greig, No. 522A.]

Ostensible Letter: Leaving to S.'s discretion whether to publish the *Dialogues concerning Natural Religion* or not; and, if so, when.

II, 59. Adam Smith, Kirkcaldy. Edinburgh, 15 Aug. 1776. *Signed.* [1 p. + 7 pp., 19 × 23 cm.; Hill, 364; Greig, No. 538.]

About the posthumous publication of the *Dialogues.*

II, 60. Adam Smith, Kirkcaldy. Edinburgh, 23 Aug. 1776. *Dictated to David Hume the Younger.* [1½ pp. + 2½ pp., 18·5 × 23 cm.; Burton, II, 515, inc.; Greig, No. 540.]

Leaves his MSS. [the *Dialogues*] to his nephew David. Empowers S. to make additions to *My Own Life.* His rapidly declining health.

II, 61. [A. S.]. [?] 3 June [?]. [½ p. + 1½ pp., 11 × 15 cm.; Greig, No. 544.]

Note inviting S. to dinner.

To [Prof. John Stewart, Edinburgh].

II, 62. [J. S.]. [Edinburgh] Tuesday Forenoon [Feb. 1754]. *Signed.* [3 pp. + 1 p., 19 × 23 cm.; Burton, I, 97–8, and II, 453–5; Greig, No. 91.]

On disputes which had arisen in the [Philosophical] Society [of Edinburgh]. S. has treated Lord Kames badly. H.'s own theories have been misinterpreted. His presumption in printing his *Treatise.* Desires to drop the question of the hostile attacks on his own views.—*P.S.* Encouraging the sale of Blacklock's poems.

To [?].

II, 63. [? .] [Paris, summer, 1764.] *Copy, corrected by H.* [2¼ pp. + 1¾ pp., 18·5 × 23 cm.; Greig, No. 250.]

An official letter from Paris about the negotiations which had led to the Peace of Paris, and about the share taken in these by Mr de Pinto.

LETTERS TO HUME.

From Jean le Rond d'Alembert (and occasionally Julie de Lespinasse).

III, 1. Hume, St Germain. [Paris] "ce mercredi 20" [1764–5]. *Signed.* [1 p. + 3 pp., 16·5 × 22 cm.]

Recommending to H. a literary man, Grosley, who is anxious to make H.'s acquaintance.

III, 2. [H.]. [Paris] "ce samedi 14" [1764–5]. *Signed.* [2½ pp. + 1½ pp., 16·5 × 22 cm.]

Begs H. to interest himself in a Canadian family living in the Isle de Rhé, who wish to return and live under English rule.

III, 3. Hume, Leicester Fields, London. Paris, 28 Feb. [1766]. *Signed.* [2½ pp. + 1½ pp., 17·5 × 21·5 cm.; E.P., 183–4.]

Thanks H. for procuring a pension for Rousseau.—The section of H.'s *Hist.* dealing with the War of Succession is bound to be diverting, but d'A. would have preferred an ecclesiastical history. "Il est, ce me semble, plus curieux de voir les hommes s'égorger pour des impertinences théologiques, que pour des provinces & des royaumes qui en valent un peu plus la peine." He bears a grudge against any history that deprives him of H.'s company.—Mlle de Lespinasse sends a similar message. Her health improves. The value of inoculation, and imitation of the English in introducing it. "Mais les anglois font des expériences, tandis que les francois disputent." -The excommunication of the Duke of Parma. It accelerates the ruin of the Papacy. "Que de fous dans ce meilleur des mondes possibles!"

III, 4. [H.]. [From d'A. and Mlle de L. jointly.] [Paris] 6 July [1766]. *Unsigned.* [4 pp., 15·5 × 19·5 cm.; E.P., 184–6; Greig, App. K.]

Mlle de L.: - Begs to know the exact story of the quarrel with Rousseau, largely in order that she may defend H. against the R. fanatics, though "la douceur de vos mœurs et l'honeteté de votre caractere forment un grand prejugé contre lui avant meme qu'on sache en detail de quoi il est question."

D'A.:—Expresses a similar curiosity. Can already hear Voltaire say in triumph: "De quoi diable aussi se meloit-il?" He is persuaded that R. must have been in the wrong, but warns H. to be careful about publishing the correspondence. -Mentions an abridgement of ecclesiastical history by the King [of Prussia], "grand serviteur de dieu ainsi que vous et moi." Yet he wishes it had been written by H.

III, 5. [H.]. Paris, 21 July [1766]. *Signed.* [8 pp., 17 × 21·5 cm.; E.P., 186–90; Greig, App. K.]

D'A. and H.'s Paris friends send him advice about how to act in the Rousseau affair: to take great care throughout, yet, since circumstances demand it, to print the letters at once in order to justify himself before the publication of R.'s Memoirs; to print nothing but the essential facts; to show great moderation; and, finally, to deny all share in the so-called letter from the King of Prussia. Let H. beware, above all, of those who would dissuade him from publishing through (i) cowardice, (ii) false generosity, or (iii) fanaticism for R. Mlle de L. and other friends advise him not to print merely a few copies for private distribution. This would have an underhand appearance.—Mlle de L.'s health. She is marked by the smallpox.—D'A. will send an account of the affair to Voltaire, which will cause him some amusement.

III, 6. [H.]. [Paris] 4th Aug. 1766. *Unsigned.* [12 pp., 17 × 21·5 cm.; E.P., 191–9; Greig, App. K, inc.]

Mocking at Rousseau's ascription of Walpole's letter to H. and himself, and

at R.'s suspicions aroused by H.'s stare. R. has revealed his true motives. R. writes of virtue, but refuses to believe in the virtues of others. D'A.'s feelings on reading R.'s long letter [of July 10]. H. should publish nothing, but wait till he is attacked. R. will be most annoyed if the matter is allowed to drop into oblivion.—D'A. asks H. to let R. have a note he is sending. Expresses, as at first, his disapproval of Walpole's letter. Mme du Deffand, he thinks, must have inspired it. Helvétius would have nothing to do with it. Walpole should write to R. explaining the true circumstances. His letter ought to apologise to d'A. R. is " un mauvais et maladroit logicien." Voltaire's anger against R. caused by the latter's having condemned him for his plays as " l'empoisonneur de la patrie."—The drastic punishment awarded to five young men by the Parlement de Paris. Voltaire's anger. Some of the judges themselves ashamed.—The affair of M. de la Chalotais.—Recommends Frisi to H.

III, 7. Hume, Leicester Fields, London. Paris, 10 Aug. [1766]. *Unsigned.* [1½ pp. + 2½ pp., 16·5 × 21·5 cm.]

Recommends Frisi, Prof. of Mathematics at Milan, to H. F. will give H. the latest news of France: the general attitude towards Rousseau, " & de beaucoup d'autres sottises telles que ce pays-ci en fournit abondamment."

III, 8. [H.]. Paris, 1 Sept. [1766]. *Unsigned.* [4 pp., 17 × 21·5 cm.; E.P., 200–2; Greig, App. K, inc.]

Having read Rousseau's long letter to H., d'A. declares it " un *commerage* plat et ennuyeux, digne d'une femme du peuple, mais à la verité d'une femme méchante et dangereuse." Walpole's letter was largely to blame. Of this he says: (i) it is of little worth in itself; (ii) it was wrong to attack a man who had done the writer no injury; (iii) W. ought to have on his conscience all the trouble he has since caused.—Mme du Deffand really hates H., as she does R., " premierement parce qu'elle hait tout le monde, & surtout les gens de merite, secondement parce qu'elle sait que vous aimez des gens qu'elle n'aime pas." Mlle de L. alone restrains a more violent opposition to her.—The real cause of R.'s discontent was the coldness of his welcome in England. Walpole ought to write telling him that he alone was responsible for the King of Prussia letter. D'A. laughs at all, " et des charlatans comme Rousseau, et des poltrons comme Mr Walpole."

III, 9. Hume, Lisle Street, Leicester Fields, London. Paris, 6 Oct. [1766]. *Signed.* [2½ pp. + 1½ pp., 17 × 21·5 cm.; E.P., 202–4; Greig, App. K, inc.]

Suard, with help of d'A. and d'Holbach, has already translated H.'s papers, with the addition of an advertisement and the small " declaration " d'A. previously sent to Rousseau. Walpole is really responsible for the trouble. Mme de Boufflers and Mme de Verdelin will not be mentioned in the publication.

III, 10. Hume, Lisle Street, Leicester Fields, London. Paris, 10 Nov. 1766. *Signed.* [3 pp. + 1 p., 17 × 19·5 cm.; E.P., 204–5.]

Suard and he have translated H.'s "factum contre ce vilain fou, soi disant philosophe," giving their reasons for some slight alterations. It has persuaded every reasonable person of H.'s goodness and of Rousseau's madness. D'A. has added a note similar to the one he sent to R., in spite of what Walpole may think of it. W. was really the cause of the trouble.—Wishes that H. would return to Paris at once.

III, 11. [H.]. Paris, 6 Apr. 1767. *Signed.* [4 pp., 17 × 19·5 cm.; E.P., 206–7.]

Owing to the weakness of his eyes he is employing an amanuensis. H. has too high an opinion of the consideration shown to literary men in France.—Rousseau is said to have received a pension from the King of England in such a way as not to be under a burden of gratitude to H.—Begs H. while in official employment to help Mr de Catt, who has served England for 20 years.—Marmontel's *Bélisaire* is being condemned by the "canaille théologique" for having said that Trajan, Socrates, etc., may not have been damned to hell. Wherever Socrates may be, d'A. hopes one day to sup between him and his good friend H.—Mlle de L. has been ill almost all winter.

III, 12. Hume, Lisle Street, Leicester Fields, London. Paris, 14 May 1767. *Signed.* [3 pp. + 1 p., 17 × 19·5 cm.]

Knows nothing of M. Graffigni.—In accordance with H.'s request he will not publish a letter written to H., though it would entirely establish his innocence.—The expulsion of the Jesuits from France.

III, 13. [H.]. [From d'A. and Mlle de L. jointly, but all in d'A.'s hand.] Paris, 8 June 1767. [4 pp., 16 × 21·5 cm.; E.P., 208–9.]

Mlle de L. [*signed*]:—Has seen H.'s kind letter about Rousseau addressed to Turgot. General agreement that R. is absolutely mad.—Asking for complete set of H.'s works.—Mme de Boufflers is in Paris, but is soon going to Pougues.

D'A. [*unsigned*]:—Dr James is taking H. a new set of d'A.'s works. Calls particular attention to Vol. V, which is new.—Expects the condemnation of *Bélisaire* by the Sorbonne, the whole two hundred of whom have about as much sense as four. Hopes the King of England will banish H. for ever, and that he will then come to France.

III, 14. Hume. Paris, 11 June [1767]. *Signed.* [3 pp. + 1 p., 16·5 × 21·5 cm.; E.P., 214–5.]

Recommends M. l'Abbé de Vauxcelles, who wishes to cross to England, to have the pleasure of crying "Wilkes and Liberty" with H.—Deplores the way

in which the *Hist.* keeps H. in England, and begs him to come to France when he is bored.—*P.S.* Mlle de L., already possessing H.'s *Hist.*, wishes the philosophical works.

III, 15. Hume, Lisle Street, Leicester Fields, London. Paris, 13 July [1767]. *Signed.* [2½ pp. + 1½ pp., 17 × 21 cm. ; E.P., 210–1.]

Recommends Mr de Catt. Rejoices, with Mlle de L., that H. may lose his post, if it means that he will return to France to console himself.—D'A. will always consider Rousseau as "un fou très dangereux."—A packet of books, sent for H.'s edification.

III, 16. Hume, London. Paris, 9 Oct. [1767]. *Signed.* [3 pp. + 1 p., 17 × 21·5 cm. ; E.P., 211–2.]

Thanks H. on Mlle de Lespinasse's behalf for the copy of his *Hist.* [John] Pringle and [Benjamin] Franklin are in Paris. Results of the publication of *Bélisaire* : praise from the King of Prussia, while the Sorbonne "sue sang & eau." The Sorbonne would like to be less tolerant, but dares not oppose the nation.

III, 17. [H.]. Paris, 8 Dec. [1767]. *Signed.* [2 pp. + 2 pp., 17 × 22 cm. ; E.P., 212–4.]

Recommends M. de Catt of Berne, who deserves a pension much more than the "chien de Diogene" [Rousseau].—R. has retired to Trye, where he tyrannises over the peasantry. He has just published a dictionary of music, where he attacks French music as he attacks everything that has befriended him.—The Sorbonne has laid its censure on *Bélisaire.*—Both Mlle de L. and himself in bad health.

III, 18. [H.]. Paris, 29 April [1768]. *Unsigned.* [1 p. + 3 pp., 13·5 × 18·5 cm.]

Apparently enclosing a letter from M. de Catt, thanking H. for the kindness he has shown to de C.'s cousin.

III, 19. Hume, London. Paris, 14 Jan. [1769]. *Signed.* [2½ pp. + 1½ pp., 17 × 21·5 cm. ; E.P., 215–6.]

Death of Lord Morton leaves a vacancy among the "Foreign Associates" of the French Academy. If the Academy intends to elect an Englishman, d'A. is willing to support Lord Stanhope, though most of his colleagues will support the usual plan of electing the President of the Royal Society. Otherwise he will vote for M. de la Grange.—His absolute indifference about Rousseau and his Memoirs. Mlle le Vasseur is thought to have caused R.'s departure from the Prince de Conti. "On assure qu'il l'a epousée ; belle conclusion d'un plat Roman."

III, 20. Hume, London. Paris, 30 May [1766–9]. *Signed.* [1 p. + 3 pp., 16·5 × 22 cm.]

Recommending M. Bruyres of Lyons.

III, 21. [H.]. Paris, 1 May 1773. *Signed.* [3 pp. + 1 p., 16 × 20 cm.; E.P., 217–8.]

Apology for delaying his reply. Illness of Mlle de Lespinasse. Their anxiety to see H. again. They fear he is devoted to rest and idleness. D'A., too, is unable to work, being attacked by insomnia. Strictness of the censorship in Paris.—The author of *Histoire du commerce des Européens dans les Indes* thanks H. for his praise. D'A. will never be consoled for the loss of the ecclesiastical history he has so often asked H. to write.

From David Anderson.

III, 22. [H.]. Edinburgh, 30 Oct. 1767. *Signed.* [2 pp., 19 × 23 cm.]

Financial matters. Advises H. to wait for a good security till Candlemas, and meantime to put his money in the hands of a banker. H. is growing too rich for a philosopher and the fact must be concealed from Rousseau. Yet H. is not anxious about acquiring more.

From d'Angiviller.

III, 23. Hume, Compiègne. [Paris] "ce mardi 30" [1764]. *In third person.* [1 p. + 3 pp., 13·5 × 19·5 cm.; Burton, II, 216–7.]

Informing H. of d'Alembert's recovery from his illness.

From Lavoyessierre, Seigneur de Bainsville.

III, 24. Hume, rue St Dominique, Paris. Rue Notre Dame des Victoires [Paris], 23 Jan. 1764. *Signed.* [1 p. + 3 pp., 18 × 22·5 cm.]

Letter from a young man of twenty-two, of a philosophical and poetical character, expressing his admiration of H.'s works.

From the Marquise de Barbentane.

III, 25. [H.]. L'Isle Adam, 9 Mar. [1765]. *Signed.* [2 pp., 14·5 × 19 cm.]

Warns H. of the change in Mme de Boufflers's sentiments after the Prince de Conti's misfortune. She has determined to give up both her pretensions and her dark and tragic notions. This letter is to be kept strictly secret.—Hopes to see H. soon.

III, 26. Hume, Paris. Villers-Cotterets, 27 June 1765. *Signed.* [2 pp. + 2 pp., 14 × 19 cm.]

Congratulating H. on attaining an independent position, which he has long deserved. Rejoicing that Mme de Boufflers intends to visit England, since the

respect she will receive there will raise her in her own esteem after her disappointment [in not becoming Princesse de Conti].—Hopes to see H. at Villers-Cotterets soon.

III, 27. Hume, Compiègne. Villers-Cotterets, 18 July 1765. *Signed.* [2 pp. + 2 pp., 14·5 × 19 cm.]

Hopes H. will not be affected by the change of Ministry in England, and that the Duke of Bedford will not be troubled by it. She hardly expects H. to visit her, owing to the pressure of his business.—Anxious for his future comfort; but the influence of such distinguished talents as his should affect those around him.

III, 28. Hume, Hôtel de M. l'Ambassadeur d'Angleterre. Paris, 25 Aug. [1765]. *Unsigned.* [1 p. + 3 pp., 14 × 18·5 cm.]

Glad that H. is satisfied with his new position. His moderation makes him easily contented, but surely he would not accept a post which would prevent his return to France.—Invitation to visit her.

III, 29. [H.]. Paris, 5 Mar. 1766. *Unsigned.* [4 pp., 14·5 × 18·5 cm.]

Glad that H. shares her feelings towards Rousseau, who differs from most people by expressing well only what he feels. R. should not persist in staying away from London. Solitude is attractive only when we can put an end to it at will.—She promises H. a cordial welcome if he returns to France.—On the "oraison funèbre" on the Dauphin pronounced by the Archbp. of Toulouse. "Elle n'a pas été generalement applaudie; on lui reproche de n'être pas assés touchante, cependant elle est en tout belle, bien écrite, et pleine d'esprit."—The Queen appears to be recovering from an illness which was expected to prove fatal.—News about H.'s friends.

III, 30. Hume, London. [Paris] 12 Mar. 1766. *Unsigned.* [1½ pp. + 2½ pp., 16 × 20 cm.]

Rejoices to inform H. that M. de Barbentane has been appointed Minister at Florence.—The difficulty H.'s friends have of finding the means of seeing him.

III, 31. [H.]. Villers-Cotterets, 3 Aug. 1766. *Signed.* [1 p. + 3 pp., 16 × 20 cm.; Greig, App. K, inc.]

Her sorrow at the trouble Rousseau has caused H. Her opinions coincide entirely with those expressed by Mme de Boufflers.—She has difficulty in believing R. a monster, and greater difficulty in defending him.

III, 32. [H.]. Villers-Cotterets, 3 Sept. 1766. *Unsigned.* [4 pp., 15·5 × 19·5 cm.; Greig, App. K, inc.]

Believes in H.'s honesty in his dealings with Rousseau, whose head she believes to be physically deranged.—The affair of M. de la Barre: the sentence

was inevitable, considering the fixed nature of French law and the violence of the offenders themselves, inspired by the *Dictionnaire philosophique.* Their punishment was necessary for the preservation of religion.—H.'s Italian Abbé [? Galiani] and the Court of Tuscany, which is religious yet gay.—She is going to Provence and hopes to see H. in Paris on her return.—*P.S.* The Parlement recommended the prisoners to mercy, but the King refused.

III, 33. [H.]. Paris, 8 June 1767. *Signed.* [4 pp., 15·5 × 19·5 cm.]

Laments the death of Lord Tavistock and the events that will prevent H. from crossing again to Paris. Rousseau is near Paris. His madness is manifest, though she still admires him as he once was.—Her retirement to a convent. She feels the loss of independence more than that of pleasures.—Begs H. to see whether some promised articles were sent by the late Lord Tavistock to her husband in Florence.

From Col. Isaac Barré, M.P.

III, 34. Hume. Rochefort, 3 Aug. 1764. *Signed.* [2 pp., 19 × 23 cm.; E.P., 34–6.]

His relatives, supposing him to be dead, have divided among them a legacy due to him. Consulting a lawyer about it. Asks H.'s advice on the matter.

III, 35. Hume. Toulouse, 4 Sept. [1764]. *Signed.* [2¼ pp. + 1¾ pp., 19 × 23 cm.; E.P., 36–8.]

Asks H. to secure an authoritative opinion from a Paris lawyer concerning the legacy.—Fears H.'s style is becoming effeminate through the influence of Parisian society.

From de Bastide, " auteur d'une maison d'éducation."

III, 36. Hume, chez M. l'Ambassadeur d'Angleterre. Rue de Richelieu, Paris, 20 Sept. 1764. *Signed.* [2 pp. + 2 pp., 19 × 23·5 cm.]

Thanks H. for the interest the latter has taken in him, and asks for an audience [about the placing of Gilbert Elliot's sons in a Paris school].

From Lord Beauchamp.

III, 37. Hume, Hôtel de Brancas, rue de l'Université, Paris. Forges, 17 Aug. [1764]. *Signed.* [2 pp. + 2 pp., 18·5 × 23 cm.]

Gratitude to his acquaintances whose letters relieve the monotony of his present life and save him from hanging himself.—The Duchesse de Choiseul has invited H. to visit her, and she would, B. believes, at least be gratified by a written excuse.—The blank in H.'s letter must represent either Mme de Boufflers or Mme de Gacé.

III, 38. [H.]. [London] 29 Mar. [1771]. *Signed.* [3½ pp. + ½ p., 18·5 × 23 cm.].

His regard for H. and willingness to show it to H. or his relations when possible. Knowing H.'s disgust at English politics, he hesitates to touch on the question; but "the City of London is now in a greater combustion, if possible, than it ever was at the time of the Middlesex Election." The commitment of the Mayor and Oliver unavoidable; the mobs put Lord North in danger of his life. The old sterling good sense of the English is wearing out.—Critical situation in Europe. The terms of the accommodation with Spain are not yet known, but the peace must be precarious "till we can be brought to renounce the idea of making establishments in the South Seas." Disarming is difficult when a lack of confidence remains.—It is also rumoured that Austrian troops are moving towards Poland "to make the Empress's mediation between the Turks and Russians more effectual." The advantages of such a feat.

III, 39. [H.]. London, 12 Dec. [1771–4]. *Signed.* [2 pp., 18 × 22·5 cm.]

Obliged to [John] Crawford for his friendly mention of B. to H. Regrets it is not in his power to serve his friends, but hopes for a continued correspondence.—At present the Opposition in Parliament is completely overawed by the big majority. But there may be unexpected happenings in America, where the Congress has adjourned to watch the movements of Parliament.

From the Marquis de Beccaria.

III, 40. Hume, rue de Colombier, Hôtel du Parc royale. [Paris] "mardi" [Dec. 1765]. *In French. Unsigned.* [½ p. + 3½ pp., 13 × 19 cm.]

Sends copies of his translation of the *Delitti* for H., Mme de Boufflers, and Rousseau, the "Socrate persécuté." Asks to be allowed to see R. before he leaves France.

From J. Becket, Bookseller.

III, 41. Hume, Edinburgh. London, 11 Nov. 1766. *Signed.* [3 pp. + 1 p., 18 × 22·5 cm.]

Suard has sent him for publication some copies of the *Exposé succinct.* He has therefore entrusted the translation to the translator of Rousseau's other works. Receiving notice of H.'s letter, he was able to make terms with [Andrew] Millar, and to hand H.'s paper over to the translator. Especial care is being taken with the work, and such false charges of R., as that H. suppressed the letters of M. Peyrou, will be entirely refuted.—Regrets Millar's reflections on Strahan's part in the publication.

From Lydia Becquet.

III, 42. Hume, Secretaire de l'Ambassade d'Angleterre, rue de l'Université. Paris, 5 Aug. 1765. *Signed.* [*Two documents as under :*—]

A. Letter to H. [1½ pp. + 2½ pp., 15·5 × 20 cm.]

Complains that she has lost the favour of the Countess de Boufflers. Her

distress at this fact. Begs H. to come to her help. Encloses a letter which she proposes to send to Mme de B., and which she begs H. to examine.

B. Enclosure. [1 p. + 3 pp., 18·5 × 23 cm.]

Recognises her inability to deserve the favour of her mistress, and begs the latter to declare openly whether or not she is tired of the writer.

III, 43. Hume, Secretaire de l'Ambassade d'Angleterre, Compiègne. Paris, 8 Aug. 1765. *Signed.* [2 pp. + 2 pp., 15·5 × 20 cm.]

On her fears lest she has lost favour with the Countess [de Boufflers]. She has been advised not to refer to the subject and to persuade H. to keep silence also. Confesses that she is partly suffering from sensitiveness and an unreasonable pride.

From Octavie de Guichard, Mme Belot, afterwards Mme de Meinières.

III, 44. Hume, rue de l'Université, Hôtel de Charot. [Paris] 15 Dec. 1764. *Signed.* [1 p. + 3 pp., 10·5 × 16·5 cm.]

Peace has been re-established between two ladies, Mme Geoffrin having admitted being at fault. Requests H. to return the MS. of *Emma*, which he has been looking over.

III, 45. Hume, rue de l'Université, Hôtel de Charot. [Paris] 15 Mar. 1765. *Signed.* [2 pp. + 2 pp., 16·5 × 22 cm.]

Her translation of H.'s *Hist.* has been well received in France. The narrowness of her fortune has compelled her to seek a livelihood in literature. The Marquise de Pompadour procured her a pension, which has proved insufficient. Begs H. to use his credit and to extol the merit of her translation in order that she may obtain an increase. A false rumour that he has obtained such a pension for her proves that there is nothing extraordinary in her request.

III, 46. Hume, rue de l'Université. [Paris] 9 Aug. 1765. *Signed.* [1 p. + 3 pp., 10·5 × 16·5 cm.]

Regrets that H. is leaving France, yet rejoices in the likelihood that the move to Ireland will prove of advantage to him.—Hopes that he will correspond with her, his translator.

III, 47. [H.]. Rue poissonnière, Paris, 7 July 1766. *Signed* "Guichard de Meinières." [3½ pp. + ½ p., 15·5 × 20 cm. ; Greig, App. K, inc.]

Admiration for H. and interest in his welfare in return for the influence his *Hist.* has had on her destiny.—Begs exact information on the quarrel with Rousseau. R. must have committed some really serious crime to arouse so bitter an outburst in a man of H.'s character. A quarrel like this in the world of letters is to be lamented. "Vous étiés au milieu d'eux tous, comme un colosse inébranlable, et vous voila en mouvement."

III, 48. [H.]. [Paris, end of July 1766.] *Signed.* [4 pp., 15·5 × 20 cm.; Greig, App. K, inc.]

She has shown H.'s letter [on the Rousseau quarrel] to M. de Montigny, who expressed the liveliest interest in H.'s welfare. Copies in translation have also been shown to other friends.—H. need have no fear about the figure he will cut in R.'s Memoirs. H. has facts, R. merely vague suspicions. H. is loved, R. feared or admired, but not loved.—Hence H. would be well advised to present the bare facts without reflections or reproaches, and to leave the reader to draw the inevitable conclusions. He must beware of his supporters. Some are anxious to help him, others only to injure R. by means of him.—Inquires whether H. intends to return to France, and whether he is continuing his *Hist.* Reflections on the eternity of H.'s fame compared with the transitoriness of R.'s sophisms. R.'s pride and ingratitude alienate all his acquaintances. As for H., "vos mœurs douces, votre bonté, votre sublime modestie vous rendent cher a tout ce qui vous connoit."

From Thomas Blacklock.

III, 49. [H.]. Edinburgh, 24 June 1766. *Signed.* [4 pp., 19 × 24 cm.; Burton, II, 399–400.]

Thanks H. for his letter. Satisfaction at H.'s success in France.—H.'s nephew [Joseph]. B.'s fears of the vivacity of his temper are calmed by his amiability and diligence. His restraint in company: without losing his dignity. His present studies.—B.'s own situation: having left a parish where he would always have been unhappy, he hoped for a better reception in Edinburgh; yet he has been coldly ignored. Therefore all the more anxious to see H.—Compliments from Mrs B.

III, 50. [H.]. Edinburgh, 2 May 1767. *Signed.* [3 pp. + 1 p., 19 × 23 cm.]

Glad to hear of H.'s health, and to receive his promise of correspondence. Overwork has caused a return of B.'s nervous complaints, but he is now a little better. His boarding establishment is completely successful, though it gives him no opportunity to save money for his old age. Resources from Kirkcudbright uncertain. H. has hinted at a possible church settlement, but B.'s previous experience of such life persuades him it would be fatal.—H.'s nephew continues his good progress.—*P.S.* [*From Mrs B.*] Her fears are aroused by B.'s close study. Suggests the possibility of a professorship of Poetry, with a moderate salary.

From the Rev. Hugh Blair.

III, 51. [H.]. Edinburgh, 29 Sept. [1763]. *Signed, initials only.* [5 pp. + 3 pp., 18·5 × 22 cm.; Burton, I, 468–70.]

On Ossian. B. is convinced of the authenticity of the poems, though most men of letters think otherwise. Macpherson is incapable of forgery. Some of

the MSS. sent to M. passed through B.'s hands, and several M. translated "in a manner under" B.'s eye. Claims confirmed by Highlanders. Improbability of such a forgery. Even interpolation is unlikely. B. is applying to various authorities for their opinions. This will retard Becket's new edit. of the poems, with B.'s *Dissertation*. The necessity for propitiating Macpherson for having apparently doubted his veracity.—H. is sure to be almost worshipped in France. But his *Dialogues*, though they might increase his popularity, had better remain unpublished. H. must not hesitate to accept this step towards advancement.

III, 52. [H.]. Edinburgh, 6 April 1764. *Signed.* [3 pp. + 1 p., 18·5 × 23 cm.]

Recommends Col. Lesly to H. His pleasure at H.'s preferment.—Changes in the staff at Edinburgh College. Increased interest in French literature among Edinburgh scholars.—B.'s *Dissertation on Ossian.* Possibility of his delivering a second course of lectures to the medical students. Pleasure on becoming H.'s tenant [in James's Court].—*P.S.* Fairholm's bankruptcy, and its effects.

III, 53. [H.]. Bruntsfield [Edinburgh], 1 July [1764]. *Signed.* [4 pp., 18·5 × 23 cm.; Burton, II, 229, inc.]

On Arthur Masson, the bearer of the letter.—All H.'s clergy friends wish to be remembered to him. Robertson's illness.—Political revolutions in Scotland.—H.'s philosophical attitude towards Parisian society.—Prosperity of Edinburgh College. Possibility that the son of the Duke of Northumberland may be sent there.—Excellence of French *Gazette littéraire.* B. has sent H. a copy of his *Dissertation on Ossian.* Claims to have established the authenticity of the poems. Begs H. to write.

III, 54. [H.]. Edinburgh, 15 Nov. 1764. *Signed.* [4 pp., 18·5 × 23 cm.]

In recommending Col. Lesly to H., he did not expect too much from H. He now recommends the consumptive son of Mr Bowman, Provost of Glasgow, who will pass through Paris on his way to Toulouse.—Life in H.'s house. [James] Oswald's son as a lodger. Changes and events in Edinburgh College.—Rumours of H.'s social successes have reached Scotland. Letters from him are very welcome.

III, 55. [H.]. Edinburgh, 8 Oct. 1765. *Signed.* [3¼ pp. + ¾ p., 18·5 × 23 cm.]

Thanks H. for sending copies of *Gazette littéraire* relating to B.'s *Dissertation on Ossian.* Flattered by the praise therein.—H.'s fortunes of great interest to his friends. Congratulations on having escaped "from the boisterous region of Irish politics." Few people have been more fortunate than H. After his brilliant life at Paris he can now retire to "philosophic ease and tranquillity."—B. continues to live in H.'s house for the present. An exceedingly good house.—Everyone of taste and discernment in Edinburgh admires Sir James Macdonald [of Sleat] as much as H. does. B. considers him as "next to a prodigy."—

John Home's project [of standing for Parliament] wild and extravagant, but was soon dropped.—News of other friends. "Jardine regrets much that the hint of the Princess Amelia was not taken. He thinks you would have made an admirable Irish bishop."

III, 56. [H.]. Edinburgh, 24 Feb. 1766. *Signed.* [4 pp., 18·5 × 22 cm.]

Character of Rousseau: "amiable, but whimsical; strong sensibilities joined with an oddly arranged understanding . . . sceptic from the turn of [his] mind, and yet an enthusiast from the turn of [his] heart." Wishes to know whether the *Nouvelle Héloïse* is founded on R.'s real life.—Rejoices that H. intends to return to Scotland again. B. will not accept any more young men into his household.—On Ferguson's book [*Essay on Civil Society*]: Robertson and B. agree there is something lacking in the style and in the disposition of the parts, yet there is "so much ingenuity & uncommonness in the matter, so much of the hand of a master often appearing on many occasions," that both R. and B. thought the beauties far outweighed the blemishes. The only way to persuade F. to delay the publication would be for H. to write and ask to read through the book.

III, 57. [H.]. Edinburgh, 13 May [1766]. *Signed, initials only. Fragment. Beginning missing.* [1½ pp. + ½ p., 18 × 22·5 cm.]

Thanking H. for long, interesting letter about Rousseau.—B. is about to leave H.'s house.—John Home says they are soon to see H. in Scotland, and that he is likely to settle there, which is very agreeable news.—Mrs Jardine has been dangerously ill, and J. himself has turned asthmatic.—Robertson has taken Lord Bute's son into his house as a pupil.

III, 58. Hume, Lisle Street, Leicester Fields, London. [Edinburgh] 12 June 1766. *Signed.* [3 pp. + 1 p., 18·5 × 24 cm.]

Dr Drysdale agrees to take Mr Dodwell's son.—B.'s deep sorrow at the death of Jardine. Begs H.'s interest in gaining for him the Deanery of the Chapel Royal, rendered vacant by J.'s death. Robertson is supporting Drysdale for this, and if the office is already filled, B. hopes his own name will be remembered at the next occasion.

III, 59. Hume, Lisle Street, Leicester Fields, London. Edinburgh, 10 July 1766. *Signed. Torn.* [2½ pp. + 1½ pp., 18·5 × 23 cm.; Greig, App. K, inc.]

B. is at a loss to account for Rousseau's actions. Begs H. not to publish the correspondence too hastily. The world relishes a quarrel between two eminent men. H. need have no fears about H.'s previous letters to B. [praising R.]; if any copies have been taken, they can easily be recovered.—B.'s regard for Drysdale makes him contented with the latter's appointment, but he hopes his own claims will not be overlooked at the next vacancy.

III, 60. [H.]. [Edinburgh] 24 Mar. 1767. *Signed.* [3 pp. + 1 p., 19 × 24 cm.]

Rejoices at the success of Ferguson's book, which he foretold. Has changed his opinion about the style. Not only is it not careless, but it may be accused of excessive artificiality in places.—Rousseau is " a sad and contemptible creature." Will R. prepare for war, or " sit down patient and contented under the load of infamy ? "—B. hopes H. has provided for Oswald's eldest son, as it is rumoured. —Offers to introduce Thomas Percy [editor of the *Reliques*], " a good sort of man, of no very enlarged views however."

III, 61. [H.]. Edinburgh, 4 June [1767]. *Signed, initials only.* [3 pp. + 1 p., 18·5 × 23 cm.]

Settlement of the affair is to be left to Chief Baron [of the Scottish Exchequer]. B. did not seriously expect it. Rousseau always considers his own interests, even in his madness. " He is a curiosity ; but a humbling one for human genius." —Success of the General Assembly.—Begs H. to support Mr Clark's application for a Crown presentation at Humbie in East Lothian. C.'s character. Increase of H.'s " party " among the ministers of the Church.—Sir James Steuart's book [*Enquiry into the Principles of Political Economy*] " a piece of the most ponderous lumber I think I have ever looked into."

III, 62. [H.]. [Edinburgh] 11 March 1769. *Signed.* [4 pp., 18 × 23 cm.; Burton, II, 421, inc.]

Recommending his cousin, Robert Blair.—Glad H. is likely to return to Scotland. Foolishness of the people H. is living amongst. " Does history present any more stupid and infatuated people, or any more weak and incapable governours ? "—B.'s admiration for [John Home's] *Fatal Discovery.* Garrick's approbation. Praise of Robertson's *Charles V.* " Your *Hist. of England,* and his as an introduction to the history of Europe, furnish a perfect historical library."—Miscellaneous questions.

III, 63. Hume, Bath. [Edinburgh, May or June 1776.] *Date and signature torn off.* [2½ pp. + 1½ pp., 18 × 23 cm.]

Pleasure at the improvement in H.'s health. Advises him to remain at Bath over the winter. H. was fortunate to meet with John Home. H. should, for his own safety, avoid all intercourse with the Methodists.—H. will be home in time to support Robertson's son for a church office.—The moderation of the General Assembly. News of H.'s friends.

III, 64. Hume. [Edinburgh, ?.] *Signed.* [1 p. + 3 pp., 14·5 × 18·5 cm.]

Thanks H. for a copy of *Hist.* and *Essays.* Asks to see him before he leaves.

From Marie-Charlotte-Hippolyte de Campet de Saujeon, comtesse de Boufflers.

III, 65. [H.]. Paris, 13 Mar. 1761. *Signed.* [4 pp., 15·5 × 20 cm.; Burton, II, 97–9; Greig, App. E.]

Her admiration for H.'s *Hist.* makes her desirous of corresponding with him, despite the possible accusation of presumption, against which she defends herself. Praise of the *Hist.*—*P.S.* Hopes H. will come to France at the Peace.

III, 66. [H.]. Paris, 12 July [1761]. *Signed.* [1½ pp. + 2½ pp., 15·5 × 20 cm.; E.P., 222–3, inc.]

Emphasises the sincerity of her former praises. Her admiration, though worth little, is honest and sincere. Offers H. rooms if he comes to France, and if he declines them, a warm welcome whenever he visits her.—*P.S.* She has ordered Robertson's *Hist.* on H.'s recommendation.

III, 67. [H.]. Paris, 29 May 1762. *In English.* *Signed.* [1 p. + 1 p., 15·5 × 20 cm.; Burton, II, 101–2.]

She has received the continuation of H.'s *Hist.*, and flatters herself it was he who sent it.

III, 68. Hume, London. Paris, 14 June 1762. *Signed.* [2¼ pp. + 1¾ pp., 15·5 × 20 cm.; Burton, II, 107–8, inc.; Greig, App. E, inc.]

She has been attracted by H.'s merits, and though she feels that H. has had no reason to return this interest, ventures to ask a favour.—Rousseau, after printing his work on education, has been forced to flee the country. She recommends him to H. R.'s character: "Il a le cœur droit, l'ame noble et desinteressée, il craint toute espece de dependance, et pour cette raison, il a mieux aimé etant en France, gagner sa vie en copiant de la musique, que de recevoir les bienfaits de ses meilleurs amis . . . je ne crois pas qu'il y ait nulle part un homme plus doux plus humain plus compatissant aux peines des autres et plus patient dans les siennes."

III, 69. [H.]. [Paris] 30 July [1762]. *In English.* *Signed.* [3¼ pp. + ¾ p., 15·5 × 20 cm.; Burton, II, 110–3, inc.; Greig, App. E, inc.]

She is unworthy of H.'s praise. "A great part of my youth is over. Some delicacy in features, mildness and decency in countenance, are the only exterior advantages, I can boast of. And as for interior, common sense, improved a little, by early good reading, are all I possess." Her knowledge of English.—She has sent the P.S. of H.'s last letter to Rousseau. Afraid R. will not accept H.'s proffered help. Her admiration for R.—She hopes H. will not be deterred from coming to France. Points out a possible error in H.'s change of "Godeffroy de Bouillon" into "Godeffroy de Boulogne." Asks his opinion on Rousseau's latest work [*Emile*].

III, 70. Hume, London. [Paris] 11 Sept. 1763. *Signed.* [1½ pp. + 2½ pp., 15·5 × 20 cm.]

Her flattering reception in England, marred only by her not having seen H. Her good opinion of Rousseau still holds.

III, 71. Hume, Paris. L'Isle Adam, 16 Oct. 1763. *In English, in Lydia Becquet's hand. Signed by Mme de B.* [1 p. + 3 pp., 15·5 × 20 cm.]

Having just recovered from measles, she is unable to meet H. herself.

III, 72. Hume, Paris. [Paris] "ce dimanche au soir" [winter, 1763–4]. *Unsigned.* [½ p. + 3½ pp., 11·5 × 17·5 cm.]

Inviting H. to visit her. She will take him to the Temple [the Prince de Conti's residence].

III, 73. Hume, Paris. [Paris] "ce 18" [winter, 1763–4]. *Unsigned.* [½ p. + 3½ pp., 11·5 × 17·5 cm.]

Inviting H. to visit her.

III, 74. Hume, Paris. [Paris] "ce 27" [May 1764]. *Unsigned.* [½ p. + 3½ pp., 15·5 × 20 cm.]

Has received H.'s letter. The death of M. de Luxembourg has made her too sad to write a long reply.

III, 75. [H., Compiègne.] [? L'Isle Adam] 6 July [1764]. *Unsigned.* [9½ pp. + 2½ pp., 16 × 20 cm.; E.P., 223–5, inc.; Greig, App. E, inc.]

H. is her "maitre de philosophie." She owes him what learning she has. Yet she ventures to say that she is not pleased with his friend's tragedy [*Douglas*]. English tragedy and Shakespeare. "Sakespear tout monstrueux qu'il est, pouvoit donner quelqu'esperances. Il est souvent admirable, et jamais ennuyeux. Ses beautes apartiennent a son genie, ses fautes a son ignorance, et a son siecle."—Detailed criticism of *Douglas*.—Her admiration and affection for H. will lead her to despise the rest of her acquaintance. She fears to lose for ever friends who leave her.—Her reading: H.'s *Essays*, Middleton's *Life of Cicero*, Plutarch.—H. and she must try to stop loving each other, but as they have not yet begun, "je puis me permettre pour aujourd'huy, de vous assurer, que je vous aime de tout mon cœur."

III, 76. Hume. [? L'Isle Adam] 21 July [1764]. *Unsigned.* [8 pp., 15·5 × 20 cm.; E.P., 226–9; Greig, App. E, inc.]

Asks whether H. believes her to be "un cœur tendre, bienfaisant, et sensible a l'amitié." Yet he suspects her friendship for him. She too is subject to jealousy, or rather "exces de delicatesse." Her affection for him is sometimes hampered by absence and by duties that a still older attachment has imposed on

her. The sanctity of these duties. But they do not take up all her time, and she is ready to devote the rest to H.—She explains her reasons for saying she could not live long. She has both the troubles and the advantages attending too much "sensibilité." She disliked criticising *Douglas*. She feels her own mediocrity, yet has her pride, and therefore devotes herself to a path in which she finds few stronger rivals: trying to perfect certain qualities of heart that nature has endowed her with.—She expects to be in Paris about the same time as H.

III, 77. [H.]. [? L'Isle Adam] 30 July [1764]. *Unsigned.* [3 pp. + 1 p., 15·5 × 20 cm.; E.P., 230–1, inc.; Greig, App. E, inc.]

Reprimanding H. for sending so short an answer to her former letter. Her opinions on mankind in general. Her pride, which she begs H. not to excite.

III, 78. [H.]. [?] 15 [Aug. 1764]. *Unsigned.* [2 pp. + 2 pp., 15·5 × 20 cm.; Greig, App. F.]

Has received H.'s letter to Lord Elibank, and Mr [Alexander] Murray's letter to H. Approves of the first, and deprecates the violence of the second. But defends Murray, in order to effect a reconciliation.

III, 79. [H.]. [?] 18 Aug. [1764]. *Unsigned.* [3 pp. + 1 p., 17 × 22 cm.; Greig, App. F.]

Protests her innocence of the charges brought against her by Murray. Leaves her justification to time. Only those will believe the charges who are prejudiced by envy or hatred, from both of which H. is free. Regrets his suspicions of her, but is convinced of his sincere affection.—Her remarks on H.'s letter to Lord Elibank were meant to refer to the epithet towards the end. E.'s love for Mary Stuart, though extravagant, should not be treated with scorn.

III, 80. [H.]. [Paris] "ce 15" [autumn, 1764]. *Unsigned.* [½ p. + 3½ pp., 15·5 × 20 cm.]

Returns Rousseau's letter, along with another [from Alexander Murray], which will give H. a better opinion of him and his affair.

III, 81. Hume, Compiègne (readdressed to Paris). [Paris] "ce 13" [Sept. 1764]. *Unsigned.* [½ p. + 3½ pp., 15·5 × 20 cm.]

Unwelcome news from England. She will not be able to see H. before the 23rd.

III, 82. Hume, Fontainebleau. [Paris] "ce vendredi" [autumn 1764]. *Unsigned.* [2 pp. + 2 pp., 11·5 × 17·5 cm.; E.P., 231.]

Illness has prevented her from writing.—The death of M. de Boufflers [her husband] has given her many things to think about. Invites H. to visit her in Paris.

III, 83. Hume, Paris. [Paris, autumn, 1764.] *Unsigned.* [½ p. + 3½ pp., 15·5 × 20 cm.]

Her health is good and her mind apparently tranquil. Her present situation absorbs all her attention.

III, 84. Hume, Paris. [1763–5.] *Unsigned.* [1 p. + 3 pp., 15·5 × 20 cm.]

Rousseau's money in the will of M. de Luxembourg, etc.

III, 85. [H.]. "Ce jeudi a 7 heures du soir" [1763–5]. *Unsigned.* [1 p. + 3 pp., 15·5 × 20 cm.]

Sends a recommendation for Mr Murray, though she has too much contempt for him to address one to the Governor.—Encloses a packet for H. Her disappointment at not seeing him.

III, 86. Hume, Paris. "Ce dimanche" [1763–5]. *Unsigned.* [1 p. + 3 pp., 11·5 × 17·5 cm.]

The time of her return to Paris is uncertain. Her journey has been pleasant.

III, 87. Hume, Paris. Wednesday [1763–5]. *In Lydia Becquet's hand. Unsigned.* [½ p. + 3½ pp., 11·5 × 17·5 cm.]

She has been ill. Hopes to see him.

III, 88. Hume, Paris. [1763–5.] *Unsigned.* [1 p. + 3 pp., 13·5 × 19 cm.]

Invitation from Mme de B. and the Prince de Conti.

III, 89. Hume, Paris. "Ce lundi au soir" [1763–5]. *Unsigned.* [1 p. + 3 pp., 11·5 × 17·5 cm.]

Invitation to visit her. She has bad news to tell him.

III, 90. Hume, Compiègne. London, "ce 22" [summer, 1765]. *Unsigned.* [1½ pp. + 2½ pp., 15·5 × 20 cm.]

Her interest in English affairs has been aroused by her fear that H. may leave her. Her sorrow at the thought of losing him.

III, 91. Hume, Calais (forwarded from Dover to London). [Paris] 7 Jan. [1766]. *Unsigned.* [1½ pp. + 2½ pp., 15·5 × 20 cm.; E.P., 231–2.]

Inquires about his journey. Is a letter purporting to be written by the King of Prussia [to Rousseau] genuine or not? "On dit qu'elle est pleine d'ironie, qu'il paroist douter de la bonne foy de ses principes, et qu'il luy mande entr'autres choses luy offrant un azile, que s'il veut etre persecuté il le sera, que s'il veut etre puni, comme roy, il le punira."

III, 92. [H.]. [Paris, Jan. 1766.] *Unsigned.* [6½ pp. + 1½ pp., 16 × 20 cm.; E.P., 232–4, inc.]

H.'s journey with Rousseau. She deprecates R.'s plan of settling in Wales. Difficulty about the pension. If he accepts, he will be accused of ingratitude towards the King of Prussia, or, if he refuses, of extravagance and presumption. The friends and enemies he has made.—[Horace] Walpole's reconciliation with Mme de B. after his letter against R.—A lawsuit in England by certain Frenchmen, on whose behalf she asks H. to thank "Mr Waterborn" [? Alexander Wedderburn], their advocate.—The sufferings of Rousseau, in spite of his courage to conceal them.—Impatiently awaiting H.'s return.

III, 93. [H.]. [Paris] "ce 18" [Feb. 1766]. *Unsigned.* [4 pp., 15·5 × 20 cm.; E.P., 235–6, inc.]

Pleased to learn of the pension intended for Rousseau. Hopes nothing will retard H.'s return to Paris.—The Duke of Richmond's quarrels with the Princes of the Blood.—Her illness. Yet, out of sheer indifference, she takes her part in social affairs.—Everything is ready for H.'s return.—*P.S.* "L'affaire de M. de Rougemont" still undecided.

III, 94. [H.]. Montmorency, 18 Apr. [1766]. *Unsigned.* [4 pp., 15·5 × 20 cm.; Greig, App. E.]

H.'s continued absence recognised as necessary on account of his indebtedness to Lord Hertford. She has changed her dwelling. Begs H. not to give himself up to a life of idleness.—Mention of Rousseau and of H.'s Paris friends. She has been putting her affairs in order.—Mme Geoffrin's journey to Poland. Mme de B. refuses to mock at the King of Poland's affection for Mme G.—Asks for Addison's *Journey to Italy.*

III, 95. [H.]. [Paris] 6 May [1766]. *Unsigned.* [6½ pp. + 1½ pp., 15·5 × 20 cm.; E.P., 237–9, inc.]

Prepared for H.'s arrival.—Wishes to know the reason for Rousseau's annoyance, which she ascribes to the influence of Thérèse le Vasseur. Her plans for the next few weeks. She is going to Lyons. If H. comes to France in time, why should they not travel together to Marseilles and Toulon?—Has met Adam Smith and is reading his *Theory of Moral Sentiments.*—The Prince of Brunswick: his charm.—She has heard that Walpole is writing a pamphlet against Rousseau. The injustice of such an attack.—Mme de Barbentane is occupied with arrangements. The advantages for Mme de Boufflers's son of his trip to Italy. The trouble he has caused her.

III, 96. [H.]. Pougues, 22 July 1766, with P.S. from Paris, 25 July 1766. *Unsigned.* [14 pp. + 2 pp., 16 × 20 cm.; E.P., 239–46; Greig, App. K.]

Reproaches H. for leaving her so long in ignorance about the quarrel with

Rousseau. Surprised at the vehemence of his anger as revealed in his letter to the Baron d'Holbach. Wonders he did not triumph over R. by dignity and magnanimity.—R.'s letter is atrocious. Yet he had some grounds for anger. She disapproves of Walpole's letter and of the share which she supposes H. to have had in it. To accuse R. of malice aforethought, as H. has done, is groundless.—That R. had joined the political Opposition is similarly incredible.—With full control of his reason R. would never have premeditated injury. But he has lost control of himself through his violent passions. His pride has been hurt by H.'s scoffing remark in Walpole's letter. Had H. been near her, he would have shown more compassion for R. She has difficulty in advising H. whether to publish the letters or not.—*P.S.* Having seen a further letter from H., which shows that he is carried away by passion, she feels it is useless to give him any advice.—Reply to H.'s letter about M. de Rougemont.—Begs H. to be careful. Adam Smith, like her, fears the violence of H.'s resentment against R.

III, 97. Hume, London. [Paris] 6 Aug. 1766. *Unsigned.* [2½ pp. + 1½ pp., 16 × 20 cm.; Greig, App. K.]

Has received H.'s apology for keeping her in ignorance of the quarrel with Rousseau, but had already pardoned him. She feels, however, that he ought to have shown greater moderation, and that he is himself responsible for any evil the matter may have caused. She has read R.'s letter. She agrees with those who think his wits affected, but not with H. in thinking his style as good as ever.—Her son's charms.

III, 98. [H.]. [Paris, 1767 or 1768.] *Unsigned.* [1 p. + 3 pp., 15·5 × 20 cm.]

Considers it unworthy of H.'s merit that he should return to France as a mere perpetual secretary to the Embassy; and this, in spite of her desire to see him.

III, 99. [H.]. [Paris] 25 May 1768. *Unsigned.* [4 pp., 15·5 × 20 cm.]

The joy and peace of mind H.'s letter has caused her has decided her to make another voyage to England. Believes she has deserved the continuance of his friendship.—Her son not yet arrived. She is a little disappointed at his match.—Notes on H.'s Paris friends.

III, 100. Hume, London. [Paris] 15 July 1768. *Unsigned.* [2 pp. + 2 pp., 15·5 × 20 cm.]

Rejoices at the prospect of H.'s visit to her. She has now recovered from her illness. Outline of her plans for the next few months, with suggestions about their meeting.

III, 101. [H.]. [Paris] 7 Jan. 1769. *Unsigned.* [4 pp., 15·5 × 20 cm.; E.P., 248–50, inc.]

Grief on hearing that H. has postponed his visit to France. She had supposed

his affection for her great enough for him to have endured such a journey.—Her son's marriage. His prospects.—Her friendship with H. is the great duty and pleasure of her life.—She has no more intercourse with Rousseau. His marriage with Mlle le Vasseur, and proposed journey to England.

III, 102. [H.]. [Paris] 29 May 1769. *Unsigned.* [4 pp., 15·5 × 20 cm.; E.P., 250–1, inc.]

Her daughter-in-law has recovered after her inoculation against smallpox.—She is sorry her previous letter angered H. She merely wished him to perceive her feelings clearly.—The perfectly free life she leads. She does nothing but what she desires. Though once overcome by a violent passion, which has now passed into friendship.—H. is unjust to the Duke of Grafton, whom she likes.

III, 103. [H.]. [Paris, 28 May 1775.] *Unsigned.* [7 pp. + 1 p., 14 × 18·5 cm.; E.P., 246–8, inc.]

Delighted at the possibility of H.'s return to France. Offers him a house in the Temple and a country residence near the Bois de Boulogne.—Mme de Bussy separated from her husband, with a pension.—Mme de Barbentane's quarrel with the Prince de Condé. Marriage of her daughter.—Death of Princesse de Conti. Mme de B.'s respect for her.—Mention of other Paris friends, especially Turgot and Trudaine.

From Lucie Boullier.

IV, 1. Hume, London. Lausanne, 20 Nov. [1766]. *Signed.* [2 pp + 2 pp., 16·5 × 22 cm.]

Sorrow at the quarrel between H. and Rousseau, and at seeing "peut-etre, les deux plus grands homes de l'univers briser avec fureur des liens qui devoient faire les delices de leur vie." Begs for their reconciliation.

From de Brand, Master of the Horse to the Prince of Prussia.

IV, 2. Hume, Edinburgh. Berlin, 7 Aug. 1756. *In English. Signed.* [1 p. + 3 pp., 19 × 23·5 cm.]

His pleasure at reading H.'s works, especially "those upon Human Understanding." He sends in return a composition by one of his countrymen.

From John Brett.

IV, 3. Hume, London. Bullyhoo, near Ardee [Ireland], 14 Apr. 1767. *Signed.* [5½ pp. + 2½ pp., 18 × 23 cm.]

Believes that men must follow their own opinions, and H. has at least forced him to think. Takes great pleasure in H.'s works, and regrets the political advancement that endangers the man of letters. Wishes a sinecure had been found to enable H. to devote himself to his studies.—Panegyric on H.'s *Hist.*,

which, however, contains some blemishes. Too partial a picture of some of the chief actors; *e.g.* the Duke of Hamilton. H., in his attachment to the Stuarts, defends the constitution rather than the individuals themselves.—Repeats expressions of gratitude for H.'s labours.

IV, 4. [H.]. Bullyhoo, 15 June 1767. *Signed.* [1 p. + 3 pp., 18·5 × 23 cm.]

Lacks H.'s confidence in Burnet. The treatment of Hamilton he thought unpardonable in H.—Fears there is little probability that the *Hist.* will be continued.

From President de Brosses.

IV, 5. Hume, Hôtel de Grimbergh, Paris. Dijon, 17 Jan. 1764. *Signed.* [3 pp. + 1 p., 16 × 20 cm.; E.P., 274–6.]

Apologises for delay in answering H. Thanks H. for praise of his book on Egypt. Its debt to H. B.'s remarks on the Sphinx have been resented by the blind devotees of antiquity. Admiration for H. "Notre siecle a produit un historien tel que ceux de Rome et de la Grece; tant pour la beauté, l'energie, et la clarté du stile, que pour l'admirable verité du tableau historique, et la fidelité des peintures." The first real historian since Tacitus.—Anxious to hear H.'s opinions on French affairs; especially on the Parlements.

IV, 6. [H.]. Dijon, 14 Apr. 1770. *Signed.* [4 pp., 17 × 21·5 cm.; E.P., 276–80.]

Having done his best to serve some Englishmen recommended by H., he now in turn recommends M. Dorgeux to H. Is engaged in collecting the fragments of Sallust, and requests H. to send him the passage in Fitzstephen referred to in his *Hist.*, in order to discover whether the original was still extant in the 12th cent.—Robertson, in his *Charles V*, tries too often to make events fit into his plan. Even H. has sometimes been accused of a prejudice in favour of monarchical government. There are definite advantages in feudal government, in that it provides a hierarchy from king to subjects.—All Europe is awaiting another book from H.

From George-Louis Leclerc, Comte de Buffon.

IV, 7. [H.]. Montbard, 4 Aug. 1774. *Signed.* [2 pp. + 2 pp., 16 × 20 cm.; E.P., 305–6.]

Recommends M. Septchene to H. Offers to send H. the vols. of the *Natural Hist.* that he lacks. Reads H.'s work with great admiration.

From Mme de Buffon.

IV, 8. Hume, Paris. [Paris] "dimanche 24" [1763–5]. *In third person.* [1 p. + 3 pp., 11 × 17·5 cm.]

Invitation to dine.

IV, 9. Hume, Paris. [Paris] 24 Dec. [1765]. *In third person.* [1 p. + 3 pp., 11 × 16 cm.]

Invitation to dine, and to persuade Rousseau to accompany him.

From Archibald Campbell.

IV, 10. Hume. [London] "Thursday, Noon" [?]. *Signed.* [2 pp. + 2 pp., 20 × 32 cm.]

Account of the precariousness of his life aboard ship, depending as it does on the temper of the Captain. Finds it impossible to become purser on a big ship.—His scheme for taxing plays and for dealing with the public debt. —Perhaps, but for misfortunes, there would not have been the same difference in eminence between H. and him. C. has been a martyr to a religion that he does not remember ever having believed.—His life is at a crisis, and he depends largely on H.'s efforts on his behalf.

From Principal George Campbell.

IV, 11. [H.]. Capefield, near Aberdeen, 25 June 1762. *Signed.* [3 pp. + 1 p., 18·5 × 23 cm.; Burton, II, 119–20, inc.]

Thanks H. for his letter. His admiration for H. in spite of differences in opinion. "No religious prejudices (as you would probably term them) can hinder me from doing justice to that goodness and candour, which appear in every line of your letter."—Pleased to have earned H.'s praise in the controversy [on miracles] between them; but regrets the accusation of vehemence, which he defends on the ground of the natural desire for victory inspiring every controversialist without altering his good opinion of his antagonist. Approves of H.'s resolution not to reply, since that might inconveniently prolong the controversy. H.'s attack being general, he is better able than C. to ignore a challenge.

From J. G. Catt.

IV, 12. H. Berne, 24 Nov. 1767. *Signed.* [1½ pp. + 2½ pp., 19 × 23 cm.]

A piteous letter begging H. to attempt to procure for C. a small pension in England to enable him to struggle against death a little longer.

IV, 13. H. Berne, 13 Jan. 1768. *Signed.* [2½ pp. + 1½ pp., 18·5 × 22·5 cm.]

Expresses gratitude for the good hopes held out by H. of his request's being granted.—M. Norton has written to Lord Shelburne as directed by Gen. Conway.

From the Comte de Rohan-Chabot.

IV, 14. [H.]. Samac, 17 May [1764]. *Signed.* [3 pp. + 1 p., 15·5 × 20 cm.]

His pleasure and instruction in reading H.'s works. Hopes a correspondence will be maintained between them. "Nous sommes nés d'un pays different, et d'un âge inégal, mais Monsieur, nous sommes cytoyens du monde, et votre facilité de caractere vous portera a vous rapprocher de ma jeunesse."—Thanks H. for his advice.

IV, 15. [H.]. [?] 17 July [1764 or 1765]. *Signed.* [6 pp. + 2 pp., 15·5 x 20 cm.]

Thanks H. for his letter. Endeavouring to gain a good knowledge of English. Thanks H. for his advice, at a time when a young man most requires it.—An account of his mode of life.

IV, 16. [H.]. Samac, 29 July [1764 or 1765]. *Unsigned. Almost illegible.* [4 pp., 19 x 23 cm.]

Distressed because he has not had a reply from H.—Constantly making humiliating reflections on the amount of time he wastes. Regards H.'s interest in him as a great blessing for a young man of his age. Asks advice for the correction of his faults.—Waiting for H.'s *Hist. of the Stuarts* and *of the Tudors*, which his brother has not yet returned to him. Meanwhile has been reading the history of France.

IV, 17. Hume, Paris. Samac, 24 Aug. [1764 or 1765]. *Signed.* [1 p. + 3 pp., 15·5 x 20 cm.]

Regrets H.'s silence. Hopes he has not offended him in any way. Declares his need for advice.

IV, 18. [H.]. Samac, 5 Jan. 1765. *Unsigned.* [3 pp. + 1 p., 18 x 23·5 cm.]

Cannot help breaking the silence H. has wished for. H. has nothing but trouble to gain from writing to him, but he has everything to gain if H. does so.—Since he last wrote he has travelled a good deal, visiting the coasts of France: Bordeaux, La Rochelle. Going to Versailles in August in the service of M. de Beauvau.—The reports that he has been indulging in dissipation are false. Living a very quiet life in the country.—Has been reading, making extracts from his reading, and arranging them alphabetically.—Has read H.'s *Hist.* almost through and been charmed with it. Its truth and impartiality, "hors une certaine partialité cependant non seulement pardonable, mais indispensable."

From M. de Chanron.

IV, 19. [H.]. Avignon, 22 Aug. 1763. *Signed.* [3 pp. + 1 p., 16 x 20 cm.]

Having met Lady [Wortley] Montague in Paris, de C. had become convinced "qu'elle joignoit à une ame forte, un jugement trés sain qui chassoit bien loin d'elle tous ces miserable prejugés." Has been informed that she died a Roman Catholic. This may have been caused by a weakening of her reason through illness. Would like to know the truth about it. But his chief aim is to express his veneration for H.

From the Chevalier de Chastellux.

IV, 20. Hume, Secretaire d'embassade, Hôtel de Brancas. [Paris, 1764–5.] *Signed.* [1 p. + 3 pp., 15·5 × 20 cm.]

Asking for letters of introduction for a M. Grollé who is going to England; formerly an advocate, now a scholar, who has written in the manner of Swift. G. has written about a voyage to Italy. The name of D. H. is as respectable among men of letters as that of Jehovah among the Hebrews.

IV, 21. [H.]. Paris, 2 Mar. 1766. *Unsigned.* [4 pp., 16 × 20 cm.]

All H.'s friends, who were becoming worried by his silence, are rejoiced at having heard from him again, and at learning the news of Rousseau. Praise of H.'s good heart. Rousseau should not complain that the fools and the wicked exceed in number the enlightened and the honest people in the world; if it were not so, philosophers would have no vocation. Regrets R.'s quarrels with worthy friends in the philosophical world. Philosophers ought to combine against the common enemy, and not fight among themselves.—Affairs in Paris: the Parlement and the King's Declaration, which has greatly pleased the general public and has turned their sympathies away from the Parlement to the Court.—News of recent publications.—News of friends.

IV, 22. [H.]. Choisy, 25 Oct. 1766. *Unsigned.* [4 pp., 16 × 21·5 cm.]

Apologises for not having answered H.'s letter sent by Col. Wedderburn. Assures H. of his tender and faithful attachment. Distressed to hear that H. has settled in Edinburgh. Knows Paris is not the best place to work in. But it is the best to receive the fruits of work in. Wonders whether H. should be encouraged to continue his *Hist.* H. will follow truth as before, but men's passions are excited by recent events, and they do not judge coolly. His chief motive, however, for discouraging H. is his desire to see him again. "*David Hume can do no wrong.* C'est ce qu'on a dit avant de voir les pièces du fameux procès, c'est ce qu'on a repeté après les avoir vües." It seems that the present century wishes to render to H. the same justice as he has rendered to previous centuries. Compares H. to Michael Angelo's or Raphael's St Michael.—Speaking of crabbed and jealous people, has H. seen the correction of Smollett that de C. has inserted in the *Gazette littéraire*?—Their group in Paris often talk of H. The latest recruit to it is Beccaria.

IV, 23. Hume, London. Paris, 27 Mar. 1767. *Unsigned.* [2¼ pp. + 1¾ pp., 14 × 18 cm.]

Has heard nothing of H. for 8 months. H. has not answered his last letter, but is not occupied in public affairs and so has less excuse for his silence. Reference to Rousseau affair. Hopes H. has not given up writing for ever. Compliments from all friends.

IV, 24. Hume, Brewer Street, London. Paris, 8 Jan. 1769. *Unsigned.* [2 pp. + 2 pp., 16 × 19 cm.]

H. is the laziest of all the philosophers. It is quite wrong to regard H. as the oracle of the century, for it is the business of oracles to give answers. Introduces the Duc de Rochefoucault, asks H. to be kind to him, to invite him to tea, to invite him to small dinners, and to introduce him to Pringle, Lord Hertford, Lord Mansfield, etc. Expressions of attachment to H. The death of Mr. Trudaine.

From the Duchesse de Choiseul.

IV, 25. [H.]. [Paris, 1763-5.] *In third person.* [1 p. + 3 pp., 16 × 20 cm.]

Inviting H. to visit her.

IV, 26. Hume. [Paris, 1763-5.] *Unsigned.* [½ p. + 3½ pp., 16 × 20 cm.]

Had hoped to be the first to inform H. of the arrival of Mme de Boufflers. But regrets her loss of memory.

From the Comtesse de Coaslin.

IV, 27. Hume, chez M. l'Ambassadeur d'Angleterre, rue de Bourbon, Paris. Cambray, 11 Aug. 1764. *Signed.* [1½ pp. + 2½ pp., 18 × 23·5 cm.]

Apologises for not having written to H. as she had promised. She is not the author of the " grifonage " he attributes to her, though she slightly suspects Lord Beauchamp.

From Alison, Mrs Cockburn.

IV, 28. Hume, Paris. Castle Hill [Edinburgh], 20 Aug. 1764. *Signed.* [3 pp. + 1 p., 18 × 22·5 cm.; Burton, II, 230-1.]

H. is the victim of a false fame; but for all his unbelief, etc., God has stamped on him the image of benignity, which the labours of his head have not effaced. He is merely the idol of a people naturally too prone to do evil.—Mr Burnet [afterwards Lord Monboddo] is similarly engaged on a quest for truth.—Extracts from a letter concerning Sir John Stewart's death.—*P.S.* Family news, with a toast made by a lady on her son.

IV, 29. Hume. Castle Hill, Edinburgh, 21 Sept. 1765. *Signed.* [2½ pp. + 1½ pp., 18·5 × 23 cm.; E.P., 120-2.]

Laments the tempering of her enjoyment of H.'s friendship by continual applications for introductions to him. Now recommends Mr Scot, at the request of Mr Pringle. Lord Marischal [Keith] was himself partly responsible for the coolness of his reception in Scotland: being old, he sought retirement. H. was out of humour in his last letter. The only part that pleased her was his desire

to return home to Scotland.—Description of her Highland expedition. Desires H. to accompany Hertford [to Ireland] and break the hearts of the French women.—*P.S.* Concerning Mrs Hamilton.—A visit to Balcarres, and the Earl's interest in H.—The Fife elections.

IV, 30. Hume, at Mr Coutts, London. Edinburgh, 1 Feb. [1766]. *Signed.* [3 pp. + 1 p., 18 × 23 cm.; E.P., 123–6, inc.]

Forgives H. for his lack of faith in return for his care of Rousseau. Her heart similar to R.'s in everything but peevishness. The impatience of those who believe in a higher existence, by comparison with those who merely follow reason.—Willing to believe H. would have helped her son, had he been able to.—Glad at the more certain prospect of H.'s return. How is she to entertain him? She can play at quadrille and sleep with him. Begs him to bring R. to Scotland. She and R. will have complete sympathy with each other.—The happiness of her cousin Baird.

IV, 31. Hume, London. Ravelston [near Edinburgh], 1 June [17]67. *Signed.* [1 p. + 3 pp., 18 × 23 cm.]

Recommends Mr Keith to H.—Begs H. to visit Scotland during the summer.

IV, 32. Hume, Brewers Street, London. [? 1767–8.] *Signed.* [3 pp. + 1 p., 19 × 24·5 cm.; E.P., 126–9.]

Congratulates H. on his health and general contentment, which is, however, due more to nature than philosophy. If God had made him, there would not be the present lack of a moving principle.—Seeking a house for H. Does not like the one he suggested. Favours George Street.—The great expansion of Edinburgh. Mentions Scot of Harden's house, and undertakes to find H. a wife. Cannot reconcile him and Lord Monboddo, but will suggest a means for doing so.—Edinburgh society: card-playing and drinking.—Intends a visit to Sir H. Dalrymple and cousin Baird. Requests Bogle's Black Rod pamphlet. Begs H. to be good to Rousseau.—List of H.'s friends.

IV, 33. Hume, James's Court. [?] 21 Mar. 1770. *Unsigned.* [1 p. + 3 pp., 16·5 × 20·5 cm.]

A rumour has reached her on "a far distant shore" that H. is to quit his immortal historic muse for a mortal wife. Warns him against such a marriage; or at least, demands a detailed explanation "in the Boswalian manner."

From the Abbé S. Colbert, V.G. of Toulouse.

IV, 34. Hume, Hôtel de Grimberghen, rue St Dominique. Toulouse, 4 Mar. [1764]. *Signed.* [2 pp. + 2 pp., 18·5 × 23 cm.]

Admiration for [Adam] Smith, who has been recommended to him by H.

Will do his best to make the sojourn of S. and his pupil, the Duke of Buccleugh, agreeable.—Has H. met Mme la Comtesse de Gacé ?

IV, 35. Hume, Paris. Toulouse, 22 Apr. 1764. *Signed.* [2 pp. + 2 pp., 16·5 × 20·5 cm.]

Recommends an officer, Mr Mackay, a captain in the French service, who desires to go to the East Indies. High praise of Smith and the Duke.

IV, 36. [H.]. [Toulouse] 28 Feb. 1765. *Signed.* [3 pp. + 1 p., 15 × 20 cm.]

Surprised at the attitude of the Court towards M. David on account of his treatment of Col. Forester and Gen. du Roure after their death.—"Quand un Protestant meurt en France, il se fait chés lui une descente de Justice ; savoir, le Juge de Police, les gens du Roi et le greffier. Ces officiers dressent un procés verbal de l'etat du corps et les gens du Roi donnent leur conclusions pour qu'il soit permis de l'interrer et le juge rend son ordonnance qui accorde cette permission. L'expedition du Procés verbal ne doit couter que dix sols, le juge n'a rien, mais les gens du Roi se font payer leurs conclusions ce qu'ils veulent parce qu'il n'y [a] aucune loi qui les fixe. Si la taxe est cependant exorbitante on s'en plaint et on la fait reduire." Forester and Roure were exorbitantly taxed, but David, the judge, was not responsible. This event has caused a sensation in Toulouse, because David is a great police judge and the only one who has been able to keep order in the town. If the English Embassy has taken part in the agitation, it should beware of injuring a man who has been of great service to Englishmen.—C. begs to be remembered to his friends in Paris.

IV, 37. [H.]. Toulouse, 10 Apr. 1765. *Signed.* [4 pp., 15·5 × 20 cm.]

C. recognises how contemptible a man David is, as "le premier auteur du meurtre d'un pere infortuné." But in the Forester and Roure affair he was wrongly accused and unjustly punished; at the instigation of the British Embassy, it is currently believed in Toulouse. C. felt himself obliged to protest.—*P.S.* The fanaticism of the people of Toulouse is such that they believe Calas, and not his son, guilty.

From Charles-Marie la Condamine.

IV, 38. [H.]. Paris, 6 Dec. 1765. *Signed.* [4 pp., 16·5 × 22 cm. ; E.P., 293–6.]

Recommends a young Spanish nobleman from America ; recounting his misfortunes, his financial difficulties, and his fatal desire to go to Portugal. Asks H. to dissuade him from his intended course.

From Gen. Henry Seymour Conway.

IV, 39. [H.]. London, 20 Sept. 1765. *Signed.* [1½ pp. + 2½ pp., 18 × 23 cm.; E.P., 265.]

Thanks H. for his personal attention. Anxious to learn the results of the late

French Act concerning the clergy, which at other times would have caused an outbreak of religious fervour.—*P.S.* Interested in H.'s Canada negotiations, and has great hopes of his success.

IV, 40. [H.]. Park Place [Henley-on-Thames], 16 June 1776. *Signed.* [2½ pp. + 1½ pp., 18·5 × 22 cm.; E.P., 266–7.]

Anxiously inquires after H.'s health. Disappointed to find him gone so soon.—Some news about local acquaintances.—Latest news from Quebec seems to show that the Americans do not think their cause worth fighting for.—Good wishes for H.'s health, and a desire to see him again when possible.

From James Coutts.

IV, 41. [H.]. [London, ? 1764.] *Signed.* [4 pp., 18·5 × 23 cm.; E.P., 169–70, inc.]

Has received H.'s quarterly pension from Mr Grenville.—Glad to hear that H. is so happy in France, and in greater favour among the ladies than so old a bachelor deserves. Considers the rumour false that H. has three or four French countesses in keeping.—Regrets H.'s keeping company with Wilkes.—Parliamentary affairs have been interesting, and C. has nearly turned M.P. Recognising, however, his lack of education, he asks for a method of remedying this defect without needing to study. Has a serious intention of visiting Paris.

From John Crawford of Auchenaimes.

IV, 42. Hume, Edinburgh. London, 29 Nov. 1766. *Signed, initials only.* [1 p. + 3 pp., 18 × 22·5 cm.]

[Horace] Walpole is well pleased with H. and himself after receiving H.'s message. C. will not repeat H.'s jokes about certain philosophers, because he cannot find them out. All H.'s friends are awaiting him, and expect him not to spend the winter in Edinburgh.

IV, 43. [H.]. London, 9 Dec. 1766. *Signed.* [4 pp., 18 × 22·5 cm.]

H. has been elected to "a certain very infamous society in Pall Mall" [Almack's].—The settlement between C. and his father, with regard to the estate. C.'s "peevish and sour humour" still remains, but is somewhat justified by an attack of gout.—Regrets the lack of cordiality between H. and Mme du Deffand, who is, he claims, much better than Mme de Boufflers. Is H. kept in Scotland by a former passion of his? But she must now be old and ugly, and C. promises him a young beauty in London, with whom H.'s conversation will have great weight.—Compliments from Lord Ossory.

IV, 44. [H.]. London, 20 Jan. 1767. *Signed, initials only.* [3 pp. + 1 p., 18·5 × 22·5 cm.]

C.'s peevishness is the result of ill-health, which, indifferent at best, is now

worse than ever.—Hopes H. will not settle in Scotland. Believes the prejudice against Scotsmen and men of letters is an illusion. "You are more universally loved by all ranks of people than any man I ever knew, and I never met with any person who could pretend to any degree of taste and sense, who did not look upon your works to be as entertaining and as instructive as those of almost any other author which the world has ever produced." C.'s own opinion, expressed in couplets of Pope.—A letter from Voltaire to the Duc de Choiseul concerning Pompignon.

IV, 45. [H.]. Bath, Wednesday [early 1767]. *Signed, initials only.* [3 pp. + 1 p., 19 × 23 cm.]

His ill-health, which he has begun to bear with resignation.—His concern about Lord Tavistock.—Congratulates H. as Under-Secretary of State. London is a better place for him than Edinburgh. Better society there, and not necessarily consisting of men of letters.

From Crébillon Fils.

IV, 46. Hume, London. [Paris] 23 Nov. 1768. *Signed.* [1 p. + 3 pp., 16 × 19·5 cm.; E.P., 306–7.]

Knowing the tolerance of H.'s philosophy, he is venturing to dedicate a novel to him. He is sending some hundreds of it to London, so that it will appear simultaneously there and in Paris.

From the Comte de Creutz.

IV, 47. [H.]. Madrid, 4 Feb. 1765. *In French. Signed.* [7½ pp. + ½ p., 19 × 23·5 cm.]

The Crown Prince of Sweden, a young man of great promise, has taken great pleasure in reading H.'s works. "Je l'ai vu verser des larmes de sensibilité à la vue d'une verité neuve et hardie qui interessoit l'humanité."—The recent progress in Europe has not been paralleled in Spain, where philosophy languishes.—C.'s impressions of Voltaire, "ce phœnomene de notre ciecle": the brilliance of his conversation; his vision of a world guided by reason, without prejudice and slavery, to be attained in fifty years. His admiration for H., whom he calls his "St David."—H. is thoroughly hated in Spain. The few who possess his works would be burned if the Inquisition had any power. Yet the Duc de Medina Sidonia and the Marquis d'Olavide love him.

From Peter Crocchi.

IV, 48. [H.]. Siena, 27 Sept. 1765. *In English. Signed.* [2 pp. + 2 pp., 18·5 × 23·5 cm.]

Sending an Italian translation of Robertson's *Hist. of Scotland*, Bk. I. C. mentioned H. in the preface because of the friendship between Robertson and H., a strange phenomenon in the literary world.—This letter is to be delivered by

[James] Boswell, " one of the most polite, sensible, and accomplished Gentleman of your Country, and whose company I have enjoy'd for a month during his stay in this Town of Tuscany."

From Sir John Dalrymple.

IV, 49. Hume, Leicester Fields, London. Edinburgh, 1 Aug. 1768. *Signed.* [1 p. + 3 pp., 19 × 30 cm.; E.P., 107.]

Intends to visit Paris to examine the Memoirs of James II, and asks H. for directions. The strangeness of Barillon's conduct.—D.'s brother has offered to raise a company for the East India Co., if he is given the command of it.

From Matthieu Dandalo.

IV, 50. Hume, London. Venice, 2 May 1762. *In French.* *Signed.* [1 p. + 3 pp., 17 × 23·5 cm.]

Sends a copy of the *Political Discourses*, which D. has translated into Italian. Translation of the *Essay on the Populousness of Ancient Nations* is in the press.

From M. de Dangent.

IV, 51. [H.]. Paris, 2 Jan. 1766. *Signed.* [4 pp., 18·5 × 22·5 cm.]

Sends a packet of papers which he begs H. to defer reading till his journey. One of these is a skit on an announcement in a newspaper.—Another is on some questions of economic administration, a subject that interests him very much.—Asks H. to keep his authorship a secret, especially in France.—Wishes Rousseau a good journey and happiness in England.—Asks if H. knows an excellent book called *Essay sur la nature du commerce en général*, traduit de l'anglois, Londres (Paris), 1755. The author is a Mr Cantillon, father of Lady Stafford. Certain calculations have been left out in the French edit. Can H. procure the English edit. for him ?

From Richard Davenport.

IV, 52. [H.]. Wootton, 14 May 1766. *Signed.* [1 p. + 3 pp., 16 × 20 cm.]

Rousseau is in perfect health. " He seems to like the place, amuses himself with walking, when the weather is fair, if raining he plays upon the harpsichord, or writes—he is very sociable and an excellent companion."—Asks H. to call, assuring him of a welcome.

IV, 53. [H.]. Davenport, 23 June 1766. *Signed.* [2 pp. + 2 pp., 18·5 × 23 cm.]

The rains have prevented him from seeing Rousseau. His astonishment at R.'s refusal of the pension. Hopes H.'s letter will have the effect his care for R. deserves. R. is too sensitive. Perhaps he hesitates to accept from one king what he has refused from another. He is busy writing at present.—Success of D.'s plough, making his land six times as good as before.

IV, 54. Hume, Lisle Street, Leicester Fields, London. Davenport, 30 June 1766. *Signed.* [2 pp. + 2 pp., 19 × 24 cm.; Burton, II, 335–6; Greig, App. K., inc.]

Has been prevented so far from going to Wootton. His great uneasiness at the letters he has received from H. and Rousseau. Expects R. will tell him something that will resolve the mystery, when he sees him. Till then it seems "an heap of confusion."

IV, 55. [H.]. Davenport, 6 July 1766. *Signed.* [3 pp. + 1 p., 15·5 × 20 cm.; Burton, II, 336–7; Greig, App. K.]

After a long discussion with Rousseau, the latter has agreed to write an answer to H.'s queries. R. denies ever having refused the pension, but expresses his veneration for the King and his gratitude to Conway.—Extreme sensitiveness of R. He is inspired by jealousy at H.'s friendship with writers whom he considers his enemies.

IV, 56. [H.]. Davenport, 19 July 1766. *Signed.* [1 p. + 3 pp., 16 × 20 cm.; Greig, App. K.]

Fears H.'s judgment on Rousseau is true. If H. will visit him, he will relate some anecdotes of the same stamp as R.'s letter.

IV, 57. Hume, Lisle Street, Leicester Fields, London. Davenport, 29 July 1766. *Signed.* [1½ pp. + 2½ pp., 16 × 20 cm.]

Rousseau's ill-health and depression of spirits.—Directions for finding D.'s house.

IV, 58. Hume, James's Court, Edinburgh. Davenport, 21 Nov. 1766. *Signed.* [1½ pp. + 2½ pp., 16 × 20 cm.]

Regrets H. is having so much trouble with Rousseau, to whom he will communicate what H. desires. D. partly pities R., and wishes it had not been necessary to make the affair public. Cannot see any reason why R. should have refused the pension. Hopes to see H. in town.—Difficulty in getting the plough delivered.

IV, 59. [H.]. Davenport, 4 May 1767. *Signed.* [2 pp. + 2 pp., 16 × 20·5 cm.]

Has not yet seen Rousseau. Is at present confined to his room with gout. If Lord Holdernesse sends someone over, he will give him all the information he can [about D.'s special plough]. R. wrote some time ago telling him that Mlle le Vasseur was beginning to decline.

IV, 60. [H.]. Davenport, 6 May 1767. *Signed.* [1½ pp. + 2½ pp., 16 × 20 cm.]

D. has lost his philosopher [Rousseau]. R. and Mlle le Vasseur have taken

chaise for London. R. left D. as strange a letter as he wrote to H. Actually, R. is uneasy at D.'s maintaining a correspondence with H.

IV, 61. [H.]. Davenport, 13 May 1767. *Signed.* [2 pp. + 2 pp., 16 × 20·5 cm.; Burton, II, 367–8.]

Unable to discover where his wild philosopher has flown. They [Rousseau and Mlle le Vasseur] have taken nothing but what they could carry. D. had been anxious to inform R. about his pension, which had been all arranged. H. will soon be able to see R.'s letters.—*P.S.* Pities R., concluding that " his head is not quite right."

IV, 62. [H.]. Davenport, 18 [May] 1767. *Signed.* [2½ pp. + 1½ pp., 16 × 20 cm.; Burton, II, 368.]

Has just received a melancholy letter from Rousseau, from Spalding. R. apparently desires to return to Wootton. D. directed him to appoint someone to receive his pension, now due.—R. claims to be the most miserable man that ever existed. But his miseries are due to his own temper, which nothing can change. His passion for botany has probably left him, and he will take up his pen again.

IV, 63. [H.]. Davenport, 25 May 1767. *Signed.* [3 pp. + 1 p., 16 × 21 cm.; Burton, II, 368–70.]

Glad Rousseau will receive the pension, and hopes he has shown some gratitude to Conway. Begs H. to attempt to save R. from prison in Paris.—Extract from a letter of R.'s from Spalding, declaring that though liberty is preferable to captivity, captivity at Wootton is preferable to any other form of captivity; and desiring to be allowed to return. Without waiting for a reply, he set off to Dover, and wrote again from there.—D. will hand over what he has of R.'s money to the person authorised to receive his pension.

IV, 64. [H.]. Davenport, 11 June 1767. *Signed.* [2 pp. + 2 pp., 16 × 21 cm.]

The man sent by Lord Holdernesse has a good knowledge of agriculture, and approves of the plough.—The amazing letter written by Rousseau to Conway must have been written while D. was in London, for R.'s pen was otherwise engaged when D. left. D. suspects that R. has gone towards Orleans, where his gouvernante has inherited some small property. R. has left three trunks at Wootton, and may perhaps return.

IV, 65. [H.]. Davenport, 4 July 1767. *Signed.* [1 p. + 3 pp., 16 × 20 cm.; Burton, II, 370.]

Has received a letter from Rousseau, dated from Fleury, in which he thanks D. for his hospitality and promises to inform him when he is settled. The style is calm in comparison with the wild talk of captivity in the earlier letters.

IV, 66. Hume, at Miss Elliot's, Brewer Street, Westminster. Davenport, 6 July 1767. *Signed.* [2 pp. + 2 pp., 19 × 24 cm.; Burton, II, 370–1.]

Has given directions to Rousseau through M. Rougemont about receiving the pension. The old woman that Mlle le Vasseur quarrelled with was D.'s nurse. R.'s gouvernante has a great influence over his actions. R. probably repents of his quarrel with H. and has arranged to communicate with H. At present he is occupied with a work that will fill 12 vols.—H. can see R.'s letter, but a copy cannot reasonably be made.—D. is ready to give anyone instruction in the use of the levelling plough.

From J. Davidson.

IV, 67. Hume. [Edinburgh] Thursday morning [early 1763]. *In third person.* [1½ pp. + 2½ pp., 18·5 × 22·5 cm.; Burton, II, 168–9.]

Enclosing copy of part of a letter from Andrew Stuart to Wm. Johnstone, dated from Paris, 16 Dec. 1762:—H. is worshipped at Paris, and should go there. The amiability of M. and Mme Helvétius. Stuart's reputation largely depends on a prophecy of H.'s coming to Paris, which he begs H. to execute.

From the Marquise du Deffand.

IV, 68. Hume, chez M. l'Ambassadeur d'Angleterre, Compiègne. Paris, "ce mardy" [? 7 Aug. 1764]. *Unsigned.* [1 p. + 3 pp., 16 × 20 cm.; E.P., 178 (wrongly ascribed there to Mlle de Lespinasse); *Mod. Lang. Review*, Oct. 1929.]

"L'objet de vos amours, la charmante Néolé [Mme de Boufflers] vous ordonne, vous commande avec sa petitte voix flutée, de souper chés moi samedy onze de ce mois."—Mme du D. will then tell him all the good and all the ill she thinks of him.

IV, 69. Hume. [Paris, 1763–4.] *Unsigned.* [1 p. + 3 pp., 10·5 × 16·5 cm.; *Mod. Lang. Review*, Oct. 1929.]

Inviting H. to supper.

IV, 70–1. Hume. [Paris] "ce jeudy" [1763–4]. *Unsigned.* [*Two documents as under:*—]

A. [From Mme du D.—½ p. + 1½ pp., 15·5 × 20 cm.; *Mod. Lang. Review*, Oct. 1929.]

Has heard through the Chevalier Macdonald of H.'s adventure at the Duchesse de Choiseul's. Has written to her, and encloses her reply.

B. [From the Duchesse de Choiseul to Mme du D.—1 p. + 3 pp., 10 × 15 cm.]

Regretting that she had forgotten to warn her Swiss that she was *at home* to H.

IV, 72. Hume, Lisle Street, Leicester Fields, London. Paris, 13 Aug. [1766]. *Unsigned.* [2½ pp. + 1⅔ pp., 16 × 20 cm.; E.P., 199–200 (wrongly ascribed there to Mlle de Lespinasse); *Mod. Lang. Review*, Oct. 1929; Greig, App. K.]

Lamenting her disappointment at finding H. was not the friend to her that she had thought him. Rousseau's actions do not surprise her, though she regrets that H. has been his dupe. She is awaiting the day when nothing will separate her from H.—Refuses to put at rest H.'s jealousy of [John] Crawford.

From George Dennison.

IV, 73. Hume, Edinburgh. Kirkwall, 7 Jan. 1775. *Signed.* [1 p. + 1 p., 19 × 30 cm.]

Asks H. whether he has "the hope of Heaven, and immortal life," and of the resurrection of the body, which will give joy and calm to the soul.—Sends an Address to the King, on which he first of all desires to have H.'s opinion.

From George Deyverdun.

IV, 74. [H.]. London, Denmark Street, Soho Square, 18 Nov. 1766. *In French. Signed.* [4 pp., 18·5 × 23 cm.; E.P., 297–9.]

Explains to H. a difficult point in the Rousseau affair. D. wrote two letters to the *St James's Chronicle* against R. Was surprised to find R. accusing Walpole of having written the first, and stating that the second had been written from materials supplied by H. Truth is, D. has never met H., much as he covets the honour. Prepared to take any steps H. suggests. Would prefer not to make his name public, but will give way if necessary.

From Sir Alexander Dick of Prestonfield.

IV, 75. [H.]. Prestonfield, 27 July 1764. *Signed.* [4 pp., 18·5 × 23 cm.; E.P., 104–6.]

Invites H., when possible, to join a tour in France projected by Mr Sargent and some ladies, and asks him to give them what aid he can. Civilities are plentiful in France, but some *je ne scais quoy* is needed to begin with.—Local news. A visit to the Lord Privy Seal, from whom he has received support in his project of building a new road. Lockhart is to propose the demolition of Scottish entails.

From the Rev. David Dickson.

IV, 76. [H.]. Newlands, 20 May 1767. *Signed.* [3¼ pp. + ¾ p., 18·5 × 23 cm.]

As the King's Address to the General Assembly is to pass through H.'s hands, D. ventures to transmit to him a singular case. The letter from the Throne in 1763 recommended great attention to the stamping out of vice and infidelity. This was superceded by a warrant to proceed against the "Reverend malefactors."

Yet the non-jurors have still maintained their independence. The last case, at Jedburgh, was not proceeded with, being tried before Lord Kames.

From Denis Diderot.

IV, 77. [H.]. [Paris, ? 1763-70.] *Unsigned.* [1 p. + 3 pp., 9·5 × 16 cm.]

Glad to send on a letter [from H.] which will do good. Thanks for Maty's letter.

[On the two blank pages H. has written alternative versions of a quatrain :—]

Approach ; no farther peril dread, /Peril
My bow unbent, my darts conceal'd, /The /the
Pleas'd with the conquest I had made,
Triumphant now I quit the field.

IV, 78. [H.]. Paris, 22 Feb. 1768. *Signed.* [4 pp., 11·5 × 19 cm. ; E.P., 282-6.]

Has had an illness, beginning with gout and resulting in a kind of deafness.—Thanks H. for his kindness to M. and Mme de Neufville ; especially recommends the latter. Deprecates national exclusiveness. " Je me flatte d'etre comme vous citoyen de la grande ville du monde."—Begs H. to return to work again. His own love of work has been disturbed by distractions. The dangers of fanaticism at bay.—Rousseau's *Confessions* : the longer it is, the less he will speak about himself and the better it will be. It is dangerous when a man like R. writes confessions.

IV, 79. Hume, London. . Paris, 17 Mar. 1769. *Signed.* [2 pp. + 2 pp., 16 × 20 cm. ; E.P., 286-7.]

Everyone is anxious to see H.'s round and smiling face, especially a young Pennsylvanian whom D. recommends to H. Everything is in a muddle. D. bemoans the fate of philosophers. " Nous prechons la sagesse a des sourds." Still far from the era of reason, which a contemporary Machiavellian sect maintains will never come.—Begs H. to prevent the young American from injuring himself by his medical experiments ; and to shun idleness himself.

IV, 80. [H.]. [Paris] 24 Nov. [1766-9]. *Signed.* [1 p. + 3 pp., 16·5 × 21 cm. ; E.P., 280-1.]

Recommends a friend of Mme D.'s, the more so as she is unfortunate. After all, one is better pleased with a good action done than a fine page written.

From Rev. John Douglas, afterwards Bp. of Salisbury.

IV, 81. [H.]. London, 18 Nov. 1760. *Signed.* [3 pp. + 1 p., 18·5 × 23 cm. ; E.P., 16-9.]

Concerning Lord Glamorgan's letter, which Mr Powney asks H. not to print till the whole collection [of the Clarendon Papers] is published.—Marquis of

Ormond is not mentioned in the letter.—Charles I's aims as revealed in the letter. —Advises H. to keep back his new edit. till D. has published his various curious letters, to which he hopes to add those possessed by the Duke of Queensberry. He has been definitely promised some of Gen. Monk's.—Perhaps some light may be thrown on the *Icon Basilike* by a letter of Charles I's to the Prince of Wales. No proof of Charles I's being a Catholic.

IV, 82. [H.]. London, 25 June 1765. *Signed.* [2 pp., 19 × 23·5 cm.; E.P., 19–20.]

Recommends Mr Grandley, a friend of Sir George Saville. Regrets Millar's illness. Mallet's widow is returning to France. D. rejoices at the rumour that H. is to succeed Sir Charles Bunbury.—*P.S.* Clarendon State Papers are in the press, and will probably be ready in time for H.'s new edit.

From Mme le Page Duboccage.

IV, 83. Hume, Secretaire de l'Ambassade d'Angleterre. Paris, 27 Sept. 1764. *Signed.* [2 pp. + 2 pp., 15·5 × 19·5 cm.; E.P., 300–1.]

Sends a collection of her works to H. Asks him to help her to find Mr Wedderburn, who has been recommended to her.

IV, 84. [H.]. [? , ?]. *Signed.* [2 pp. + 2 pp., 15·5 × 20 cm.; E.P., 301.]

Recommends two members of the Academy at Lyons who are anxious to meet H.

From Mme Dupré de St Maur.

IV, 85. Hume, Edinburgh. Paris, 15 Dec. 1757. *Signed.* [1½ pp. + 2½ pp., 16 × 20 cm.; Greig, App. C.]

Admiration for H. She represents the ignorant, the most numerous class of readers. Hopes H. will visit Paris when peace is concluded.

IV, 86. Hume, Lisle Street, Leicester Fields, London. Paris, 16 May 1759. *Signed.* [2 pp. + 2 pp., 16 × 20 cm.]

Gratitude for favours received from H. The joy with which M. de Montigny received H.'s approbation of de M.'s translation [of the *Nat. Hist. of Religion*].—H. need fear no troubles in France. He will receive nothing but homage.—*P.S.* Saw Mr Stuard [John Stewart] a short while ago, but his business took him away.

IV, 87. Hume, Hôtel de Grimberg, rue St Dominique, Paris. Chatillon, Wednesday, 4 Jan. [1764]. *Unsigned.* [1 p. + 3 pp., 14·5 × 18 cm.]

Invites H. to dinner, to meet Turgot.

IV, 88. Hume, Hôtel de Lauragais, Paris. Chatillon, "ce vendredi" [late autumn, 1765]. *Unsigned.* [½ p. + 3½ pp., 12 × 17·5 cm.]

Asks for news of H.

IV, 89. [H.]. Paris, 6 Mar. 1766. *Signed.* [4½ pp. + 1½ pp., 14 × 18·5 cm.]

H.'s absence has not yet chilled the enthusiasm of his friends, but he ought not to rely too much on their constancy.—Glad at the efforts H. is making on behalf of Rousseau. Love of fame, which R. shows, should be humoured when it injures no one. R. will be happy in his retreat, but must not cease to write.—News from Paris: "Le Roy est venu a Paris sans s'etre fait annoncér pour y presider a une assemblee des Chambres du parlement, il y a parlé avec une dignité et une fermeté tres imposantes, depuis ce jour, on s'est souvent assemblé sans rien resoudre, enfin on a envoyé ce soir les gens du Roy lui demander le jour ou il permettroit au parlement entier de se presenter devant lui, je ne scais ce qu'ils diront mais il me semble qu'il ne leur reste d'autre parti a prendre que celui de la soumission, le discours du Roy a beaucoup de succes dans le public, une plus longue resistance de la part du Parlement ne seroit pas bien receue, il seroit cependant bien fâcheux que l'usage des commissions devint frequent."—Funeral oration on the Dauphin by the Abbé de Boesmont.—Lord Forbes's actions at Paris.—All H.'s friends are anxious for him to return.

IV, 90. [H.]. Montigny, 20 Aug. 1766. *Signed.* [5 pp. + 1 p., 14 × 18·5 cm.; Greig, App. K.]

The interest aroused in Paris by the quarrel with Rousseau. Everyone sides with H., who should write or print nothing, but instead rejoice that in losing R. he has lost but little.—*P.S.* The only reasonable course is for H. to place the correspondence in the hands of some prudent friends.

IV, 91. Hume. Paris, 28 Feb. 1767. *Signed.* [2¼ pp. + 1¾ pp., 14 × 18·5 cm.]

Regrets that H. is not likely to return to France, and that he has not replied to her letter.

IV, 92. [H.]. Paris, 6 Mar. 1768. *Signed.* [2½ pp. + 1½ pp., 16 × 19·5 cm.]

Introduces M. d'Anjeule [?]. Recalls to H. all his French friends. Urges him to visit France again.

IV, 93. [H.]. Montigny, 3 July [?]. *Signed.* [1½ pp. + 2½ pp., 15·5 × 20 cm.]

Assures H. of the goodwill of his Paris friends.

IV, 94. [H.]. Paris, 17 Apr. [1769]. *Unsigned.* [3½ pp. + ½ p., 14 × 18·5 cm.]

Thanking H. for his sympathy on the death of her friend [M. Trudaine], whose patient suffering on his death-bed she describes.

IV, 95. Hume, Edinburgh. Paris, 4 Sept. [1775]. *Signed.* [3 pp. + 1 p., 15·5 × 20 cm.]

Urges H. to return to France. Announces the death of her husband. "Il avoit 80 ans, et nulle incommodité, il a joüi de la vie jusqu'au bout, et travailla encore 10 heures la veille de son accident, aussi a-t-il laisse 27 vol. in follio ecrits de sa main que j'ai remis a la bibliotheque du Roy."—The rest of Mme D.'s family. H.'s nephew has visited H.'s friends in Paris. "Que n'avez vous eté temoin de leur tressaillement de joye lorsque l'on annonça Mr Hume que l'on crut etre vous." Impression made on her by H.'s nephew.

From Colonel James Edmonstoune of Newton.

V, 2. [H.]. Geneva, 30 Dec. 1763. *Unsigned.* [1 p. + 1 p., 18 × 22 cm.]

Asks H. what he is doing at Paris. Some of the rumours E. has heard concerning H.

V, 3. [H.]. Geneva, 26 Mar. [1764]. *Signed, initials only.* [4 pp., 19 × 23 cm.; Burton, II, 185–6; Greig, App. C.]

Uncertainty of his own future. Consults H. on the character to be assumed by a Mr Vivian, a clergyman who has imbibed some of H.'s ideas and is "very very low Church." Can such a man, without pricks of conscience, accept an ecclesiastical appointment? The difficulty of gaining a political post.—Hopes to hear of H.'s appointment as Secretary to the Embassy, though he fears delays.—Compliments from Lord Mountstuart, who has taken pleasure in reading H.'s *Hist.* Asks about this rival Macaulay *Hist.*—Wishes H. could come to Geneva. The friends he would find there: M. and Mme Tronchin. Suggests that H. could do something to help Rousseau; *e.g.* print his works in England.

V, 4. Hume, Paris. Geneva, 13 June 1764. *Signed.* [1½ pp. + 2½ pp., 19 × 23·5 cm.]

Begs H. to send M. d'Eon's letters to M. Tronchin, and to write a page telling the latter how pleased he is to have the chance of obliging him. T. is a great friend to E. and H. Encloses two of T.'s performances.—E. is setting out for Turin.

V, 5. Hume, New Town, Edinburgh. Newtoun, Stirling, 17 Mar. 1773. *Signed.* [1 p. + 3 pp., 19 × 23 cm.]

Inability to come to Edinburgh owing to his bad health. No recollection of having sent a Plautus to H. Sends a salmon.

V, 6. Hume, New Town, Edinburgh. Newtoun, 12 Apr. 1775. *Signed.* [2 pp. + 2 pp., 18·5 × 23 cm.; Burton, II, 474, inc.; Greig, App. C.]

H. is an obstinate creature. E. defends his own house against H.'s aspersions

on it.—It is paradoxical to hear of H., who is reputed a *contemptor divum*, talk of keeping vows. An anecdote of Pope Benedict XIV. Invites H. to come on a visit along with Baron Mure.

V, 7. Hume, New Town, Edinburgh. Linlithgow, Wednesday [7 Aug. 1776]. *Signed, initials only.* [1¼ pp. + 2¾ pp., 16 × 18·5 cm.; Burton, II, 510, inc.]

A touching note bidding H. farewell.

From Patrick, Lord Elibank.

V, 8. Hume, Ninewells. [? Edinburgh] 11 May 1763. *Signed.* [1¼ pp. + 2¾ pp., 18·5 × 23 cm.; E.P., 44–5.]

Mme de Boufflers has come to England specially to meet H., who can hardly fail to gratify "this flattering curiosity, perhaps passion, of the most aimiable of God's creation." He promised to write from Paris, but found he would be in England as soon as his letter.—"No author ever yet attained to that degree of reputation in his own lifetime that you are now in possession of at Paris."

V, 9. [H.]. [? London, 1764.] *Signed.* [2 pp. + 2 pp., 18·5 × 23 cm.; E.P., 45–6.]

Anxious to desert England and politics for Paris.—Effect of the Dutch bankruptcies on the stocks; dealers must keep larger sums of money by them.—Begs H. to perform a piece of business for him.

V, 10. Hume, Paris, readdressed to Compiègne. Balancrief, 9 July 1765. *Signed.* [2 pp. + 2 pp., 18 × 22 cm.; Burton, II, 260; Greig, App. F.]

Deplores what he considered to be coldness in H.'s letter. The value he sets on H.'s friendship. H. should not have suspected him of siding with his brother [Alexander Murray] against a friend of thirty years' standing. Has always perceived the greatness of H.'s talents, and endeavoured to arouse a similar admiration in others.

From Sir Gilbert Elliot of Minto.

V, 11. [H.]. London, 14 July 1762. *Signed.* [4 pp., 18·5 × 22·5 cm.; E.P., 38–9, inc.]

Will discover whether Rousseau is in London or not. If so, will show him every attention possible. Doubts if a pension could be procured.—E.'s admiration for R. Criticism of *Emile* and of R.'s work in general.

V, 12. [H.]. London, 25 Apr. 1764. *Signed.* [4 pp., 18 × 22·5 cm.; Burton, II, 278, inc.; Greig, App. C.]

The question of the Secretaryship to the Embassy. Lord Hertford is doing the proper thing. H. should not be despondent about his success. E. has used

what little interest he has, but Lord H. has been beforehand with him. H. need have no fear; he shall have at least Christian burial, and perhaps a niche in Westminster Abbey. Yet admonitions are useless; H. has been rendered vain by the attentions of the French ladies, and has shown that posthumous fame alone will not satisfy him.

V, 13. [H.]. Brussels, 15 Sept. 1764. *Signed.* [4 pp., 16·5 × 22 cm.; Burton, II, 233–5.]

Made a sudden departure from Paris. Will send his boys to H., if any suitable school can be found, and hopes to have them back with their own native simplicity and some of the graces of Parisian society. Desires them to be cleared of prejudices, but feels strongly that love of one's own country is one of the greatest virtues. H. is on the very brink of a precipice.—Delight at the gentle manners of the French.

V, 14. [H.]. Minto, 19 Oct. 1764. *Signed.* [2½ pp. + 1½ pp., 18 × 23 cm.; Burton, II, 245.]

Thanks H. for taking such trouble about his boys. They are leaving for London in a day or two.—E. is standing for Roxburghshire and feels confident of success. Has sent out his letters, and is eating county feasts, which compare very unfavourably with those of Paris.

V, 15. [H.]. Minto, 21 Oct. 1764. *Signed.* *Torn.* [2½ pp. + 1½ pp., 18 × 23 cm.; E.P., 40–1.]

Letter to be delivered by [Robert] Liston, his sons' tutor. H.'s long letter has not yet arrived. Plans for the boys in Paris: study of Latin and French; visits to Mme Geoffrin, etc., and occasionally to the French comedy.

V, 16. [H.]. Edinburgh, 6 Nov. 1764. *Signed.* [4 pp., 18 × 23 cm.; Burton, II, 239–40 and 245.]

All H.'s letters have at last arrived.—The Secretaryship would naturally fall to H. Difficulty lies in finding an equivalent post for Bunbury. H. must not, like Rousseau, allow excessive independence to prevent him from making solicitations, by which alone political preferment can be secured. The present prejudice against Scotland, which to some extent hampers both E. and H., may give way to some other, and H. become a bishop and E. a minister.

V, 17. Hume, Paris. London, 25 Mar. 1765. *Signed, initials only.* [3 pp. + 1 p., 18·5 × 24 cm.]

Knowing H.'s numerous engagements and correspondents, has not troubled him with many letters. Asks H.'s advice about Liston and the boys, and wishes to know the effect of Paris life on them.—Grenville will read the Budget

to-morrow. Harrison is to receive the reward for discovering the longitude, and Dr Meyer for improvements anticipated by Newton. H.'s success in the negotiations about the French prisoners.

V, 18. [H.]. Minto, 11 July 1768. *Signed.* [2 pp. + 2 pp., 18 × 22·5 cm.; Burton, II, 415–6.]

James Balfour is said to be the author of the book H. referred to. Gilbert Stuart is impatient to receive H.'s letter. He is an admirer of H., though E. has taken care lest he carry this admiration too far in some points.—E. has no correspondents, fearing to correspond with any of the Ministry.—Expensiveness of farming. A day's wage is now 1s.—Hopes H.'s love affair and his King William flourish.

V, 19. [H.]. [? , ? .] *Signed.* [1 p. + 3 pp., 18·5 × 23 cm.]

Rumours of H.'s ill-health have reached him through Lord and Lady Hertford. Begs H. to contradict them.—East India affairs the topic of business. Contemporary literature is enough to occupy everyone for two years to come.

From M. d'Ennesy.

V, 20. Hume, London. Fort royal, Martinique, 20 June 1766. *Signed.* [1 p. + 3 pp., 16 × 20·5 cm.]

Has received H.'s letter recommending the interests of Mr Stewart, as represented by Messrs Nelson and Burnet. Will do all he can for them.

From Sir Harry Erskine of Alva.

V, 21. Hume. Spa, 14 Sept. 1764. *Signed.* [6 pp., 19 × 23 cm.; E.P., 164–7.]

Hopes H. will get the post he deserves [the Secretaryship of the Embassy], and asks to be informed if there is a chance of Lord Hertford's proving successful. E. himself can do absolutely nothing.—Has been driven to Spa by two military fevers, but is now much recovered. His sons will be future secretaries of embassies. H. and he have discovered that brains are not needed for politics.—Very pleased to hear of H.'s conversion, though Lady E. has some doubts about it. Offers to entrust a son to be educated by H., but he must hold no converse with Wilkes.

From Fenouillot de Falbaire.

V, 22. [H.]. Paris, 18 Nov. 1767. *Signed.* [2 pp. + 2 pp., 18·5 × 23·5 cm.]

Sends H. a play written by F. on the Calas case. It cannot be performed in Paris, because they still tolerate certain very intolerant priests there. Hopes to have it performed in England with the help of H. and Garrick. A pleasure to work for a nation he so much admires.

From the Chevalier de Fauseuil.

V, 23. Hume, Edinburgh. Oxford, 18 Dec. 1766. *Signed.* [3 pp. + 1 p., 18 × 26 cm. (1 p. cut).]

H. has acted like a real philosopher in his dealings with Rousseau, lavishing his kindness on the ungrateful and, like Diogenes, seeking an honest man. F. had given up the search and had retired from the world, when unexpectedly he found what he wanted in H. H., too, may find an honest man if he continues the search. He has at least an altar in F.'s heart.

From Professor Adam Ferguson.

V, 24. Hume. Edinburgh, 26 Nov. 1763. *Signed.* [4 pp., 18·5 × 23 cm.; Burton, II, 175–6, inc.]

Account of H.'s nephew Joseph's education. Question of his proceeding to Greek and completing his Latin studies. His daily programme: *e.g.*, in the evening reading amusing books like his uncle's *Hist.*—H.'s reception in Paris. F. hopes H. will end his days in Scotland.—Praise of Lord Marischal [Keith]. The conversion of the Lord Marischal's Mohammedan girl, Ahmet Ulla.

V, 25. [H.]. Edinburgh, 17 Apr. 1767. *Signed.* [3 pp. + 1 p., 18 × 22·5 cm.]

Begs H. to support the petition of his colleagues. Pleased that the last representation in favour of [John] Home is likely to prove successful.—Thanks H. for the review and the author for his kindness. Gratitude to Mrs Montagu for her notice of him. Believes, however, that among the whole list of creation no being acts his part or fulfils his destiny better than *man.* Hopes H. will argue in this way when he sees Mrs Montagu. Knows that both she and H. are against him in preferring the Athenians to the Spartans.

V, 26. Hume, St Andrews Square, Edinburgh. Geneva, 6 June 1774. *Unsigned. Slightly torn.* [3 pp. + 1 p., 18·5 × 23 cm.]

Hopes to draw his money more regularly and repay H. what he owes him. His journey pleasant, though not adventurous. A month's tour in Switzerland. Strangeness of the Swiss Constitution. The character of his pupil [Lord Chesterfield].—At present at Calvin's former house, and in view of Voltaire's castle. Voltaire is "worn to a shadow but has all his Vivacity and his Genius intire." Voltaire's correspondence with Lord Chesterfield; a dispute with a lady on the Trinity; and a philippic against France.

From M. Fleury.

V, 27. [H.]. [Paris] 11 Sept. 1764. *Signed.* [3 pp. + 1 p., 18·5 × 23 cm.]

Presents H. with a copy of his new books. If they obtain H.'s approval, they will be worthy of that of the whole English nation.—*P.S.* On some apartments in F.'s house which would suit the gentlemen referred to [Elliot's sons].

From Thomas Forrest.

V, 28. Hume, Brewer's Street [London]. Strand, London [?]. *Signed.* [3 pp. + 1 p., 18·5 × 23 cm.]

Anxiety to write to H. and to converse with him. Though H. states that the Italian language is the softest of all, F. declares Malay to be softer ; and is thereby led to mention something of his own life in the East. Malayans call the sun *Eye of the Day.*—Admiration for H. and desire for his acquaintance.

From Benjamin Franklin.

V, 29. Hume. Coventry, 27 Sept. 1760. *Signed.* [3 pp. + 1 p., 18 × 23 cm. ; Franklin, *Works*, ed. Sparks, Boston, 1840, VII, 208 ff., inc.]

Thanks H. for his approval of some pieces sent to him, though F. was hardly at all responsible for the one on Pennsylvania.—Lack of earnestness shown towards the expedition by all except the Duke of Bedford. The Army being left idle at Albany. Importance for England of an understanding of the Colonies. —Excellence of H.'s essay on *Jealousy of Trade.* Promotes the interest of humanity. But F. hopes, in particular, that the essay may result in an abatement of the jealousy of Colonial trade.—Accepts the condemnation of words *pejorate* and *colonize* as not in common use, and of *unshakeable* as low. Regrets the inability of the English language to form compound words. Hopes " we shall always in America make the best English of this Island our Standard, and I believe it will be so." Increased public in America for English works.

V, 30. Hume, Edinburgh. London, 19 May 1762. *Signed.* [3 pp. + 1 p., 20 × 32 cm. ; Franklin, *Works*, ed. Sparks, VII, 236 ff.]

Pleased to hear that his paper on preserving buildings from lightning was accepted by the Philosophical Society [of Edinburgh]. Discusses the use of a leaden spout as a conductor. His experiment in sending an electrical charge into a glass tube filled with water.—Lord Marischal's position as president in a theological dispute, with an illustrative anecdote concerning the erection of a maypole.—F. thanks H. for the compliment about his wisdom. He will carry it where there is less wisdom than in England. His regret at leaving England.

V, 31. Hume. London, 21 July 1762. *Unsigned and incomplete.* [2 pp., 20 × 32 cm.]

Directions about the best way to preserve a house from lightning. Its success in America.

From S. Fraser.

V, 32. [H.]. George Street, Hanover Square, London, 27 Nov. 1775. *Signed.* [1½ pp. + 2½ pp., 18 × 22·5 cm.]

Had been regretting the troubles of patronage, when he was overjoyed to

receive from H. a request that he could grant. Expects to be able to give the lieutenancy as H. desires.

From Thomas Rockwood Gage.

V, 33. [H.]. Coldham Hall, 26 June 1764. *Signed.* [4 pp., 18 × 23·5 cm.].

Thanks H. for his letter and invites him to Coldham Hall on his return to England. Compliments from his brother and Mr Dennett.—His action about his debt on the Jesuits. Account of the papers sent to his attorney in Paris. Regrets the latter's dilatoriness, and invites H. to inspect the papers and to take them over when the attorney is finished with them.

V, 34. Hume, Paris. London, 16 Aug. 1765. *Unsigned.* [2¼ pp. + 1¾ pp., 18·5 × 22·5 cm.]

Thanks H. for attending to his affair. Asks advice about allowing credit to M. Bertan. Invites H. to Coldham on his return to England.

From Mme Geoffrin.

V, 35. Hume, Compiègne. [Paris] "ce mercredy" [1763–5]. *Unsigned.* [1 p. + 3 pp., 14·5 × 19 cm. ; E.P., 289.]

Her anxiety to please H. and her pleasure at his regard for her. Hopes he will return from Compiègne "aussi coquin" as when he set out.

V, 36. Hume, chez M. l'Ambassadeur d'Angleterre. [Paris] "ce samedy matin" [16 Mar. 1765]. *Unsigned.* [1 p. + 3 pp., 16 × 22 cm. ; Burton, II, 211, inc. (facsimile) ; E.P., 287–8 ; Greig, App. C.]

Upbraids him for having sent her so sumptuously bound an edit. of his works, which has put all her other books to shame.

V, 37. Hume, London. [Paris] 1 Feb. 1766. *Unsigned.* [½ p. + 3½ pp., 14·5 × 19 cm. ; E.P., 288–9.]

Cannot allow Col. Gordon to depart without taking along with him a note "pour mon gros Drole," to express her love for him and impatience to see him.

V, 38. Hume, London. Paris, 17 Apr. [1766]. *Unsigned.* [1 p. + 3 pp., 15 × 19 cm. ; E.P., 290–1.]

H. has turned himself into a "parfait petit maitre" by acting "le Beau Rigoureux" in not answering one of her love letters. His excuse of not knowing enough French will not do for her. She anxiously awaits his return.

V, 39. Hume. Paris, 25 Sept. 1767. *Signed.* [1 p. + 3 pp., 15·5 × 19·5 cm. ; E.P., 290.]

Thanks H. for making her acquainted with Pringle and Franklin ; though

their stay was short.—H. has bought a house in spite of his promises to return to France. " Je voudrois bien pouvoir vous oublier, mais je ne puis."

From the Abbé Georgel.

V, 40. Hume, Hôtel de l'Ambassadeur d'Angleterre, rue St Dominique. [Paris] " samedy 14 " [Jan. 1764]. *In third person.* [1 p. + 3 pp., 10 x 15·5 cm. ; Burton, II, 221.]

Invitation to H. to dine with Prince Louis de Rohan.

From Edward Gibbon.

V, 41. [H.]. Brereton, 4 Oct. 1767. *Signed.* [1½ pp. + 2½ pp., 18·5 x 23 cm. Burton, II, 410–1.]

Having determined to write the history of the Swiss, he was at first compelled to put the idea aside on account of his ignorance of German. By the aid of a translator this difficulty was removed. Begs H. to glance through the pages already written. At H.'s word (though at no other man's) he is prepared to burn them.

From John Grant.

V, 42. Hume. [?] 13 Mar. 1777. *Signed.* [1 p. + 1 p., 20 x 29 cm.]

Recognising that H. is ill and little likely to recover, he begs to be remembered in H.'s will, in order to compensate him [for some transaction unmentioned].

From Fulke Greville.

V, 43. Hume, Leicester Fields, London. Wilbury, near Andover, 10 June 1759. *Signed.* [2½ pp. + 1½ pp., 18·5 x 23 cm.]

Disappointed that he is not to have the chance of the " rare . . . felicity of freely Philosophising among my Shrubs & Coppices, with one who knows & loves Truth, and was willing to describe her Charms."—Therefore begs H. to visit him, if H. can leave London. H. alone is necessary to make Wilbury completely delightful.

V, 44. Hume, Paris. Spa, 6 Sept. 1764. *Signed, initials only. Torn.* [3 pp. + 1 p., 19 x 23·5 cm.]

Was the author of " a brochure, which reflected upon some political and some poetical questions," shown to H. by Dr Burney. Begs H. to send him some censure or approbation that can be relied on. As the brochure was printed anonymously and by an obscure bookseller, it has not met with the same success as it did in MS. Asks H.'s candid opinion of it.—*P.S.* Hears that H. does not greatly admire Churchill ; hopes, therefore, he will not accept the opinions given in the pamphlet merely out of politeness.

V, 45. [H.]. Brussels, 24 Sept. 1764. *Signed.* [4 pp., 19 × 23 cm.]

Would have liked to have had H.'s remarks on the pamphlet.—Feels compelled to say something against Churchill, "the professd Satyrist of the age, and the first Subject of Satyre in it." Agrees with H.'s censure of him.—French ideals are as restrained as the English are unrestrained; and so Voltaire's criticism of Shakespeare is partly just, partly excessive. Limited outlook of V. and the French generally.—*P.S.* Amiability of Mme Belot, who is said to be translating some of H.'s works.

From Mrs Frances Greville.

V, 46. [H.]. Wilbury, 11 Jan. 1768. *Signed.* [2 pp. + 2 pp., 18·5 × 24 cm.]

Begs H. to forgive a lady for writing to him merely because her "gazzeteer," Sir G. Macartney, is occupied in ministerial and matrimonial negotiations. Macartney claims to be completely happy. Asks H. to act as her intelligencer.—Recommends Mr Mortimer, Vice-Consul at Ostend. Mortimer must not be judged on his *Hist. of England*, which he wrote merely for bread.—Begs H. to spare her some of those qualities that make solitude so delightful to him.—A message to Mr Crawford.

V, 47. Hume, Brewer Street, London. Wilbury, 6 Feb. 1768. *Signed.* [1½ pp. + 2½ pp., 18·5 × 23 cm.]

H.'s being now a fallen minister is the reason why she has not written; she wished to save him the postage on her letters.—Thanks him for news. He has omitted to mention the destination of George Pitt, whom she advises to go to Turin rather than Germany.—Asks H. to continue his *Hist.*, and to bring it down to George I's time. Does he believe Commodore Byron's claim to have discovered a race of giants called Patagonians?

From the Comte de Guines, French Ambassador to England.

V, 48. [H., Edinburgh.] Inverary, 7 Oct. 1772. *Signed.* [2 pp. + 2 pp., 15·5 × 20 cm.]

Recalls the days he and H. spent together in France, and is unwilling to visit Scotland without visiting H.

From the Earl of Hardwicke.

V, 49. [H.]. London, 1 Aug 1764. *Signed.* [4 pp., 19·5 × 24 cm.; E.P., 108–10.]

Thanks and congratulates H. on resolving the mystery of James II. Offers H. his assistance in correcting the facts as given in the first edit. of his *Hist.*—The "noble stand" made by Parliament during the Cabal Ministry, in introducing anti-Catholic measures. Lord Arundel of Wardour was the negotiator of the Secret Treaty. Lord H. had imagined it was the Duchess of Orleans.—Begs for

information about the Popish Plot, the Exclusion Bill, etc.—The Memoirs [of James II] are said to have been seen by Lady Holdernesse, and read by Lord Bolingbroke.

From Claude-Adrien Helvétius.

V, 50. [H.]. [Paris] 1 Apr. 1759. *Signed.* [6 pp. + 2 pp., 16·5 × 21·5 cm.; E.P., 6–9, inc.]

Admiration for H., whose works he would have quoted oftener [in *De l'Esprit*] but for the severity of the censorship. Has been persecuted on account of the book.—Answers some objections made by H. Claims that public esteem is inspired by pain and pleasure, or interest.—The Abbé Prévost is translating H.'s works, of which 2 vols. are already printed. Helvétius thanks H. for the gift of his works, and begs, if possible, to be elected *agrégé* of the Royal Society. But for the war, would visit H. Encloses copy of his own book.

V, 51. [H.]. Lamigny, 12 July 1759. *Signed.* [4 pp., 16·5 × 21·5 cm.; E.P., 10–2.]

Decides to let the question of the Royal Society drop. Perhaps Prévost may not be able to include everything in his translation of H.—Studying English. Bolingbroke's style. H.'s superiority in clearness.—Thanks whoever may have sent him the works of Robertson and Ramsay.—The persecution against him in France has died down. His enemies consider his good health one of his crimes.

V, 52. [H.]. Lamigny, 2 June 1763. *Signed.* [3 pp. + 1 p., 18·5 × 23 cm.; E.P., 13–4.]

Praise of H.'s *Hist.* Begs H. to write the history of the Church, a debt he owes to the Universe.

V, 53. [H.]. Voré, 28 June 1767. *Signed.* [4 pp., 19 × 23 cm.; E.P., 14–5.]

Does not know Graffigni, but will attempt to learn more about him. Has had gout, but is now writing again. But the difficulty is to get it printed. The Inquisition has passed from Spain to France.—Congratulates H. on his ministerial post; expresses admiration for him. Hopes to see him again, but begs for Walpole to be sent meantime.

From Mme Helvétius.

V, 54. Hume, Paris. Voré, 6 May 1764. *Signed.* [1 p. + 3 pp., 16·5 × 21·5 cm.]

Begs H.'s influence in gaining a living or a pension for a friend.

From the President Hénault.

V, 55. Hume, chés l'Ambassadeur d'Angleterre. [Paris] "mardi 26" [June 1765]. *Signed.* [1 p. + 3 pp., 10·5 × 16 cm.; E.P., 291.]

Congratulates H. on his appointment [as Secretary to the Embassy].

From the Earl of Hertford.

V, 56. [H.]. London, 5 Aug. 1765. *Signed.* [5½ pp. + ½ p., 18·5 × 23 cm.; Burton, II, 289–90, inc.]

Has accepted the Lord Lieutenancy of Ireland, though against his real inclination, for the sake of the King, his family, and his party. He will consider the Irish prejudices and indulge them when there is no material or public inconvenience in doing so. Lord Kildare is the only opponent he has to fear. Would have preferred H. as Secy., but the prejudice against the Scotch is too strong. A provision for H. is, however, the condition of Lord H.'s acceptance.—On the Duke of Richmond, the next Ambassador to France.—State matters.—Asks H. to express his gratitude to the French.

V, 57. [H.]. London, 16 Aug. 1765. *Signed.* [6 pp., 18·5 × 23 cm.; E.P., 112–5.]

M. de Guerchy is misleading the French Court if he declares that England is indifferent about the demolition of the Dunkirk fortifications, and if he declares that in the affair of the Canada Bills the English merchants will accept less than 50 per cent. in the French funds. Lord H. hopes the French will accept this arrangement.—The King has settled a pension of £400 a year on H. from the time when he ceases to be Secy. of Embassy. Ushership of the Black Rod in Ireland is in Lord H.'s disposal, and produces £800 or £900 a year. H. can hold the office and pay someone £300 a year to perform the duties. Lord H. cannot open his ministry by requesting a pension on his own account. Situation in Ireland is extremely delicate. He will do what he can.—H. will not be at liberty till Nov., when Lord H. hopes to see him in Dublin. Lord H. regrets leaving France, and proposes to visit it as often as he can, if possible with H. as a companion.

V, 58. [H.]. [London] 5 Sept. 1765. *Signed.* [4 pp., 18·5 × 23 cm.; E.P., 115–6.]

Glad H. approves of the efforts made in his favour. The office H. mentioned [Secretary to the Order of the Thistle] was immediately obtained for Mr Dempster, a steady friend of the Ministry's.—The Black Rod must be disposed of as Lord H. thinks best. The pension he hopes to transfer in part to an Irish establishment for greater security, though the introduction of a Scotch name must not open his Irish ministry.—Has spoken to his brother about the Canada Bills.—H.'s apartment in Dublin Castle will be ready for him when he comes.—*P.S.* Begs to be remembered to his French friends, and asks H. to keep silent about the Irish pension.

V, 59. [H.]. London, 20 Sept. 1765. *Signed.* [3 pp. + 1 p., 18·5 × 23 cm.; E.P., 116–8.]

Can do nothing for Capt. O'Donnell, who by entering the French army without permission has rendered himself liable to trial for felony, without benefit of

clergy.—Urges H. to attempt to persuade the French Court to accede to the demands of the Canada merchants; and hopes H. will, for his own honour, succeed in reaching a settlement before the Duke of Richmond arrives.

V, 61. [H.]. Dublin Castle, 10 Dec. 1765. *Signed.* [3 pp. + 1 p., 18·5 × 23 cm.; E.P., 118, inc.]

Could not believe H. would continue unpopular in Ireland, though warned that H.'s presence might endanger his own government.—Sorry he will not see H. before next summer. Asks him about his plans. Lord H.'s administration in Ireland is made easy by the good opinion of the people. Opposition just great enough to show off the good qualities of Lord Beauchamp in Parliament.

V, 62. [H.]. Dublin Castle, 10 Feb. 1766. *Signed.* [2½ pp. + 1½ pp., 18·5 × 23 cm.; E.P., 119.]

Clamour about the alteration of the Corn Law has subsided. Lord H. is as free from anxiety as the readily troubled state of the country will allow. The possible danger from the factious spirit in England.—Approaching marriage of Lord H.'s eldest daughter to Lord Drogheda, and continued success of his son in Parliament.

V, 63. [H.]. Dublin Castle, 13 Mar. 1766. *Signed.* [1½ pp. + 2½ pp., 18·5 × 23 cm.]

Thanks H. for his account of a singular parliamentary debate. It is rumoured that Lord Temple will give a further account. Begs H. to continue his communications, if the scene continues.—Lord H.'s affairs in Ireland are going well. There are gainsayers, " but the Irish confidence is so strongly with me that they can only protract, but not disturb." Hopes to meet H. in London at the beginning of May.

V, 64. [H.]. Dublin Castle, 15 May 1766. *Signed.* [2½ pp. + 1½ pp., 18·5 × 23 cm.]

Thanks H. for his accounts of what is passing in England.—In Ireland the Speaker, contrary to Lord H.'s advice, prepared an Address to the King on the Septennial Bill; but now repents and promises absolute compliance. The Opposition, realising Lord H.'s strength, have attacked him in print, though their methods are generally condemned.—Will visit Paris in October, and hopes to see H. in London.

V, 65. [H.]. D[ublin] Castle, 7 June 1766. *Signed.* [1 p. + 1 p., 18·5 × 23 cm.]

Has just prorogued the Irish Parliament. " I am now perhaps the most popular man in this kingdom."—" The Session has ended most honorably; the House of Commons have acknowledged the King's goodness in reproving them for their former error."—Soon to embark for England.

V, 66. [H.]. London, 12 Nov. 1766. *Signed.* [1½ pp. + 2½ pp., 18·5 × 23 cm.]

Sends H. the printed account of his quarrel with Rousseau. R. is generally condemned in France. Lord H. was surprised to see it in print, but will inform the King of H.'s reasons.—Successful opening of Parliament; no opposition. George Grenville unsupported.

V, 67. [H.]. London, 22 Jan. 1769. *Signed.* [1½ pp. + 2½ pp., 18·5 × 23 cm.]

Does not think Mr Shackleton is dying. His appointment was due to the influence of the Devonshire family. If he dies, Lord H. will take care to learn the King's wishes and Mr Ramsay's pretensions.—Regrets the loss of H.'s society. His Irish circle is restricted to a few friends. Sir Gilbert Elliot says he has written to laugh at H.'s philosophy, and to advise dissipation; which advice H.'s French friends would second.

From Lady Hervey.

V, 68. [H.]. Bill Hill, 8 Oct. 1763. *Signed.* [2½ pp. + 1½ pp., 15 × 18 cm.; E.P., 24–5, inc.]

Thanks H. for his letter and hopes to see him before he leaves London. Glad H. is going to so pleasant a place as Paris, and may see him there, if gout permits. Sends some letters of introduction. Advises him to ask Mme de Boufflers to read him a play of her own.—Charm of the conversation of Mme d'Aiguillon and Mme Geoffrin.

V, 69. [H.]. St James's Place [London], 1 Jan., continued on 9 Jan. 1764. *Unsigned.* [2½ pp. + 1½ pp., 18·5 × 23 cm.; E.P., 26–7, inc.]

New Year greetings.—She has been very ill. Praises Mme de Boufflers for her judgment and talents. Fears H. has seen but little of Mme Geoffrin, whom Lady H. praises highly.—A swelling in her wrist prevented her continuing. She sympathises heartily with H. in his admiration of the French. They admire him too. She hopes to make him acquainted with the Duchesse d'Aiguillon.

V, 70. Hume, Lisle Street, Leicester Fields [London]. Old Windsor, 23 July 1766. *Signed.* [2 pp. + 2 pp., 16 × 20 cm.; E.P., 27–8, inc.; Greig, App. K, inc.]

The madness of Rousseau is becoming dangerous. She has informed Mme Geoffrin of the circumstances, so that no one else may be treated as H. has been. The union of madness and genius, virtue and wickedness, in R.—Intends to go to Bill Hill. Sends compliments to the Ramsays, and again warns H. against R.

V, 71. [H.]. St James's Place [London], 6 Sept. 1766. *Signed.* [1 p. + 1 p., 16 × 20 cm.]

She tried to get the Secretaryship to Lord Bristol for Mr Larpent, but the latter refused to go except as second Secretary, which could not be granted.

From Baron d'Holbach.

V, 72. [H.]. Paris, 22 Aug. 1763. *In English. Signed.* [2 pp. + 2 pp., 19 × 23 cm.; E.P., 252–3.]

Has wanted to become acquainted with "one of the greatest philosophers of any age, and of the best friends to mankind."—Realised that even English freedom might not allow the publication in England of a book H. has mentioned. —Hopes, along with all the philosophic minds in Paris, to see H. soon.

V, 73. Hume, Hôtel de Brancas, rue de Bourbon, Paris. [Paris] 24 June [1765]. *In French. Signed.* [1 p. + 3 pp., 19 × 23·5 cm.]

Congratulates H. on his appointment [as Secy. to the Embassy], and rejoices that merit has for once received due recognition.

V, 74. [H.]. Paris, 16 Mar. 1766. *In English. Signed.* [4 pp., 18·5 × 23·5 cm.; E.P., 253–6.]

Anxiety to see H. again in Paris. Would like to see the *Hist.* continued, but H.'s "stock is plentifull enough to oblige the world some other way."—Glad H. has not so far had any trouble with Rousseau. Himself, he has had nothing to complain of with regard to R., but he wishes he could say the same of some friends.—Compliments from H.'s Paris friends. Mme Geoffrin is about to set out for Poland.

V, 75. [H.]. Paris, 7 July [1766]. *In French. Signed.* [4 pp., 18·5 × 23·5 cm.; E.P., 256–8; Greig, App. K, inc.]

Laments the actions of Rousseau. H. should avoid a literary war, which is endless and of which the public is a bad judge. He must follow the example of many of his Paris friends who have suffered at the hands of R., but have not answered.—H. has acted honourably, and this will be recognised.—D'H. has learnt from a friend of de Rougemont's that R. received 10,000 livres from the Duc de Luxembourg.

V, 76. Hume, Lisle Street, Leicester Fields, London. Paris, 18 Aug. 1766. *In French. Signed.* [3 pp. + 1 p., 19 × 23 cm.; E.P., 259–61; Greig, App. K, inc.]

Regrets H.'s trouble with Rousseau. No one in Paris hesitates to acquit H. of blame. R.'s letters justify d'H.'s prophecy that R. would act the madman to get out of the affair. H. should remain silent till forced by R. to speak, and then overwhelm him with the proofs.—Regrets the death of Sir James Macdonald. Has not yet given up all hope of seeing H. in Paris.

V, 77. Hume, Lisle Street, Leicester Fields, London. Rue Royale, Butte St Roch [Paris], 1 Sept. 1766. *In French. Signed.* [3 pp. + 1 p., 18 × 23·5 cm.; E.P., 261–3; Greig, App. K, inc.]

Changes his mind and advises H. to publish Rousseau's letters, because the

latter is reputed to have sent a challenging letter to Guy, and has a large number of fanatics throughout Europe.—But H. must show moderation, and stick to the naked facts. R. is preparing his Memoirs, and must be anticipated. R. will have on his side those whom his rhetoric has dazzled; H., the friends of truth.

From the Earl of Holdernesse.

V, 78. Hume. Sion Hill, 26 July 1764. *Signed.* [3 pp. + 1 p., 19 × 23 cm.; E.P., 72–3.]

Apologises for not having seen H. before leaving Paris. His uneventful journey. Social life at Paris is entirely suited to his character. The inability of the English to produce such a society. H. ought to enjoy it while he can. He will have sufficient time for philosophy in England.

V, 79. [H.]. London, 2 July 1765. *Signed.* [4 pp., 18·5 × 24 cm.; E.P., 73–4; Greig, App. C.]

Has received the letter from H. with the message from the Prince [de Conti]. He will observe the utmost secrecy even towards the lady [Mme de Boufflers]. He can only help her if she introduces the subject [the question of her marriage with the Prince] herself. He will then encourage her to think of the three options before her. He must wait till he hears her arguments before expressing an opinion himself. Unable to visit France during the summer. Pleased that he is remembered there.

From Agatha Home (Mrs Home of Kames).

V, 80. [H.]. Ed[inburgh], 25 Mar. 1764. *Signed.* [2 pp., 18·5 × 22·5 cm.]

Recommends a poor lad named Smith, who is studying miniature painting.—Good wishes to H. Praises of his nephew.

From John Home of Ninewells.

V, 81. Hume, Brewers Street, London. Edinburgh, 21 Nov. 1768. *Signed.* [2 pp. + 2 pp., 20 × 33 cm.]

Recommends Mr Richardson. Has been wondering whether H. will go to France or not. Hesitates to allow David [his son] to accompany him. David's talents and ability. Joseph's education and inclination.—His financial position.—Mr Campbell's desire to send his brother-in-law to the East Indies. Death of Mrs Home of Boghall, and illness of Lady Spittal.—Account of H.'s financial affairs as they will be at Candlemas 1769.

V, 82. Hume. [? , ? 1774.] *Signed.* *Fragment of a letter.* [½ p. + 1½ pp., 18 × 22·5 cm.]

David [the writer's son] wishes to go to Glasgow next winter to study under Professor Millar.

From Mme d'Houdetot.

V, 83. Hume, chés M. l'Ambassadeur d'Angleterre, Hôtel de Lassuy, Paris. [Paris] 28 Aug. [1764]. *In third person.* [1 p. + 3 pp., 10 × 15·5 cm.]

Invitation to dinner.

V, 84. Hume, Hôtel de Lassuy, Paris. [Paris, 1763–5.] *In third person.* [2 pp., 9·5 × 15 cm.]

Begs H. to gain for M. d'Houdetot the reading of the correspondence of M. d'Eon; and not to forget his promise to visit her.

V, 85. Hume, Paris. [Paris] "ce mardy au matin" [1763–5]. *In third person.* [2 pp. + 2 pp., 9 × 14 cm.]

Regrets having missed seeing H., and begs him to visit her again. Advises him to go to the Comédie française.

From M. Hulin.

V, 86. Hume, Ministre de la Cour d'Angleterre, Hôtel de Brancas, Paris. Paris, 9 Sept. 1765. *Signed.* [1 p. + 3 pp., 18 × 23 cm.]

Concerning a letter from the King of England to the King of Poland.

From Mrs (A.) Irwine.

V, 87. [H.]. Queen Ann Street [London], 6 Dec. 1763. *Signed.* [2½ pp. + 1½ pp., 18·5 × 23·5 cm.]

Recommends Mr Bickerstaff, an officer of the army, who turned to poetry and "never thought of Mars without Venus." It will be of advantage to B. to be known to the celebrated philosopher, though she refuses to think of H. in that light. Some of B.'s verses are good, since she herself is the object. She praises his prose with less qualification, and his heart most of all.—H. must not answer this letter, unless he tells her first how he avoids murders among the ladies enamoured of him.

From Colonel J. Irwine.

V, 88. [H.]. Bath, 1 Jan. 1765. *Signed.* [3 pp. + 1 p., 18·5 × 23 cm.]

Begs H. to procure him a small gold box for Spanish snuff, with a short dissertation on the taking of snuff.—Improvement in Mrs I.'s health.—*P.S.* Will think of H. every time he takes snuff. Further exact instructions on the size of box required.

V, 89. Hume, Paris. Gibraltar, 15 Sept. 1765. *Signed.* [2½ pp. + 1½ pp., 20 × 31·5 cm.]

Begs H. to recommend to the Lord-Lieut. of Ireland the Rev. Mr Barry,

Mrs I.'s brother, who has many virtues even though he *is* a clergyman. Barry wishes to go somewhere where there is more society round him than in the west of Cork.

V, 90. Hume, British Coffee-House, London. Gibraltar, 15 Nov. 1765. *Signed.* [2½ pp. + 1½ pp., 18 × 22·5 cm.]

Regrets that, anxious as he is to show his friendship for H., it is now too late to do anything for the Chev. de Narbonne, of whose fate he knows nothing except the rumour that he has been put to death after imprisonment.—Satisfied with H.'s retirement from public service, if it pleases H.

V, 91. Hume, Edinburgh. Gibraltar, 30 Mar. 1767. *Signed.* [2½ pp. + 1½ pp., 20 × 32 cm.]

Promises to interest himself on Mr Graham's behalf.—Thanks H. for sending him a copy of the correspondence with Rousseau, about which he had been in some perplexity. Felt confident that H. had been wronged. No one who has read the correspondence hesitates to pity and commend H.—Grateful for H.'s praise of his actions at Gibraltar. Hopes also to gain the approbation of the ministers; which is all he is likely to get.

From the Rev. John Jardine.

V, 92. [H.]. Edinburgh, 14 Aug. 1764. *Signed, initials only.* [7 pp. + 1 p., 16·5 × 20·5 cm.; Burton, I, 232, and II, 230, inc.; Greig, App. C, inc.]

Recommends George Drummond, who desires to pay his respects to Lord Hertford, but not to meet the Parisian noblesse.—Danger the Church of Scotland has been in has prevented J.'s writing earlier, but that has now passed away. The Church has been persecuted. Now it is *its* turn to persecute. Begs H. to inform the Gallican Church of this fact, so as to urge on a union of the two churches. The agreement should be a compromise, about which J. gives some suggestions.—H. stands in great need of J.'s advice. He is in danger of being seduced by the French ladies. They are really succubas. See Luther. Begs H. not to disgrace the Scottish Church.—John Adam's affairs are improving. J.'s satisfaction at H.'s success in Paris. Compliments from Mrs J.

V, 93. [H.]. Edinburgh, 21 Apr. 1765. *Signed.* [2¼ pp. + 1¾ pp., 16 × 21 cm.]

Recommends Dr Arthur Lee, an alumnus of Edinburgh. J. has always refused to send introductions when more than a mere recommendation to H. was desired.

From George Johnstone.

V, 94. [H.]. London, 22 Mar. [1763]. *Signed.* [2½ pp. + 1½ pp., 20 × 32 cm.]

Thanks H. for singling him out as the brother and son of greatest influence. His opinions of his sister's conduct coincide with H.'s, and he had already begun

to act as H. advised.—There has been trouble in the Commons about a tax on wine and cider. Pitt has again shown himself "like a sleepy Lyon goaded in his Den." The protestations of the Opposition against the tax. Sir Francis Dashwood is uncertain in figures, and careless in addressing members in the House.

From William Johnstone (afterwards Pulteney).

V, 95. [H.]. London, 28 Jan. 1765. *Signed.* [1½ pp. + ½ p., 18 × 22·5 cm.]

On H.'s quarrel with Lord Elibank: whatever H. may think of Lord E.'s answer, he should remember Lord E.'s many good qualities. No one has a greater regard for H. than Lord E. J. feels confident of a speedy reconciliation.

V, 96. Hume, Secy. of State's Office, London. London, 8 Oct. 1767. *Signed* (Pulteney). [2 pp., 18·5 × 23 cm.; E.P., 167–8.]

Begs H. to give evidence that P. has changed his name from Johnstone to Pulteney, and that his estate is known both as Auchenbedrigg and as Solway Bank.—Will not require such a certificate when he has made his name famous throughout Europe like Pitt, or William Pulteney his kinsman, or David Hume his friend.

V, 97. [H.]. [London, spring, 1769.] *Copy in H.'s hand, unsigned.* [½ p. + 3½ pp., 18·5 × 22·5 cm.; Burton, II, 425, wrongly ascribed to H.]

Praise of [Andrew] Stuart. P.'s pleasure in having been able to help him.

From [? the Abbé Joseph-Romain] de Joly.

V, 98. Hume. Rue de la Harpe, Paris, 29 Aug. 1764. *Signed.* [2 pp., 18 × 23 cm.]

Asks H.'s advice about a translation de J. has made of Marcus Aurelius, and begs him to inquire about a precious MS. of this work in Trinity Coll., Oxford.

From Earl Marischal George Keith.

V, 99. [H.]. [Neuchâtel] 2 Oct. 1762. *Signed, initial only.* [2 pp., 16 × 20 cm.; Burton, II, 105–6; Greig, App. D.]

Rousseau, being persecuted, has sought asylum and been granted protection in Neuchâtel. He is rapidly increasing in popularity there. He would probably be happiest in England, and wishes to know whether it would be possible to publish an edition of his works in England. Lord M. would be sorry to lose R., but encourages him to go to England, and recommends him to H.

V, 100. Hume. Touch [Aberdeenshire], 28 Oct. 1763. *Half in English, half in French. Unsigned.* [3 pp. + 1 p., 18·5 × 23 cm.; E.P., 57–9, inc.; Greig, App. D, inc.]

The Countess de Marsan is to receive the present of snuff for the Duc de

Gadagne.—Lord M. would like to creep nearer to the sun. Compares the expenses of living in Paris and in London. The restraints of Scottish life ; its bigotry and hypocrisy. The clergy abuse their power.—Lord M. can retire to Port Mahon, Venice, or Sans Souci, but none is satisfactory in every respect. Infirmities of old age increase. Would like the advice of H. and d'Alembert. Meantime undecided.

V, 101. Hume, Paris. Edinburgh, 20 Nov. [1763]. *Unsigned.* [½ p. + 3½ pp., 18·5 × 23 cm.]

Begs H. to see that a snuff-box reaches Mme de Fromont. Compliments to Mme Geoffrin, and to Helvétius and d'Alembert.

V, 102. [H.]. [Edinburgh] 4 Jan. 1764. *Part in French. Signed, initial only.* [2 pp. + 2 pp., 18·5 × 23 cm. ; E.P., 62–3, inc.]

Thanks H. for letter. Regrets unable to make the journey to Paris. Society in Berlin not disagreeable. His female convert [from Mohammedanism] and her doubts about the immortality of the soul. Lord M. himself would not suffer martyrdom for that belief. Extract from a Paris correspondent relating H.'s success among the Paris women. Lord M. hopes H. will not become a " petit maître."—Recommends Mme de Frenaye to H. and to the Ambassador.

V, 103. [H.]. [Edinburgh] 4 Feb. [1764]. *In French. Unsigned.* [2 pp. + 2 pp., 18·5 × 23 cm. ; E.P., 60–1.]

Has received H.'s letter telling of the lady who was disgraced at the French Court because she did not know who H. was.—Mme Geoffrin has given H. good advice.—Lord M. has been touched by a letter from Rousseau. " Il est trop sauvage." Lord M. has written to Neuchâtel asking friends to help Mlle le Vasseur if R. dies. Begs H. to tell this to Mme de Boufflers only. R. is too honest a fellow for this world.—Lord M. dislikes Edinburgh. Difficult to be free in Scotland. H. will tire of his duchesses and should leave political life and take to philosophy again.

V, 104. Hume. Edinburgh, 23 Feb. 1764. *Unsigned.* [1 p. + 3 pp., 18·5 × 23 cm.]

His new estate in the north of Scotland. Encloses a letter to a young man [Meuron] whom he would like to engage as tutor to Baron Mure's sons.

V, 105. [H.]. Potsdam, 11 Sept. 1764. *In French. Unsigned.* [3½ pp. + ½ p., 18·5 × 23 cm. ; Burton, II, 217, inc.]

Regrets d'Alembert's ill-health. Description of Lord M.'s " hermitage," to which he invites d'A. Regrets did not know Mme de Boufflers was in Holland. Rousseau was about to take the waters in Savoy, but had to return to Motiers-Travers. Voltaire an enthusiastic anti-Christian. Lord M.'s life at Potsdam.—

Recommends David Floyd to H.—D'Argens is leaving for Provence, where he will find no happiness.—Wishes H. would continue instructing the world in truth, and desires him to investigate a claim made by Needham, that all generation comes from fermentation.

V, 106. Hume, Paris. Potsdam, 20 Feb. 1765. *Unsigned.* [2 pp. + 2 pp., 18·5 × 23 cm.; E.P., 63–5.]

The attacks of Voltaire on Rousseau. They will only injure each other without either gaining the victory.—Asks after Lady Stanhope. A message to d'Alembert about his inscription.—Respect for Mme du Plessis Chatillon. Dislikes Maupertuis for his attitude towards de Torcy. Requests H. to send him some seeds by Helvétius.

V, 107. Hume, Paris. Sans Souci [Potsdam], 22 Mar. 1765. *In French. Unsigned.* [2 pp. + 2 pp., 18·5 × 23 cm.; E.P., 66–7.]

Begs H. to consult with Mme de Boufflers about the best thing to do for Rousseau, who has been bitterly attacked by the ministers of the Church. Even the protection of the King [of Prussia] excites his enemies and alienates his friends. The priests claim to be the sole judges in matters of church discipline. Lord M. advises R. to retreat either to Venice or to Cornwall.

V, 108. Hume, Paris. Potsdam, 20 May 1765. *In French. Unsigned.* [2 pp. + 2 pp., 18·5 × 23 cm.]

Afflicted at the position of Rousseau; united with Mme de Boufflers by a similar sympathy in her; yet cannot correspond with her, since he is already overwhelmed with his present correspondence. The King is building a house for Lord M., which seems to imply that Lord M. is a kind of Grand Vizier. But he is going downhill very fast, though easily.—The pastors have renewed their attacks on Rousseau, denouncing him as Antichrist in person. Difficulty in quelling the disturbance.—M. de Melfort has asked Lord M. to stand godfather to his child, with d'Alembert and other good Christians sharing in the ceremony.

V, 109. [H.]. [Potsdam, summer, 1765.] *In French. Unsigned.* [1 p. + 3 pp., 18·5 × 23 cm.]

A renewed attack on Rousseau, who has been nearly stoned. A trap laid at his door. Lord M. will welcome him.

V, 110. Hume, Compiègne. Potsdam, 3 Aug. 1765. *Half in French. Unsigned.* [2 pp. + 2 pp., 18·5 × 23 cm.]

Hume about to change from a philosopher into a minister; he will soon gain a pension and be able to retire. D'Alembert's chagrin when he was refused a pension. "He wants a part of your quiet temper, he is fire and lightning." He

would benefit by coming to Lord M.'s hermitage.—Mme de Fromont is returning to Switzerland, and requires H.'s address in order to send by him some embroidered muslin for the Duchesse de Gadagne.

V, 111. Hume, Paris. Potsdam, 16 Aug. 1765. *Unsigned.* [1 p. + 3 pp., 18·5 × 23 cm.]

Disadvantages to H. of becoming a minister; yet he will be able to retire when he pleases. Repeats request that H. will arrange for the piece of muslin to be conveyed to the Duchess de Gadagne.—Requests more seeds; and asks that Mr Gordon should be introduced to Helvétius.—Has no longer any hopes of d'Alembert's coming to Potsdam.

V, 112. [H.]. Potsdam, 10 Sept. 1765. *Unsigned.* [2½ pp. + 1½ pp., 18·5 × 23 cm.; E.P., 67–9.]

Rejoices that H. is now at liberty to devote himself to literature. But H. must have done much good while a minister. Records a good deed performed by himself at Turin.—D'Alembert lacks H.'s philosophical temper. Rousseau, harassed as he is, is the nearest approach to H. Lord M. requires some pictures to complete his collection. Describes one of Voltaire in his deshabille.—The Douglas Cause. A rumoured seduction of H. to the Duchess's side. Difficulty of proving birth. A story of the Bp. of Magdeburg.

V, 113. [H.]. [Potsdam] 4 Mar. 1766. *In French. Unsigned.* [1 p. + 3 pp., 18·5 × 23 cm.; E.P., 69–70, inc.]

Admires George III's kindness towards Rousseau, whom he advises to settle with Baron Wolf. Glad H. and R. are so satisfied with one another.

V, 114. Hume, York Buildings, London. [Potsdam] 29 Apr. [1766]. *Unsigned.* [1 p. + 3 pp., 19 × 23·5 cm.; Burton, II, 104–5 (incorrectly dated 1762); Greig, App. D.]

Wishes to see H. to clear himself of certain misconceptions about his own life. Regrets the death of Mr Floyd, who would have been of use to H. in his *Hist.*—Rousseau will never be happy in a country of whose language he is ignorant; just as Lord M. feels his own ignorance of German, though he is learning some expressions. Arranges to meet H., should he come to France in the summer.

V, 115. Hume, at Mr Stuart's, York Buildings, London. Potsdam, 15 Aug. 1766. *Unsigned.* [2 pp. + 2 pp., 18·5 × 23 cm.; E.P., 70–1; Greig, App. K.]

Regrets the quarrel with Rousseau, which he ascribes to R.'s too strong imagination rather than to conscious ingratitude. H. has nothing to fear for himself, and would be well advised not to answer, attacked as R. is by many

assailants already.—Will be happy to see H. at Potsdam, but H. must not expect too much from Lord M.'s memory. The kind of life H. must expect to find there. Directions for his journey. Compliments to Dr Juan [? Pringle].

V, 116. Hume, F.D. [Fidei Defensor]. Potsdam, 7 July 1767. *Unsigned.* [1 p. + 3 pp., 18 × 23 cm.]

H. will be "le bon David," though Lord M. will not go so far as a certain lady and prophesy that H. will die a good Presbyterian. "A good Christian, yes, for you have writ in defence of the Faith against human reasoning."—Regrets he cannot help Rousseau, as H. generously desires.—H. is the only man to reveal absolute impartiality. "To the highflyers you are therefor a sad whig, to the whigs an hidden Jacobite, and to reasonable men le bon David, a Lover of truth."—Regrets the burning of Floyd's memoirs and his own correspondence.

V, 117. Hume. Potsdam, 11 June 1768. [Written jointly with Sir Andrew Mitchell, both parts *unsigned.* 2 pp. + 2 pp., 18·5 × 23 cm.]

Lord M.:—Upbraids H. for not writing. Defends this conduct to Mitchell on the ground that H. must be continuing his *Hist.*, but knows this to be a lie, and the true cause to be laziness or French ladies. Invites H. to visit Germany.

Sir A. M.:—Cannot help forgiving H. even while he upbraids, knowing he has done so much to dispel prejudice. Has passed some time with Lord M. They both talk of H. with resentment and affection.

V, 118. [H.]. [Potsdam, ? 1774.] *In third person.* [1 p. + 1 p., 9·5 × 22·5 cm.; E.P., 65.]

Has left a private paper for him with Prof. [Adam] Ferguson. An expensive marriage. Begs H. to forward to Aberdeen his old baton of Earl Marischal.

From the Abbé Le Blanc.

VI, 3. [H.]. Paris, 25 Aug. 1754. *Signed.* [3 pp. + 1 p., 19 × 23 cm.; Burton, I, 458–9.]

Sends his French translation of the *Political Discourses*, which he begs H. to revise. This is exciting much interest in France. Likelihood of a second edit. soon. Le B. feels that H. and he were made for each other. Refers to his own *Lettres d'un François*.

VI, 4. [H.]. Paris, 1 Oct. 1754. *Signed.* [4 pp., 19 × 24 cm.; *P.S.* 1 p. + 1 p., 12 × 18·5 cm.; Burton, I, 459–60, inc.]

Thanks H. for his letter and proffered friendship. Begs him to correct the text and notes of the *Polit. Disc.*—Offers to translate H.'s *Stuarts*, but must have the book before it is published in London, in order to prevent a translation by writers who know neither French nor English.—The interest aroused by the *Polit. Disc.* in France.—*P.S.* Has heard from Montesquieu, whose modesty will not

allow him to communicate the letters he received from H. and from [Robert] Wallace. Le B. asks for copies of them.—If H. is pleased with the translation of the *Polit. Disc.* will he send a recognition in the form of a letter which can be printed in the literary journals, so as to prevent other translators from undertaking the *Hist.*?

VI, 5. [H.]. Dresden, 25 Dec. 1754. *Signed.* [4 pp., 17 × 21·5 cm.; Burton, I, 461–2, inc.]

The magnificence and virtue of the Saxon Court.—Le B. has received Vol. I of the *Stuarts*, which he will do his best to translate satisfactorily. Praise of H.'s style.—Begs H.'s remarks on the *Polit. Disc.*, and the notes appended. Agrees with H.'s judgment on Bolingbroke, who talks too much about himself. The Dutch translation of the *Polit. Disc.* [Mauvillon's] is unreadable. Le B. has been urged to print a Dresden edit. Constantly engaged in spreading H.'s fame through Europe.

VI, 6. [H.]. [Paris, autumn, 1755.] *Signed.* [6 pp., 17 × 22 cm.; Burton, I, 462, inc.]

Regrets the interruption of their correspondence. His translation of H.'s *Hist.* is advancing slowly. A hack writer has already prepared one in Holland.—Preparations are on foot for a fine new edit. of the *Polit. Disc.* in Saxony, and for a third edit. in Paris.—Extracts from letters of de Maupertuis praising the *Polit. Disc.* and wishing for a translation of H.'s other works.—Le B. is trying to get ahead of the Dutch translation. Admires the first vol. of the *Hist.*—H. was correct in prophesying that commerce would in future direct peace and war in Europe. But the Muses in all nations remain friends.

VI, 7. [H.]. [Paris, summer, 1757.] *Signed.* [4 pp., 17 × 22 cm.; Burton, I, 460–1.]

Laments that the War has prevented their corresponding. Intentions of translating the *Stuarts* have been thwarted, and he has arranged for one of his friends to do it instead.—H.'s *Polit. Disc.*, like Montesquieu's *Esprit des lois*, have aroused great interest, and caused the publication of many similar works. The most important of these is *L'Ami des hommes*, by the Marquis de Mirabeau. Praise of this author, who opposes H.'s system on luxury, though with proper regard to H.'s eminence. Le B. offers to hold back a new edit. of the *Polit. Disc.* to enable H. to reply to Mirabeau.—Duclos is an admirer of H. "Lui & moi nous sommes convenus vingt fois qu'il étoit heureux pour notre siécle qu'au moment où sur ces Cotes Méridionales nous avons perdu un des Astres les plus brilliants qui ayent éclairé les Lettres, il en aît paru dans le Nord un autre non moins lumineux, & dont l'utilité se reconnoit à proportion que ses lumiéres se répandent de plus en plus. Vous êtes le seul dans l'Europe qui pouviés remplacer Mr le Président de Montesquieu."—Mme Dupré de St Maur, wife of the translator of Milton, has the same opinion of H., and desires two complete copies of the *Hist.*

VI, 8. [H.]. Paris, 20 Nov. 1762. *Signed.* [1½ pp. + 2½ pp., 19 × 23 cm.]

The signing of peace has re-established communications between England and France, and all the literary world in Paris is impatiently awaiting H. The Duc de Nivernois's anxiety is a sufficient proof of this statement.

From Julie de Lespinasse (see also under d'Alembert).

VI, 9. [H.]. Paris, 23 Feb. 1766. *In d'Alembert's handwriting, unsigned.* [3 pp. + 1 p., 16 × 19·5 cm.; E.P., 179–80; Greig, App. C.]

She has not been able to resist the temptation to write to H. The weakness of her eyes through smallpox. Her anxiety for H.'s return.—Advises Rousseau to write an *éloge* on the Late Dauphin, who always interested himself in R.'s welfare. Encloses a little memorandum on the Dauphin's character, written by a good hand [see **XIII, 43**]; and begs H.'s opinion, by way of Mme de Boufflers, if not direct.

VI, 10. [H.]. [Paris] 26 Mar. [1767]. *Signed.* [2 pp. + 2 pp., 14 × 20·5 cm.]

A note to remind him of her existence. Regrets his absence. He is much regretted every Friday at Mme de Boufflers'. Has read Deyverdun's letter. Rousseau will be embarrassed about what to say about it.—Will H. convey the enclosed letter to its addressee?

From Robert Liston.

VI, 11. [H.]. Over New Liston, 22 July 1768. *Signed.* [3½ pp. + ½ p., 16 × 19 cm.; E.P., 313–5.]

Surprised at the Parmese demanding a Roman Catholic for their professorial appointment; but the loss of the position will not affect him much. Pleased to have gained H.'s good opinion so completely, and hopes H. will recommend him for the secretaryship of some minor embassy.—Had intended travelling as tutor to Mr Johns, but this too has been dropped.

From Lord Lyttelton.

VI, 12. [H.]. Hagley Park, 12 Oct. 1761. *Signed.* [1½ pp. + 2½ pp., 18·5 × 23 cm.; E.P., 21.]

Thanks H. for letting him see proof sheets of his *Hist.*, especially as he wished to see H.'s work before publishing his own. Will certainly point out any Scotticisms, but hopes to improve his own style by examining H.'s.—Accepts with pleasure the office [of English critic] which H. and his countrymen offer him.

VI, 13. [H.]. London, 17 Feb. 1765. *Signed.* [2 pp. + 2 pp., 18 × 22 cm.]

Recommends the Count de Molther. Inquires about H.'s health and studies. His son's stay at Paris will be too short, and so he will not introduce him to H.

From Sir George Macartney.

VI, 14. **Hume. St Petersburg, 8–19 May 1767.** ***Signed.*** **[1 p. + 3 pp., 18 × 30·5 cm.]**

Thanks H. for his offer to correspond, but as he is returning to England, he hopes to visit H. in person. Much as he enjoys H.'s writings, he enjoys his conversation still more.

From George Macaulay.

VI, 15. [H.]. St James's Place [London], 22 Mar. 1764. *Signed.* [1 p. + 1 p., 18·5 × 24 cm. ; E.P., 111–2.]

Informs H. that he had previously transmitted to him a copy of his [M.'s] wife's ***History of England.***

From Sir Alexander Macdonald of Sleat.

VI, 16. Hume, Brewer's Street, London. Oxford, 27 July 1769. *Signed.* [6½ pp. + 1½ pp., 18 × 23 cm.]

The difference between Commoners and Gentlemen Commoners at Oxford. The exercises required of both are identical. Necessary difference in expenses amounts to £50 or £60 a year, though the chief sources of expense are the extra extravagances. M. himself was fortunate, having been elected from Westminster School. A Gentleman Commoner will require about £250 a year, and a Commoner quite £200, though he can live in retirement and obscurity on half of that. —[Joseph] Home's former companions in Edinburgh are Gentlemen Commoners, though the actual advantage is slight. University College is perhaps the most attentive to its younger members, but Christ Church has the largest society.

From Sir James Macdonald of Sleat.

VI, 17. [H.]. North Uist, 30 Aug. 1763. *Signed.* [3 pp. + 1 p., 18·5 × 22 cm. ; E.P., 46–8.]

Flattered by H.'s inquiries. Rejoices that H. is going to France, where he is valued according to his merit ; but fears the French will never allow him to return. Surprised that H. is to attend Lord Hertford, whom he eulogises. M. was himself formerly intimate with Lord H.'s son [Lord Beauchamp]. M. hopes to be in Paris before Christmas, and to see H. there.

VI, 18. Hume, Compiègne. Paris, 24 July 1764. *Unsigned.* [3 pp. + 1 p., 18 × 23 cm. ; E.P., 48–50.]

Will not give vent to his complaints to anyone but H. and " la divine Comtesse " [Mme de Boufflers]. But he left Paris because Lord Hertford's sense of his own dignity and Lord Beauchamp's forgetfulness of former friendship had left M. nothing better than an inn to stay at.—If this should change H.'s feelings towards M., M. will always remain true to the friendship he has vowed.

VI, 19. [H., Compi]ègne. Paris, 4 Aug. 1764. *Signed.* *Torn.* [2½ pp., 18 × 23 cm.; E.P., 50–1.]

H.'s letter has given him pleasure on account of the esteem it expresses. But he deprecates the accusations of passion and violence. Perfectly willing to be reconciled to Lord Beauchamp, and will give up all discontent. This willingness to act against his reason and his feelings of resentment proves his veneration for H.

VI, 20. [H.]. [Paris, ? 1764.] *In third person.* [½ p. + 1½ pp., 15·5 × 18·5 cm.]

Presents his compliments to "the great man," and requests leave to call on him and to dine with him at Chatillon.

VI, 21. [H.]. [Paris] "Tuesday Morning" [? 1764]. *Signed.* [1 p. + 1 p., 18·5 × 23 cm.]

Explains his having been out when H. called.

VI, 22. Hume, Hôtel de Brancas, Paris. London, 26 Apr. 1765. *Signed.* [3 pp. + 1 p., 18 × 23 cm.; E.P., 51–4; Greig, App. F, inc.]

Apologises for delaying to write, but the journey from Paris to London was too dull to provide material. The inconvenience of London life: conversation restricted to the clubs, to which M. does not belong. His business is therefore little advanced.—Lord Eglinton's successful speech in the House of Lords on the Scotch banks. Much discussion expected on the Regency Bill. Rumour that Lord Sandwich has gone to Ireland, and has been succeeded by Lord Northumberland in his present office.—Regrets the angry correspondence between H. and Lord Elibank, and would like to see them reconciled.

VI, 23. [H.]. London, 18 May 1765. *Signed.* [3½ pp. + ½ p., 18 × 22·5 cm.; E.P., 54–6.]

Recommends [John] Crawford.—The Regency Bill. The King's wishes. It has been passed, with an amendment. Its effect on the approaching change of ministry uncertain. No materials to form a ministry.—Riot of the silk weavers. The Duke of Bedford nearly killed by a stone. The impudence of the mob and the weakness of the Legislature.—Regency Bill not caused by any ill-health of the King, nor have the mobs anything to do with it.

VI, 24. Hume, Paris. London, 3 June 1765. *Signed.* [2½ pp. + 1½ pp., 18·5 × 24 cm.]

Regrets that unexpected business has prevented him from joining H. in Paris. A message to Mme de Boufflers about a gridiron, an inscription, and a pocketbook.—News of Oswald and Elliot. Death of Clairaut.

From the Comte de Maillebois.

VI, 25. Hume, Secretaire d'Ambassade d'Angleterre, rue St Dominique, Paris. [Paris] 26 Jan. 1764. *In third person.* [1 p. + 3 pp., 10 × 16 cm.]

Invitation to dinner.

From Lamoignon de Malesherbes.

VI, 26. [H.]. [Paris, spring, 1766.] *Signed.* [4 pp., 19 × 23 cm.; E.P., 219–21; Greig, App. C.]

Asks H. to make a slight correction in a certificate given by de M. to Rousseau.—De M. is writing to H. in case this letter should be published among the Rousseau Memoirs, which would make the veracity of the certificate more than ever suspect.—Begs H. to return to France again.

From David Mallet.

VI, 27. [H.]. [London, Apr. 1763.] *Signed.* [3 pp. + 1 p., 19 × 24·5 cm.; Burton, II, 142.]

Has at last gone through H.'s volumes with an eye to verbal errors and Scotticisms. Has not been idle himself. Intends to publish a translation at the same time as the original. Feels confident that no matter how many other historians cover the same period his own work will still be new.—Encourages H. to proceed with the work that he desired should remain a secret [an ecclesiastical history].

From the Marquis de Marigny.

VI, 28. Hume, Hôtel de S.E. M. l'Ambassadeur d'Angleterre, rue de l'Université. Paris, 26 Dec. 1764. *In third person.* [½ p. + 3½ pp., 16·5 × 21·5 cm.]

Asks for a meeting with H.

From Matthew Maty.

VI, 29. Hume, Little Warwick Street [London]. British Museum, 28 Apr. 1767. *Signed.* [2 pp. + 2 pp., 19 × 24 cm.; Burton, II, 360, inc.]

Has to return H. the papers [Rousseau's letters] that H. sent by [Allan] Ramsay. The Trustees do not think it proper to receive them. Has no doubt of R.'s character, for whom he thinks madness the only excuse. Regrets that H. had to publish, but rejoices that the majority of people are on his side.—Begs H. to send him M. Lauragais's letter.

From Andrew Millar.

VI, 30. Hume, Paris. London, 24 Apr. 1764. *Signed.* [1½ pp. + 2½ pp., 16 × 20 cm.]

M. is reprinting the *Tudors* in small 4to, and H. can therefore make the

proposed alterations in the corresponding edit. of the *Stuarts*. H. must not think of discontinuing his *Hist.*—The edit. of Coke sent to H. was the only complete one to be had.—Sir John Gordon desires a collection of the remonstrances of the Parlement during the present reign.—Mrs Mallet is wrong in her head. Millar is going to Harrogate for six weeks.

VI, 31. Hume, Paris. Harrogate, 5 June 1764. *Signed.* [1½ pp. + 2½ pp., 16 × 20 cm.; Burton, II, 202, inc.]

New edit. of the *Stuarts* will be printed before Christmas.—True story of the suppressed dissertations lent to John Wilkes.—News of H.'s friends.

VI, 32. [H.]. London, 26 Nov. 1764. *Signed.* [2 pp. + 2 pp., 16 × 20 cm.; Burton, II, 264, inc.; Greig, App. C, inc.]

Account of the sales of the *Hist.* and the *Essays*. Surprised that H. troubles himself about such matters. H. has every encouragement to proceed with his *Hist.*—Need of a new edit. of the *Essays*.

VI, 33. [H.]. London, 25 Feb. 1765. *Signed.* [3 pp. + 1 p., 16 × 20 cm.]

Pleasure at H.'s decision to continue his *Hist.* Prejudices against it will subside, and it will become the standard history. H. will have no difficulty in gaining access to documents.—Request about some of Bacon's letters from Mr Onslow. M. offers to exchange sets of Wood's *Ruins of Balbec* for the sets of Le Roy's *Plans of Athens*.—The prejudice against the Scots is directed solely against Lord Bute. The Grenville Ministry. Rise in the stocks. Rumours of reconciliation with Duke of Cumberland.—*P.S.* Inquires about Mallet, who is an agreeable man and would be much esteemed if more sincere.

VI, 34. [H.]. Kew Green, 4 Oct. 1766. *Signed.* [2½ pp. + 1½ pp., 16·5 × 20·5 cm.]

Upbraids H. for going to Scotland without seeing M. Allan Ramsay has not provided the engraving of H.'s head. Begs H. to continue his *Hist.* Has decided that it will be M.'s last new venture. Invites H. to visit him in Pall Mall. A letter from Adam Smith, who says that H. is light-headed to think of finishing his days in France. Letter from Maclaine [of the Hague], who is convinced that H. has been injured by Rousseau. R. should have been sent to Wales to live in the natural state. Maclaine desires a true account of the affair.

VI, 35. Hume, Edinburgh. Kew Green, 2 Nov. 1766. *Signed.* [2 pp. + 2 pp., 18 × 22 cm.]

Corrected copy has been found and forwarded. Quick sale of the *Hist.*, of which a new edit. will soon be required. There are as good engravers in England as in Paris.—M. urges H. to proceed with the *Hist.* Hurt that H. has left him and Cadell out of the publishing of the Rousseau correspondence.

VI, 36. [H.]. Kew Green, 22 Nov. 1766. *Signed.* [2 pp. + 2pp., 14·5 × 18 cm.; Burton, II, 392–3, inc.]

Satisfied with H.'s reasons for not entrusting him with the publishing of the Rousseau correspondence. Becket has been ungrateful enough to refuse even to add M.'s name to it. R. is mad with pride.—Adam Smith advises the continuation of the *Hist.* The excellent sales of the work. It gains the approbation of all unprejudiced readers.—The new edit. of the *Essays.*

VI, 37. Hume. Kew Green, 21 Oct. 1767. *Signed.* [1 p. + 3 pp., 16 × 20·5 cm.]

Willing to postpone reprinting the *Hist.* till H. has completed his revisions. Thinks Donaldson's drawing would be suitable for the 4to *Essays.*

From Professor John Millar.

VI, 38. Hume, St Davids Street, New Town, Edinburgh. Glasgow, Sunday [spring, 1776]. *Signed.* [3 pp. + 1 p., 16 × 20 cm.; E.P., 315–7.]

H.'s criticism of Adam Smith's style is justified, though perhaps too severe. Some of S.'s positions are, however, difficult to maintain; especially his main point, " the unbounded freedom of trade." M. admits the general incompetence of a government to deal with commerce, and the difficulty of enforcing a decision. But a government must interfere when the interests of an individual conflict with those of the public.

From the Marquis de Mirabeau.

VI, 39. [H.]. Du Bignon, 2 Aug. 1763. *Signed.* [3 pp. + 1 p., 17 × 21·5 cm.; E.P., 22–4.]

Flattered at being anticipated by so illustrious a man as H. Ignorance of English. Especially regrets inability to read Richardson, who, it seems, is not fully valued in his own country. Similarly, H. is not appreciated at home. But a historian will receive his tribute from posterity. M. has read only the *Stuarts,* and has often wished he could reason for an hour with H.

VI, 40. Hume, chez M. l'Ambassadeur d'Angleterre, Paris. [Paris] 12 Mar. 1764. *In third person.* [1 p. + 3 pp., 13·5 × 19 cm.]

Asks for an appointment to call on H., and requests him to read indulgently the book he is enclosing.

From Sir Andrew Mitchell (see also under Earl Marischal Keith).

VI, 41. Hume, Mr Conway's Office [London]. Berlin, 15 Aug. 1767. *Signed.* [1 p. + 3 pp., 18·5 × 23 cm.]

Begs for a letter from H., and sends him one that has been mislaid.

VI, 42. Hume. Berlin, 9 Jan. 1768. *Signed.* [4 pp., 18 × 22·5 cm.]

Has little respect for H. as a political prophet, and wishes that Conway would continue in an office for which he is eminently fitted.—As M.'s health is better and his election to Parliament assured, he is under no immediate necessity to return.—Begs H. to strive to obtain for Mr Campbell the situation promised by the Duke of Grafton.—Compliments from the Earl Marischal.

From F. Louis Moltken.

VI, 43. [H.]. Hôtel de Rome [Paris], 9 Mar. 1765. *In French. Signed.* [4 pp., 18·5 × 23 cm.]

Apologies for having had H. ushered into his sick room. His admiration for H. and his works.

From M. Monin.

VI, 44. [H.]. Paris, 30 June [1766]. *Signed.* [2 pp., 13·5 × 18·5 cm.]

Mme de Boufflers and he recommend M. Fantaire to H. M. regretted, on his return to Paris, to find that H. had gone. M.'s friendship with Rousseau.

From the Comtesse de Montauban.

VI, 45. [H.]. Paris, 15 Mar. 1769. *Signed. Torn.* [1½ pp. + 2½ pp., 16 × 20 cm.]

Recalls to H.'s memory the time they passed together at L'Ile Adam, and begs him to obtain information concerning " La maison de L'Emery " of Scottish origin.

From President de Montesquieu.

VI, 46. Hume, London. Bordeaux, 19 May 1749. *Signed.* [2½ pp. + 1½ pp., 19 × 24 cm.; Burton, I, 456–7; *Corr. de Montesquieu*, ed. by Gebelin and Morize, II, 188–9.]

Thanks H. for his letter containing many useful comments on *L'Esprit des lois*, particularly on the English and Scotch jury system. Has been reading some of H.'s essays with great admiration.

VI, 47. Hume, Ninewells. Paris, 3 Sept. 1749. *Signed.* [2½ pp. + 1½ pp., 16·5 × 22 cm.; Burton, I, 457; *Corr. de Montesquieu*, ed. by Gebelin and Morize, II, 222–3.]

Thanks H. for helping with the Edinburgh edit. of *L'Esprit des lois*, and for sending copies of his own works.—M. is aware of the reputation of Dr Middleton.

VI, 48. Hume, Edinburgh. Paris, 13 July 1753. *Signed.* [1 p. + 3 pp., 17 × 22·5 cm.; Burton, I, 457–8; *Corr. de Montesquieu*, ed. by Gebelin and Morize, II, 479–80.]

Thanks H. for copies of his works. Likelihood of a French translation by the

translator of [Robert] Wallace. M. admires these two friends, " qui font ceder d'une maniere si noble les petits interêts de l'esprit aux interêts de l'amitié," and would himself like some place in this friendship.

From Trudaine de Montigny.

VI, 49. [H.]. •Paris, 16 May 1759. *Signed.* [4 pp., 15·5 × 20 cm.; Burton, II, 167–8, inc.]

Has translated H.'s *Natural History of Religion.* It has pleased his friends, and he hopes it will lead to a correspondence with H. and a meeting at the end of the War.—Restrictions on the French press.—[John] Stewart's short stay in France. M. has received the case of books and has distributed them as directed.

VI, 50. Hume, Lisle Street, Leicester Fields, London. Paris, 16 Aug. 1759. *Signed.* [2 pp. + 2 pp., 16 × 20 cm.]

At H.'s request he is applying to M. de Caumartin in favour of Mr Coutts, and will spare no pains in the matter. Coutts will find little difficulty in exculpating himself.—H. will be welcomed in France, M. having spread the rumour of his coming.

VI, 51. Hume, Lisle Street, Leicester Fields, London. Montigny, 6 Oct. 1759. *Signed.* [2 pp. + 2 pp., 16 × 20 cm.]

Regrets that the death of his wife has prevented him from making very great exertions for the release of Coutts.—The two vols of the *Stuarts* are translated, but will not appear till the *Tudors* are ready also. M. hopes to be employed in any business that H. may have in France.

VI, 52. [H.]. Paris, 17 Mar. 1760. *Signed.* [1 p. + 3 pp., 16 × 20 cm.]

This letter is being delivered by [John] Stewart, whose account of France, M. hopes, will induce H. to come over. Coutts is at last free. M. offers to serve H. in any way.

VI, 53. [H.]. Montigny, 23 Aug. 1766. *Signed.* [3 pp. + 1 p., 18·5 × 23 cm.; Greig, App. K, inc.]

Has eagerly read through H.'s packet of Rousseau letters. French opinion is unanimously in H.'s favour, even among the devotees of R. M. himself completely condemns R. Does not consider that there is any need for the account of the quarrel to go into print. Will show his own translation of the documents to H.'s friends.—Mme de M., in spite of her love for R., sympathises with H.—*P.S.* Regrets the death of Sir James Macdonald.

VI, 54. Hume, at Mrs Eliot in Brewer Street, London. Paris, 1 Sept. 1766. *Signed.* [2 pp. + 2 pp., 16 × 20 cm.; Greig, App. K, inc.]

Account of those who have read the Rousseau papers. D'Alembert believes

it may be advisable to print them, in which case M. would suggest some slight alterations. Above all, moderation is necessary. Meantime H. should have complete confidence in his friends, who will print when it is advisable.—H.'s friends expect and hope to see him in September.

VI, 55. Hume, Under Secretary of State, London. Paris, 27 Mar. 1767. *Signed.* [1½ pp. + 2¾ pp., 16 × 20 cm.]

Congratulates H. on his new office, and envies him a country where men of letters receive government posts.—Ingratitude of H.'s protégé [Rousseau].

VI, 56. Hume, Lisle Street, Leicester Fields, London. Fourqueux, 15 June 1767. *Signed.* [6 pp. + 2 pp., 15·5 × 20 cm.]

H.'s humanity towards Rousseau. Turgot has requested the Prince de Conti to use his influence to prevent Rousseau's arrest. M. already perceives R.'s dangerous madness. "La combustion qui regne aujourdhui dans sa patrie est un de ses ouvrages." The quarrel with H. has served to warn the world of R.'s true character.—Delighted at the rumour that H. is returning to France to visit his friends. H.'s sanity about the causes of war. Peace is likely to continue between England and France. There are two points of friction between England and Spain: the Manilla affair and the establishment of Commodore Byron in the South Seas. Opinion on these. Begs to know what has been said in England. —The satisfaction to be gained from the study of literature.

VI, 57. Hume, Brewers Street, London. Paris, 21 Feb. 1769. *Signed.* [1½ pp. + 2½ pp., 14 × 18·5 cm.]

Thanks H. for his sympathy [on the death of Montigny's father]. H. has lost a good friend. Mme de Fourqueux has married the Contrôleur Général. His character: he is better than his predecessor.—Compliments to John Stewart.

From the Abbé Morellet.

VI, 58. Hume, London. Paris, 8 Sept. 1766. *Signed.* [3 pp. + 1 p., 15·5 × 20 cm.; E.P., 308–10; Greig, App. K, inc.]

Sends a copy of *Delitti* of Beccaria, and one of Lucian's dialogues, translated by himself. If H. approves of it, he will translate the rest.—Asks H. to come to Paris during the winter, and look over the *Dictionary* [*of Commerce*], which M. will begin to publish in November.—M.'s ideas about Rousseau coincide with those of d'Alembert and d'Holbach: that H. should print the letters, with a short account of the quarrel. M. was one of R.'s devotees, but has not been blinded to the extravagance and wickedness of his conduct towards H.

VI, 59. [H.]. Paris, 13 May 1769. *Signed.* [4 pp., 17 × 22·5 cm.; E.P., 310–2.]

Asks H. to send copies of M.'s latest work to certain friends, and to read and

criticise it himself. Hopes H. is continuing his *Hist.* Suard is translating Robertson's.—News of H.'s French friends.

From Durey de Morsan.

VI, 60. Hume, Paris. Neuchâtel, Switzerland, 17 June 1764. *Signed.* [3 pp. + 1 p., 19 × 24 cm.]

Teaches pupils for the sake of doing good. Describes two of his feats in pedagogy. If he can obtain any pupils, he will demand no fees beyond bare expenses, and being a man of private means will be satisfied with the success he attains.

From William Mure of Caldwell.

VI, 61. Hume, Jack's Land [Edinburgh]. Caldwell, 7 May [1753–61]. *Signed.* [1½ pp. + 2½ pp., 19 × 23 cm.]

Invites H. to visit him when M. returns from Bute. Directions for travelling.

VI, 62. Hume. Caldwell, 12 May 1764. *Joint letter with Mrs Mure. Signed "W. M." only.* [4 pp., 19 × 23 cm.]

M. :—Thanks H. for letters and for the trouble he has taken over M.'s scheme for finding a tutor for his sons.

Mrs M. : If H. wants an interesting letter, he must provide her with a subject. Lord Glasgow's appointment as Commissioner.

VI, 63. Hume, Paris. Caldwell, 16 Sept. 1764. *Signed.* [1½ pp. + 2½ pp., 18 × 22·5 cm.]

Recommends Mr Stewart, and thanks H. again for arranging the business of the tutor.

VI, 64. [H.] Edinburgh, 13 July [1767]. *Signed.* [2½ pp. + 1½ pp., 18·5 × 22·5 cm.]

Suspicions of Graffigni appear to be justified. Lord Bute is also affected by this discovery. M. is prepared to follow H.'s advice entirely with regard to his sons' education.—Both sides feel confident about the verdict in the Douglas Cause.

VI, 65. Hume. Caldwell, 17 May 1769. *Signed, initials only.* [3 pp. + 1 p., 19 × 24 cm.]

Thanks H. for his letter. M. is greatly pleased that his feelings and H.'s on the Douglas Cause agree. M. himself was beginning to feel that he was not impartial, till he heard H.'s unprejudiced opinion. The effrontery of Lord Mansfield's statement. M.'s sympathy for Andrew Stuart.—Glad that H. will soon be with him.

VI, 66. Hume, St Andrew's Square, Edinburgh. Caldwell, 25 Oct. [1773]. *Signed.* [1 p. + 3 pp., 19 × 23·5 cm.]

Has been too busy to write, but will have no rest till he sees H. himself. The death of [Ambassador] Keith.

[On outside of autograph, a draft sentence in H.'s hand for his *Hist.*]

VI, 67. Hume, St David's Street, Edinburgh. Caldwell, 27 Aug. [1775]. *Signed.* [1 p. + 3 pp., 18·5 × 24 cm. ; *Cald. Papers*, II, ii, 247-8 ; Greig, App. C.]

Begs H. to keep to his original plan of visiting the north of England, calling in at M.'s on his return, and going on to Inveraray.

VI, 68. Hume, St David's Street, Edinburgh. Caldwell, 4 Sept. [1775]. *Signed, initials only.* [2 pp. + 2 pp., 18·5 × 24 cm. ; *Cald. Papers*, II, ii, 250–1 ; Greig, App. C.]

H. will not be able to evade his promised visit to Inveraray. The company to be there. Directions for travelling.

From Edward Murphy.

VI, 69. [H.]. The Cell near the Sable (vulgarly called the Black) Rock, near Dublin, 2 June 1767. *Signed.* [3 pp. + 1 p., 18 × 23 cm. ; E.P., 171–3.]

Thanks H. for his speedy answer. M.'s cypher.—H. does M. honour and, M. hopes, justice, by comparing him with Anacharsis. The Irish are proved to have descended from Scythians or Skyths, which word has been corrupted into *Scots.* Swift was the son of a Skyth.—M. believes evil rulers and bad subjects to be equally injurious.—Difficult to entrust his cypher to Conway, or anyone but the King, though he expects considerable aid from Lord Charlemont.—Compliments.

From the Hon. Alexander Murray.

VI, 70. Hume, Edinburgh. Paris, 18 Mar. 1761. *Signed.* [2 pp. + 2 pp., 19 × 23 cm. ; Burton, II, 94.]

Encloses a letter to H. from Mme de Boufflers, " the most amiable and accomplished Lady in this kingdom, or indeed any other." The value of such an acquaintance, should H. come to Paris. M. offers him his sister's apartment. Begs to be remembered to Henry Home.

VI, 71. Hume, Charing Cross, London. Paris, 20 July 1761. *Signed.* [2 pp. + 2 pp., 19 × 23 cm.]

[Mme de Boufflers] is perhaps " the most accomplished of her age in the whole universe." H. can only refuse her offer of accommodation at her house by pleading an earlier promise to lodge with M.—M. can now come home, but prefers Parisian society, which he proposes never to quit. Asks for Robertson's *Hist. of Queen Mary.*—Directions about Montcalm's epitaph and monument.

VI, 72. Hume, Embassy [Paris]. [Paris] "Saturday morning" [autumn, 1763]. *Signed.* [1 p. + 3 pp., 18·5 × 23 cm.; Burton, II, 168–9; Greig, App. F.]

Invites H. to dine with him some day next week to gratify some of H.'s French admirers. The anxiety to meet H. is especially strong among the women.

VI, 73. [H.]. [Paris] 3 July 1764. *Signed.* [2 pp. + 2 pp., 18 × 24 cm.; Greig, App. F.]

Death of M.'s brother-in-law.—Expects soon to see the end of his lawsuit. The judges have already paid too much attention to the lies of his enemies. Asks H. to write to M. le Noir, begging him to be the *rapporteur* of M.'s affair. The unworthiness of the charges, and the agony they have caused M.

VI, 74. Hume, Paris. [Paris] 19 Aug. 1764. *Signed.* [1 p. + 3 pp., 19 × 23·5 cm.; Greig, App. F.]

Glad to learn that what he heard is untrue, but sorry to perceive H.'s superiority to himself in coolness and politeness. Will always be pleased to see H., but begs him to avoid this subject.

From Lady Elliot-Murray (wife of Sir Gilbert Elliot).

VI, 75. Hume, New Town, Edinburgh. Minto, 12 Oct. 1772. *Signed.* [4¾ pp. + 3½ pp., 18 × 24·5 cm.; Burton, II, 446–9.]

Through not taking her advice H. has had to travel 70 miles in one day merely to gratify an excessive sense of delicacy. Relates the conversation between Sir Gilbert [Elliot] and herself, when E. arrived and found H. gone. She was forced to promise that H. would return soon, and if he does not, he will be responsible for making her deceive her husband.—Sir Gilbert and she have strong feelings of gratitude towards the [French] Ambassador [the Comte de Guines], but can hardly invite him except officially, which at present is impossible.—Sends her best wishes to her "poor good Harry," who thinks H. a saint.

From Daniel O'Conor.

VI, 76. [H.]. London, 10 Feb. 1764. *Signed.* [4 pp., 18·5 × 22·5 cm.]

Encloses a printed letter deprecating H.'s prejudices against the Irish, which are unworthy of a philosopher. Hopes the reasoning in it will lead H. to expunge the offending sections of his *Hist.* The Roman Catholic Irish have suffered calumny, and have been led by the penal laws to their own destruction. "To Consider the present Roman Catholick Irish in a proper Light, You must Consider them, Sir, as a people half murthered, Chained to the ground, and constantly trod upon in this Situation, by a troop of wanton Oppressors."—The evil effects of accusations from so great a man as H. O'C. begs him to add an appendix to the new edit., retracting his errors.

From " O'Murraghoo Rex."

VI, 77. [H.]. [? , ? .] *Signed.* [½ p. + 3½ pp., 19 × 22 cm.]

Asks H. to make a tour of Ireland. The Irish will not eat him. They are too careful about the quality of their meat, to eat Scotsmen.

From Angelo Pavesi.

VI, 79. [H.]. Piacenza, 6 Jan. 1765. *In Italian. Signed.* [3 pp. + 1 p., 18 × 24 cm.]

H.'s works are admired in Italy, as in England and France. They have inspired P. to write a book in Italian on commerce, etc., about which he asks H.'s advice.

From Sam Pechell.

VI, 80. Hume. London, 5 Mar. 1765. *Signed.* [3 pp. + 1 p., 19 × 23 cm.]

Concerning some money lent by General Caillaud in Pondicherry to M. de Lally and M. de Landivisiau which has not been repaid. Asks H. to apply to the French Ministry for payment from the former's estate, and to remind the latter, if possible, of the debt. [See also **VIII, 20.**]

From the Rev. Thomas Percy (afterwards Bp. of Dromore).

VI, 81. [H.]. Northumberland House, 5 Jan. 1773 [wrongly dated 1772]. *Signed.* [8 pp., 18·5 × 22·5 cm.; E.P., 317–20.]

Thinks H.'s analysis of the *Northumberland Household Book* much too severe, and begs him to alter it before the new edit. of the *Hist.* appears. Has himself defended it in a book, a copy of which is being forwarded to H.—The Earl was liberal, not niggardly, considering the times in which he lived.—*P.S.* Suggests that H. should relate an instance of the Earl's donations.

VI, 82. [H.]. Northumberland House, 22 Jan. 1773. *Signed.* [6 pp., 18·5 × 22 cm.; E.P., 321–4.]

Thanks H. for his letter. Any alteration will not mean a sacrifice of truth, or to his case; for if the Earl's hardships were due to his parsimony, H. cannot draw general inferences about the conditions of life at the time.—Detailed criticism.—P. leaves everything to H.'s impartiality and candour.

From P. A. du Peyrou.

VI, 83. [H.]. Neuchâtel [Switzerland], 13 Feb. 1766. *Signed.* [2 pp. + 2 pp., 18·5 × 22·5 cm.]

Admiration for H. and Rousseau leads him to express his gratitude for the distinguished services H. has rendered to R. in England. Encloses a letter for R.

From the Abbé Prévost.

VI, 84. [H.]. Paris, 18 Mar. 1760. *Signed.* [2½ pp. + 1½ pp., 16·5 × 21·5 cm.; E.P., 292–3.]

P.'s sense of the honour his acquaintance with H. does him. Has translated the *Stuarts*. Regrets that he has not been able to let H. revise the translation.—*P.S.* Sends H. a copy of his own new work, *Monde Moral.*

From Richard Price.

VI, 85. [H.]. Newington Green, 24 Mar. 1767. *Signed.* [1¼ pp. + 2¾ pp., 19 × 22·5 cm.]

Thanks H. for his letter. Never allows himself to be prejudiced by a difference of opinion, but believes "that nothing is fundamental besides a faithful desire to find out and to practise truth and right."

From Sir John Pringle.

VI, 86. [H.]. London, 9 May 1765. *Signed.* [2½ pp. + 1½ pp., 18·5 × 23 cm.]

Recommends Mr Hewson, and asks that he be introduced, if possible, to M. Duhamel or any distinguished surgeon.—Satisfaction at hearing of H.'s popularity in Paris.—On Ossian. Has suspected the authenticity of the poems, but all doubts should be removed by "the cloud of witnesses" produced by Blair in his defence.—*P.S.* A foreign medical man placed Ossian almost on a level with Homer.

VI, 87. [H.]. London, 9 Aug. 1765. *Signed.* [4 pp., 18·5 × 23 cm.; E.P., 267–9.]

Has sent the particulars of H.'s case to Mr Sharpe. If he sees Sharpe in town, he will pay him; if he goes to Paris, H. should give him 2 louis d'or.—Finds H.'s remarks on Ossian unsatisfactory.—Congratulates H. on being appointed Secy. to the Embassy. He will, however, miss the enjoyments of Parisian society, "Circe's enchanted island."

VI, 88. [H.]. London, 26 Jan. [1773]. *Unsigned.* [3 pp. + 1 p., 18·5 × 23 cm.]

Opinions on the War with the American Colonies. They did not desire to break away till England attempted to impose a tax on them. "On the contrary, they considered themselves as a happy people, & took pride in being part of this great & flourishing empire." Even now Pennsylvania refuses to consider any course entailing separation. The governors sent to them could never be oppressive.—The British conquest of Canada should have discouraged the revolt. The Americans would not have fought the French to become free.

VI, 89. [H.]. London, 5 Mar. 1773. *Unsigned. Half of 2nd page missing.* [3 pp., 18·5 × 23 cm.; Burton, II, 465–6, inc. and wrongly dated.]

Thanks H. for his letter, which served as an introduction to Lord Hardwicke.

An extraordinary story of the Pretender's cowardice, contrasting with the accepted stories of his personal courage.—Would like to aid Mr Fraser, but fears will have no opportunity to do so . . . [*part missing*].—The century of uninterrupted prosperity of the country is bound to lead to some reduction in the liberties of the people . . . [*part missing*].

VI, 90. [H.]. London, 22 Oct. 1773. *Signed.* [2 pp. + 2 pp., 18·5 × 23 cm.; E.P., 269–70.]

Thanks H. for taking care of his house. Danger from the west winds. Homer's description is so true that some Scots critics may claim him as a countryman.—Pleased to hear of the marriage of one of the Miss Ordes with a Scots gentleman. The value of such marriages in softening Scots manners.

VI, 91. [H.]. London, 2 Nov. 1773. *Signed.* [4 pp., 18·5 × 23 cm.]

Detailed instructions for furnishing and decorating his house in Edinburgh. Annoyance at the prospect being obstructed by other buildings.

VI, 92. [H.]. London, 21 Dec. 1773. *Signed.* [3 pp. + 1 p., 18·5 × 23 cm.]

Asks H. to send him an account of the expenses incurred in the building of his house, and thanks him for the trouble taken.—Further instructions about the house.—Has been re-elected Pres. of the Royal Society, and has been asked to publish a discourse of his on the different kinds of air; probably to indemnify him for the manner in which he was abused by a Grub Street journal last year. Similar abuse has appeared this year.

VI, 93. Hume, St Andrew's Square, Edinburgh. London, 28 Mar. 1774. *Unsigned.* [4 pp., 18 × 22·5 cm.; E.P., 270–2, inc.]

Has decided not to continue the building of the house.—[Benjamin] Franklin's attitude towards the civil war: not inspired by love of faction but by zeal for his own country. F.'s excessive closeness, when advice would have prevented him from using a private correspondence. But F. laboured to preserve peace, and the acceptance of his advice might have prevented the mischief. The Colonies were not waiting for a pretext to revolt, but were content, till the action of Grenville.

VI, 94. [H.]. London, 7 Apr. 1774. *Unsigned.* [4 pp., 15·5 × 20 cm.; E.P., 272–3, inc.]

About a small building he had intended to annex to his new house, and his nephew's anxiety to construct it.—Doubted, when preparing the list of the Royal Society, whether the Chief Baron of the Scottish Exchequer should be called Honourable or Right Honourable. Discussed the matter with the King, who decided that on such occasions the higher title should be given.—Prophesies trouble in [American] affairs.

VI, 95. [H.]. London, 13 Apr. 1775. *Signed.* [4 pp., 18·5 × 22·5 cm.]

Contemplates with dissatisfaction the setting up in Edinburgh of a card assembly to replace the old dancing assembly. Will not lend his house for the purpose. Has empowered a cousin to sell his house as soon as possible.—Thanks H. for praising his Discourse to the Royal Society. Sets much more store by H.'s praise than by the malicious abuse of his attacker.

VI, 96. [H.]. London, 29 May 1775. *Signed.* [2 pp. + 2 pp., 18·5 × 23 cm.]

Recommends M. Leclerc de Septchênes from Paris.—Hearing of H.'s ill-health, begs him to come to London for medical advice.

VI, 97. Hume, St Andrew's Square, Edinburgh. London, 8 July 1775. *Signed.* [4½ pp. + 3½ pp., 18·5 × 23 cm.]

H. is suffering rather from a peculiarity of constitution than from a disease. He need only be careful about food, drink, exercise, etc., and not take even rhubarb. Can suggest no medicine to overcome the heats H. feels at night.—H. does well not to trouble to write any other great work, but might easily write some essays or maxims.—American situation: the Americans were formerly quite content. They subscribed to the last war and shared in its benefits. "Every thing they wanted they took from England, paid money for it, & were proud of the purchase." But the Stamp Act and the Tea Tax and the attempt to enforce them caused dissatisfaction. The American army.—The state of affairs would have been worse had the French remained in Canada. Nor is the increase in population to blame. English claims were those of an ambitious and tyrannous country. The tie of affection towards the Mother Country would have preserved the Union.

VI, 98. [H.]. London, 2 Jan. 1776. *Signed, initials only.* [4 pp., 18·5 × 23 cm.]

Has long since quarrelled with the War Office and so can do nothing for Dr Lind, but in case there should be a second request for a physician to be sent to America he will recommend L., though he has small confidence in success. The disagreeable and dangerous nature of war service. Captain Cook: preparations for a third voyage.—P.'s Discourse printed but not yet published. Copies are being sent to H. and to Lord Kames.—Suggests that H. should have his body rubbed with coarse gloves every morning.

VI, 99. [H.]. London, 25 Jan. 1776. *Unsigned.* [3 pp. + 1 p., 18·5 × 23 cm.]

Trade with America. The vast balance in favour of England. Knowing the influence H.'s opinions are likely to have, P. sends Glover's speech as a more reliable source of information.

VI, 100. [H.]. London, 25 Feb. 1776. *Signed.* [3 pp. + 1 p., 18·5 × 23 cm.]

Encourages H. again to come to London for medical advice. The greater variety of cases the doctors have to deal with there. If H.'s sole objection is that he can lie only in his own bed, he can bring that and the maid who makes it along with him. They will employ natural, not shop remedies. The journey and the sight of old friends will contribute to his recovery.—*P.S.* Some news from France. America: rumoured repulse of the Americans before Quebec. The poll is going against Wilkes, though his defeat is by no means certain. A *bon-mot* of Wilkes.

VI, 101. Hume, Bath. London, 7 June 1776. *Signed.* [2½ pp. + 1½ pp., 18·5 × 23 cm.]

Advises H. to stay some time longer in Bath, till his physician tells him he is receiving no benefit from the waters. Urges him to continue rubbing himself down.—Probability of a second defeat of the rebels before Quebec. Arrival of Howe at Halifax. No news of Cornwallis.—Begs H. not to despair of recovery. Knows it is needless to warn him against melancholy. "I doubt that the prospect of the very worst would not make you unhappy."

VI, 102. [H.]. London, 5 Aug. 1776. *Signed.* [2 pp., 19 × 23 cm.]

Regrets the increased pain H. felt on the return journey. H.'s physicians' rejection of "the very name of a mercurial course" gives P. some hope of their knowing some other way of recovery. –Begs to hear the state of H.'s health once a fortnight.

From Allan Ramsay (the painter).

VI, 103. Hume, Advocates' Library, Edinburgh. Rome, Mons Viminalis, 13 Mar. 1756. *Signed.* [3¼ pp. + ¾ p., 19 × 26·5 cm.; E.P., 30–3, inc.]

Has delayed writing from having nothing to write about. H. already knows everything about Papacy and slavery.—Sends a Greek inscription [of his own invention] which he professes to have found in the Farnese Palace, containing a living testimony to a miracle.—Glad there are people in Scotland who devote themselves to luxury.—Asks whether the premiums [offered by the Select Society] have produced an essay on taste. R. himself does not stand very high in reputation as a philosopher, being in general considered "a comical dog." Has re-read H.'s *Hist.* with great satisfaction; and particularly the character of Shakespeare as it now stands. Prides himself on having suggested the alteration.—H. cannot expect help from parties, for the lovers of truth and reason do not form parties. Sir Harry Erskine's opposition to the Ministry. Elliot's reputation causes no surprise. Asks about Home's plcy, and the farce. Compliments to Lord Kames and to Elliot. Wishes further information about the books requested for the Advocates' Library.

VI, 104. [H.]. London, 8 Nov. 1763. *Signed.* [3 pp. + 1 p., 16·5 × 19·5 cm.; E.P., 29–30.]

Recommends Lady Hervey's son, she being laid up with gout. Mrs Macaulay's *Hist.*: " a Romance called James the first, the Secret design of which is to abuse you and me." Determines on revenge. Lycurgus recommends democrats first to set up democracies in their own house, so let Wilkes marry Mrs M.—Compliments to Lord Hertford. Portraits of the King and Queen are ready.

From Michael Ramsay.

VI, 105. [H.]. London, 5 June 1764. *Signed.* [3 pp. + 1 p., 18 × 22·5 cm.]

Recommends Dr Burney, " one of the most ingenious & deserving " of musicians.—Not from lack of affection to H. that he writes so seldom. " No friend you have, and you have more real ones than any man I ever knew, w[t]out including admirers: I say, no one of them enters w[t] a purer satisfaction into every Honor and every agreable circumstance of life you enjoy than I do. I hear often of you, & no longer ago than this morning by Tristram Shandy."—For himself, he continues in his old wandering way. Has been to Edinburgh and seen H.'s nephew Josey, who would do better but for " a foolish old Woman of a Grandmother."—News of his own nephews.

VI, 106. Hume. Bannockburn, 10 Sept. 1764. *Signed.* [1½ pp. + 2½ pp., 18 × 22·5 cm.]

Recommends Mr Sargent. S.'s fortune and character; the ill-health and amiability of his wife.—News about H.'s friends, and about R. and his family.

From the Abbé Raynal.

VII, 2. Hume, Hôtel de l'Ambassadeur d'Angleterre, rue St Dominique [Paris]. [Paris] 23 Dec. [1764]. *In third person.* [1 p. + 3 pp., 12 × 17 cm.]

Sends the Hist. of the Divorce for H. to read. Suggestions for a date for H. to dine with Mlle Hellin.

From Thomas Reid.

VII, 3. Hume, Edinburgh. King's College [Aberdeen], 18 Mar. 1763. *Signed.* [2 pp. + 2 pp., 19 × 23 cm.; Burton, II, 154–6.]

Thanks H. for praising R.'s performance [*Enquiry into the Human Mind*]; and especially for advice about style.—His debt to H. " I have learned more from your writings in this kind than from all others put together." The value of H.'s work, whether accepted or not, in assailing accepted principles.—Hopes for H.'s further remarks on his work. H.'s " friendly adversaries " in Aberdeen. Though they are all good Christians, H. would be more welcome among them than St Athanasius.

From Mme Riccoboni.

VII, 4. Hume, Secretaire de l'Ambassade, rue de l'Université, Paris. [Paris] " ce vendredi " [1763–5]. *Unsigned.* [½ p. + 3½ pp., 15 × 19 cm.]

Asks H. to send for two large volumes arrived from London.

VII, 5. Hume, Hôtel de Grimberg, Paris. [Paris, early 1764.] *Signed.* [1 p. + 3 pp., 14·5 × 18 cm.; E.P., 304–5.]

Has been reading H.'s *Hist.* in English, and praises it. After this, she is ashamed to look upon " the stuff " she is sending H.

VII, 6. [H.]. [Paris, Mar. 1764.] *Signed.* [2 pp., 13·5 × 19 cm.]

Concerning an English translation of her novel *Miss Jenny.*—The effect of absence on her friendship with H.

VII, 7. [H.]. [Paris, Mar. 1764.] *Unsigned.* [3 pp. + 1 p., 10 × 15 cm.; E.P., 303–4.]

Fears H. will never have leisure to cultivate her acquaintance. *Miss Jenny* is soon to appear. Begs H. to look after the English translation.—A relative of Fielding's has desired to visit her.—Upbraids H. for writing so seldom to her.

VII, 8. Hume, Hôtel de Charost, Paris. [Paris] 2 Feb. 1765. *Signed.* [3 pp. + 1 p., 15·5 × 19·5 cm.]

Becket [the London bookseller] has informed her of the slow sale of her book in England. It has not caught the fancy of the public. She is offended by B.'s awkward praises and lack of tact.—B. has also offered to sell some French copies, and to pay her £10 now and the rest when they are sold.—Such trifles, however, can have little interest for H.

VII, 9. Hume, Lisle Street, Leicester Fields, London. [Paris] 29 June 1766. *Signed.* [2½ pp. + 1½ pp., 16 × 20 cm.; E.P., 301–3.]

Thanks H. for his letter. Her precautions against allowing [Alexander] Wedderburn to visit her. Mlle de Riancour's admiration for H. Mme R. hates mere ceremony.—Glad H. is continuing his *Hist.*, but regrets his prolonged absence.

From John Roberts.

VII, 10. [H. ?]. Curzon Street [London], 29 Sept. 1766. *Signed.* [4 pp., 18 × 22 cm.]

Account of a Negro struggle in Africa, instigated by John Cunenter [?]. C. adopted a Negro boy, whom he left with Captain Hamilton as a pledge for debt. Hamilton sold the boy in the West Indies, since C. refused to redeem him. The

boy and another one were, however, redeemed by Mr Crichton, who paraded them in England as Southern princes.

From William Robertson (not the historian).

VII, 11. [H.]. At Mr Becket's in the Strand [London], 3 Oct. 1767. *Signed.* [1½ pp. + 2½ pp., 18·5 × 23 cm.]

Pleased to hear that his book has been presented to H. His aims were to remove the religious prejudices of the world and replace them by reason and toleration.—Would like to hear H.'s opinion of his design and treatment, but hesitates to visit H., because of the importance of H.'s business and the lack of gratitude shown to R. by the *literati*.

From J.-B.-R. Robinet de Châteaugiron.

VII, 12. [H.]. Bouillon, 2 Sept. 1767. *Signed.* [*Two documents as under:*—]

A. Letter: [2½ pp. + 1½ pp., 18·5 × 22·5 cm.]

Encloses copy of a letter [*B.*] which has remained unanswered.—Has been working on the history of philosophy among the Greeks, Romans, English, and French, attempting to prove that philosophy has always been the friend of society and has been persecuted only by the wicked. His difficulty in completely understanding English philosophy has led him to appeal to H.—Has just dined with Dr Tucker, Dean of Gloucester.

B. Letter of 17 Dec. 1765, addressed to H. in Dublin. [1¼ pp. + ¾ p., 19 × 23 cm.]

Intends to prepare a complete edit. of H.'s works in translation, and desires H.'s advice on certain points. Which translation of the *Polit. Disc.* is he to follow?—6 vols. of H.'s works have appeared in Holland. Voltaire had a hand in the translation of the "essais de morale." The notes added by the translators will be suppressed. The translation of the *Hist.* is said to have been mangled in places. R. will follow the best English edit. Inquires whether the *Treatise of Human Nature* (London, 1739) and a *Tableau philosophique de l'histoire du genre humain* (on the point of being issued) are really by H. R. will also be pleased to translate any unpublished works H. may have.

VII, 13. Hume, London. Bouillon, 8 Jan. 1768. *Signed.* [2 pp. + 2 pp., 19 × 23·5 cm.]

A long illness has prevented him from answering H.'s letter sooner.—Agrees with H. that the human race will never become philosophical. Two classes of men: those who think, whose religion is philosophy, and those who do not, whose religion is superstition. The highest hope of philosophy is to discredit the apostles of falsehood. If, as H. believes, this stage has now been reached in England, the present century will form the most glorious era in English philosophy. Even in France "il y a néanmoins une fermentation dans les esprits qui présage quelque chose de grand."—Thanks H. for promise to send him new

edit. of philosophical works. Mentions a wretched compilation published in France called *Pensées philosophiques, morales, critiques, littéraires & politiques de M. Hume.* The preface is in bad taste.

VII, 14. Hume, Ecuyer, London. Bouillon, 9 Apr. 1768. *Signed.* [1 p. + 3 pp., 16 × 20·5 cm.]

Thanks H. for a copy of *Philosophical Essays.* Difficulties of the task R. is undertaking. Will begin revising the French translation next month.

From M. de Rolland.

VII, 15. [H.]. Hôtel de Reims, rue de l'hirondelle, Paris, 12 July 1769. *Signed.* [3 pp. + 1 p., 18·5 × 24 cm.]

Sends a letter from Helvétius, which unforeseen circumstances have prevented him from delivering in person. Having written a poem on the Order of the Garter, which has been highly praised by discerning friends, and having dedicated it to the Queen of England, he begs H. to tell him how he can send a specially prepared copy to her. Asks also whether it is likely to take in England.

From D'Augier des Roques.

VII, 16. [H.]. Chateau des Roques, 8 Nov. 1763. *Signed.* [3 pp. + 1 p., 18·5 × 24 cm.]

Admiration for H. Also for Count of Harcourt whose acquaintance des R. made while a prisoner in England. Now des R. is at leisure at home, desiring only, as a consummation of his happiness, the friendship of H., " le plus grand génie de la Grande Bretagne."

From Jean-Jacques Rousseau.

VII, 17–18. Hume, Edinburgh. Motiers-Travers, 19 Feb. 1763. [*Two documents as under :—*]

A. Original letter. *Signed.* [2 pp. + 2 pp., 18 × 23·5 cm.; *Exposé succinct ; Corr. de R.*, 1826, III, 129–31 ; *Corr. gén. de R.*, ed. Dufour, IX, 103–4 ; Greig, App. G.]

Regrets having put off his visit to England, but the kindness of Lord Marischal has, so to speak, enabled him to find Scotland in the middle of Switzerland.

B. Duplicate of above with slight addition. [2½ pp. + 1½ pp., 12 × 18·5 cm.]

VII, 19. Hume, Paris. Strasbourg, 4 Dec. 1765. *Signed.* [1 p. + 3 pp., 16·5 × 22 cm. ; *Exposé succinct ; Corr. de R.*, 1826, IV, 231 ; *Corr. gén. de R.*, ed. Dufour, XIV, 315 ; Greig, App. G.]

Accepts H.'s invitation, and will set off in five or six days. His only desire is a quiet place in which to end his days.

VII, 20. Hume, London. [Chiswick] " ce lundi soir " [17 Mar. 1766]. *Signed.* [1 p. + 3 pp., 15·5 × 20 cm.; Burton, II, 305; *Corr. gén. de R.*, ed. Dufour, XV, 109; Greig, App. G.]

Prays to be excused from dining with Lady Aylesbury.

VII, 21. Hume, London. Wootton, 22 Mar. 1766. *Signed.* [2 pp. + 2 pp., 16 × 20 cm.; *Exposé succinct; Corr. gén. de R.*, ed. Dufour, XV, 117–8; Greig, App. G.]

His arrival at Wootton and delight in the place. Gratitude to H.

VII, 22. Hume, Lisle Street, Leicester Fields, London. Wootton, 29 Mar. 1766. *Signed.* [3 pp. + 1 p., 16 × 20 cm.; *Corr. gén. de R.*, ed. Dufour, XV, 128–9.

Life at Wootton. Excessive attentions of Davenport. Difficulty in conversing with the servants. Asks H. to send off a present to du Peyrou. The wintry weather.

VII, 23. Hume. Wootton, 23 June 1766. *Signed.* [2 pp. + 2 pp., 16 × 20 cm.; *Exposé succinct; Corr. gén. de R.*, ed. Dufour, XV, 275–6; Greig, App. G.]

H. is accused of bringing R. into England only to dishonour him. R. will have no further dealings with him.

VII, 24. Hume. Wootton, 10 July 1766. *Signed.* [18 pp. + 2 pp., 18·5 × 30 cm.; *Exposé succinct; Corr. gén. de R.*, ed. Dufour, XV, 299–324; Greig, App. G.]

The detailed accusations against H.

From Owen Ruffhead.

VII, 25. Hume. Middle Temple [London], 1 Mar. 1763. *Signed.* [3 pp. + 1 p., 18 × 30 cm.; E.P., 41–4.]

Though greatly admiring H.'s *Hist.* as a whole, begs to point out some errors in the last vol.—Detailed criticisms.—If these are inaccuracies, begs H. to correct them. Again expresses his great debt to H.'s *Hist.* as a whole.

From " Ruth."

VII, 26. [H.], " Boaz." Edinburgh, 20 Apr. 1767. *Signed Ruth* [4 pp., 18·5 × 23 cm.]

An allegorical reproach in Biblical language to H. for allowing his affections to be attracted by Rousseau, " that false Prophet from the mountain." Praises H. for his forgiveness. Begs him not to allow his benevolence to be discouraged by his experience. A special plea for assistance to the writer.

From the Comte de Sarsfield.

VII, 27. Hume, Edinburgh. London, 13 Aug. 1775. *Signed.* [2 pp., 16 × 20 cm.]

Asks H. why he does not visit his French friends. A list of those who would be glad to see him.

From J. H. Schneider.

VII, 28. [H.]. Amsterdam, 5 Dec. 1763. *In French. Signed.* [3 pp. + 1 p., 19 × 22·5 cm.]

Sends a copy of H.'s essays in French. S. is about to publish Vol. VI of the 1st edit. and Vol. I of the 2nd edit. Had intended to begin the new edit. with a dedication to H., but feared to be presumptuous. Will correct any errors H. points out. Vols. I–IV were translated at Berlin, and Vol. V at Paris.—A vol. has also been translated by Mauvillon, and one by the Abbé Le Blanc. The latter seems the better.

From the Bishop of Senlis.

VII, 29. Hume. [? , ? 1763–5.] *In third person, in English.* [1 p. + 3 pp., 11 × 17 cm.]

The Bp. desires to see H. and to arrange to purchase regularly the *Annual Register*, "at its setting forth as ornamental to His English Library."

From Leclerc de Septchênes.

VII, 30. [H.]. Edinburgh, 10 July 1775. *Signed.* [2 pp. + 2 pp., 15 × 19 cm.]

Has called several times at H.'s house to thank him for his kindness. Has been compelled to leave before H.'s return from the country, but hopes that H. will send on letters of introduction to London.

From Adam Smith.

VII, 31. Hume, Edinburgh. Glasgow, 22 Feb. 1763. *Signed.* [1 p. + 3 pp., 20 × 32 cm.]

Recommends Henry Herbert, and invites H. to Glasgow.

VII, 32. Hume, Paris (re-addressed to Compiègne). Toulouse, 5 July 1764. *Signed.* [3 pp. + 1 p., 17 × 20·5 cm.; Burton, II, 228, inc.]

Duke of Buccleugh proposes to set out soon for Bordeaux. Requests recommendation to Duc de Richelieu and Marquis de Lorges. Duc de Choiseul has not introduced them to the fashionable world, as was expected. Their lack of progress, so far, due partly to the Duke's ignorance of French. Life is rather dull. S. has begun to write a book to pass the time. Invitation to Sir James [Macdonald] to pass a month with them.

VII, 33. [H.]. Toulouse, 21 Oct. 1764. *Signed.* [3 pp. + 1 p., 18·5 × 23 cm.]

Thanks H. and the Ambassador for recommendations to the Duc de Richelieu. Kind reception by the latter. Proposes to return soon to Bordeaux.—Begs H. to receive Mr Scott kindly. The Duke is much improved by visits to Bordeaux and Baguières. Life gayer now. When joined by Scott, they intend to visit the meeting of the States of Languedoc at Montpélier. Desires a recommendation to the Comte d'Eu, the Archbp. of Narbonne, and the Intendant.

VII, 34. Hume, Paris. Toulouse, 4 Nov. 1764. *Signed.* [1 p. + 3 pp., 18·5 × 23 cm.]

Recommends Mr Urquhart, "the only man I ever knew that had a better temper than yourself."

VII, 35. Hume, Leicester Fields, London. Paris, 6 July 1766. *Signed.* [2 pp. + 2 pp., 23 × 37 cm.; Burton, II, 350–1; Greig, App. K.]

Convinced that Rousseau is a rascal, but begs H. not to publish the correspondence. R. wants publicity, and if attacked by H., will have a strong party supporting him, including the Church, the Whigs, and the whole English nation. All H.'s friends, and especially Turgot, beg him not to publish. S. fears the influence on H. of the "English literati," who are accustomed to publish all their trifling gossip.

VII, 36. Hume, Under-Secretary for the Northern Department, London. Kirkcaldy, 7 June 1767. *Signed. Mutilated.* [2¼ pp. + 1¾ pp., 19 × 24 cm.]

Recommends Comte de Sarsfield.—Occupied in study. His amusements are long solitary walks by the sea. Never happier in his life.—Inquires about Rousseau, and the agreement between the Ministry and the East India Co.

VII, 37. Hume, Under-Secretary for the Northern Department, London. Dalkeith House, 13 Sept. 1767. *Signed.* [2½ pp. + 1½ pp., 18·5 × 30 cm.]

Indignation at the behaviour of the Bp. of Raphoe.—Recommends a cousin, Mr Skeene.—Duke and Duchess of Buccleugh have been here for nearly a fortnight. Duchess a very agreeable woman. H. would fall in love with her.—Requests a true account of Rousseau's movements.

VII, 38. Hume, St Andrews Square, Edinburgh. Edinburgh, 16 Apr. 1773. *Signed.* [1½ pp. + 2½ pp., 18·5 × 23 cm.]

Has left the care of his literary papers to H. None worth the publishing except "a fragment of a great work which contains a history of the Astronomical Systems that were successively in fashion down to the time of Des Cartes." Suspects more refinement than solidity in it. Directions where to find it.

VII, 39. Hume, St Andrews Square, Edinburgh. Kirkcaldy, 22 Aug. 1776. *Signed.* [2½ pp. + 1½ pp., 20 × 33 cm.]

Concerning the property of the MS. of H.'s *Dialogues*. Suggests S. should add a few lines to H.'s *My Own Life*, and correct the proofs for the next edit. of H.'s works. But still has hopes of H.'s recovery.

From Tobias Smollett.

VII, 40. [H.]. London, 31 Aug. 1768. *Signed.* [1 p. + 3 pp., 18·5 × 23·5 cm.; Burton, II, 418–9.]

Recommends Capt. Robert Stobo, who has an intimate knowledge of American affairs.—S. is going into "perpetual exile." Wishes health and happiness to H., whom he considers one of the best men and undoubtedly the best writer of the age.

From the Countess of Stanhope.

VII, 41. [H.]. Geneva, 4 July 1765. *Signed.* [1 p. + 3 pp., 19 × 23 cm.]

Introducing two wonderful boys, who are travelling alone. Duchesse d'Enville talks well of H. A message to Lady Hertford.

VII, 42. [H.]. [Lucerne] 3 Aug. 1765. *Unsigned.* [2½ pp. + 1½ pp., 18·5 × 23 cm.]

The people's dislike of Voltaire, who has had to decamp to France. Things are quiet at present, but a trifle might raise a flame again. "The mob with power are insolent & ungovernable, & none ever knows the turn they may take." Some change will in time be inevitable.—Staying in Lucerne. Has come for her son's education. Considers England, on the whole, the best place to live.—She has now an Irish estate, but prefers to remain Scots. A conversation with Wilkes on Scotland. Lord Mountstuart is coming. She is glad that Wilkes has gone.

VII, 43. [H.]. Geneva, 17 Feb. 1766. *Signed.* [3½ pp. + ½ p., 16 × 20 cm.]

Concerning the English representative at Geneva, a man of infamous character, who mixes in party strife, though the Council is unwilling to make a formal complaint against him. He falsely ascribes his lack of popularity to his disdain for it, and to his friendship towards England. He may cause great trouble at the present critical time. He is a liar and serves three masters. Mystery of his appointment. She begs H. to inform Gen. Conway of the situation.

VII, 44. [H.]. [Geneva] 12 Sept. 1766. *Unsigned.* [3½ pp. + ½ p., 18 × 23 cm.; Greig, App. K.]

Hears that H. intends to publish the correspondence with Rousseau, and advises him to be more careful in future in the choice of his friends. She has been fighting H.'s battle, and believes his excessive credulity has been his only fault in the affair. Had he procured R. a pension of £1000, R. might have assassinated him.—She will be careful in making use of H.'s letter. Her hatred of villany like R.'s. There are many more like him.

VII, 45. Hume, Edinburgh. [Geneva] 21 Nov. 1766. *Unsigned.* [2½ pp. + 1½ pp., 18·5 × 23 cm.]

The printing of the Rousseau correspondence in France. Sends an extract from the advertisement. Great interest generally in the affair. She has fulfilled H.'s wish.—R.'s friends hardly support him, and his enemies and the impartial are unanimously for H. No one fears a quarrel with R. It only proves that R. is under an obligation to the person he has quarrelled with. " If there is a Hell, that man will fry ; bad as you are, I think you'll not go to the same place."

VII, 46. Hume, care of Hamilton and Balfour, Edinburgh. [Geneva] 24 Nov. 1766. *Unsigned.* [1 p. + 3 pp., 16 × 20 cm.]

Has received his two letters. Has done right in disobeying his first orders. Will destroy his letter. Affairs here in a bad way.

Sir James Steuart of Coltness.

VII, 47. [H.]. Coltness, 10 Nov. 1767. *Signed.* [2½ pp. + 1½ pp., 18 × 23 cm. ; E.P., 174–6.]

Prospect of Chatham's return to power. S.'s own hopes. Begs H. to thank Conway for past favours, and to remind him of S.'s existence.—H.'s care in searching Scripture for a Papal quotation. Advice to him for his *Hist.* The English Constitution always must be changing.

From Archibald Stewart of Allanbank.

VII, 48. Hume. Bath, 9 Mar. 1775. *Signed.* [3¼ pp. + ¾ p., 18 × 23 cm.]

Having caught a cold, he was not cured till he took a journey to London. Counsels H. to come to Bath, and will engage lodgings for him. H. should bring his cook with him.—If H. prefers, S. will meet him at York.

From John Stewart.

VII, 49. Hume, London. Paris, 1 Mar. 1759. *Signed.* [5½ pp. + ½ p., 18 × 23 cm. ; Greig, App. C.]

Has been caressed in Paris as H.'s friend. Encourages H. to come to Paris.—Praise of Mme Dupré [de St Maur], M. de Trudaine, and his son [Trudaine de Montigny], who has translated H.'s *Natural History of Religion.* Turgot, the Chevalier de Chatlus [Chastellux], and Helvétius are also H.'s admirers. Compliments from other Parisian men of letters.—Asks H. to send over books listed.—*P.S.* List of further books wanted.

VII, 50. [H.]. [Paris, Mar. 1759.] *Signed, initials only.* [2½ pp. + 1½ pp., 18 × 23·5 cm. ; Greig, App. C.]

Sending H. Montigny's translation of the *Natural History of Religion.*—Asks for some additional books. By sending them H. will be serving " men who would

go to the Indies to serve " him. H. is the man they hold in highest esteem in the world.—Advises H. to write to M. and Mme Dupré [de St Maur] and to M. de Trudaine. A high value is placed on his letters in Paris.—Encloses proposals for publishing the Academy's map of France.

VII, 51. [H.]. London, 6 Mar. 1764. *Signed.* [4 pp., 18·5 × 22 cm.]

Apologises for having delayed writing. Knows H. practises " the Christian doctrine of forgiveness more religiously than any shaved head in Christendom." Will be better in future. Does such perverse delay proceed from fear or from pride ?—Asks H. to inform M. Maltête that S. has ceased to deal in Burgundy.

VII, 52. Hume, Paris. London, 28 Sept. 1764. *Signed.* [4 pp., 18 × 23 cm.]

Recommends Capt. Stewart, who is passing through Paris. The story of a scandal between him and a lady of unspotted virtue is discredited. Has seen Elliot, who says it is impossible to spoil H.—*P.S.* Glad the books are safe. Advises H. to dine occasionally at Mme Geoffrin's and at d'Holbach's. Inquires if the passion H. hints at " was excited by my friend in any fair Lady who inhabits sometimes the rue des Vieilles Andriettes and sometimes a Country Villa." If so, the passion is reciprocal.

VII, 53. Hume, Compiègne. London, 23 July [1765]. *Signed, initials only.* [3 pp. + 1 p., 18·5 × 22 cm.]

Has hesitated to write to H., because a structure, however great its appearance of solidity, has proved itself ready to collapse. Believes the present fabric rests on a stronger foundation. Pitt, however, refuses his countenance, and will not act without his brother-in-law. The great popularity of Pitt.

VII, 54. Hume, Hôtel de Beaupreau, rue de l'Université, Paris. London, 15 Nov. 1765. *Signed.* [2 pp. + 2 pp., 18·5 × 23 cm. ; Burton, II, 311.]

Difficulty of finding a place where Rousseau can lodge. An old French farmer at Fulham would accept him as a boarder, though the house will not be very comfortable.

VII, 55. Hume, Edinburgh. [London, Nov. 1766.] *Signed.* [3 pp. + 1 p., 18 × 22·5 cm.]

Surprised to hear of the publication of the Rousseau correspondence in France, in spite of H.'s firm resolution to remain silent. Suspects the influence of R.'s enemies in Paris.—Bengal revenue has produced nearly £2,000,000, which cannot be brought home in either goods or bullion. British must turn bankers in India.—How long can India suffer such an annual exportation without any return ?

[Enclosed is a newspaper cutting of the English translation of Rousseau's letter to Guy the bookseller, challenging H. to publish the H.-R. correspondence.]

From Lord Stormont.

VII, 56. [H.]. Vienna, 5 Sept. 1767. *Signed.* [2½ pp. + 1½ pp., 18 × 23·5 cm.]

Recommends Baron Revitsky, a Hungarian, an accomplished man, a traveller, and a linguist.

From William Strahan.

VII, 57. [H.]. London, 10 July 1764. *Signed.* [4 pp., 18·5 × 23 cm.; E.P., 75–80, inc.]

The printing of Mme Riccoboni's book.—Tranquillity during the parliamentary recess. Dismissal of Conway. Lord Bute's influence with the King, and the decline of the Opposition. Bute is "sincere and honest," but "too timid and irresolute." Interview between Pitt and the King. The advantage of holding Florida. Except for the National Debt there is universal contentment and prosperity in England and the Colonies.—Clive has returned to the East with full powers and a force big enough to conquer the Mogul Empire.—Robertson's *Hist. of Charles V* is said to be finished. S. urges H. to proceed with his own *Hist.*, which is generally admired.—*P.S.* Death of the Earl of Bath.

VII, 58. Hume, Paris. London, 11 Jan. 1765. *Signed.* [3 pp. + 1 p., 20 × 32 cm.; E.P., 80–6.]

Loss incurred on the English edit. of Mme Riccoboni's book [*Miss Jenny*], which has not sold. Continued peace at home. Further decline in the Opposition. Proposal to purchase the superiority of the Isle of Man. Increase of Sinking Fund to nearly £3,000,000. Wilkes's last letter has made little impression. [Charles] Townshend is likely to become paymaster to the Army. Unrest in Pennsylvania.—Detailed account of the opening of Parliament. Chief topic was the ransom of the Manilla Islands. Conway inveighed against the Ministry for his dismissal. Grenville spoke for the Ministry.

VII, 59. Hume, at Miss Elliot's, Brewer Street, London. Glasgow, 27 July 1768. *Signed.* [1 p. + 3 pp., 18 × 22·5 cm.]

Care in correcting H.'s *Hist.* Willing, if necessary, to reprint it at his own expense if there are too many errors. Hopes H. is continuing it.—Will be glad to return to England.

VII, 60. Hume, Brewer Street [London]. New Street [London], 14 May 1769. *Signed.* [1 p. + 3 pp., 18 × 23 cm.]

Promises H. his six copies when printed, and also that in future H. may direct the printing as he likes.

VII, 61. [H.]. London, 13 Jan. 1770. *Signed.* [7 pp. + 1 p., 18 × 22·5 cm.; E.P., 91–7, inc.]

Hopes to see H. eventually in London again.—Junius's Letter, which "must certainly be the Production of Wilkes, and the pitiful Junto of Desperadoes with whom he is connected."—Account of the opening of the session in the House of Lords.—Chatham's speech; the peace, America, and the Middlesex Election. Lord Cholmondeley, Duke of Grafton, and others, supported the peace. The Lord Chancellor supported Chatham.—Marchmont and Lyttelton on the Middlesex Election. Mansfield ably opposed interference by the Lords. Chatham tried to browbeat him. Other speeches by Sandwich, the Duke of Richmond, etc. The Address voted.—Similar result in House of Commons.—American Colonies the most pressing affair.—S. has lamented to Lord Stormont the weakness of the latter's uncle [Lord Mansfield] in dealing with Wilkes.

VII, 62. [H.]. London, 1 Mar. 1771. *Signed.* [4 pp., 18 × 22·5 cm.]

Proposal to form a new colony on the Ohio is under consideration. The advantages of such a position. Lord Hertford is interested in the idea.—The 8vo. edit. of the *Essays* is exhausted and that of the *Hist.* is nearly so.—Peaceful conclusion to the war with Spain. Importance to England of maintaining the peace. The Ministry is satisfactory except for weakness in Home Affairs. Lord North acts his part very well, and gains in public favour every day. Importance to the King of preventing a change of Ministry. Discontent in France.—Death of Sir Andrew Mitchell.—Regrets that H. is building a house so far from London. Increase of S.'s business.—General prosperity in England. Decline of the Opposition. Wilkes rarely heard of. Wealth flowing in from the East Indies.

VII, 63. [H.]. London, 25 May 1771. *Signed.* [4 pp., 18 × 23 cm.]

Philosophical essays are in the press. S. desires to print the new edit. of the *Hist.* as soon as possible. Preparations for a beautiful edit. of 1500. Still urges H. to continue this work, which will remain for ever "the Standard History of this Country."—Question of the Falkland Islands settled. Neither France nor Spain desires war. Financial ruin of France; prosperity of England. "Our Colonies are growing very considerable without the smallest Fear of a Separation from us."—Necessity of giving a check to the canaille in London. Yet the general situation inspires optimism.

VII, 64. [H.]. London, 23 July 1771. *Signed.* [6 pp. + 2 pp., 18 × 23 cm.; E.P., 98, inc.]

General testimony to H.'s literary merit.—Unsuccessful and contemptible remonstrances of the Lord Mayor of London, despised by the "honest spectators." Ministry appear to be firmly established, yet they show excessive pusillanimity.—King is said to have been advised to keep the government in his own hands, which is the worst possible advice. Contrast with George I and George II, who

ruled by ministers. Yet S. has been assured that North is as absolute a master of the Cabinet as Walpole was. In that case, his courage is at fault.—There seems to be a fatality attending the Ministry whenever they meddle with Wilkes. Governmental interference inflamed the mob when the elections were going against W.—Rise of the National Debt is alarming, but 20 years of peace would enable a good government to reduce it considerably. Even in the event of national bankruptcy the landowners would be called on to play their part, and the result might be an additional stimulus to trade.

VII, 65. [H.]. London, 25 Jan. 1773. *Signed.* [2¼ pp. + 1¾ pp., 18·5 x 22·5 cm.; E.P., 99, inc.]

Printing of the new edit. of the *Hist.*—Dr Percy has received a copy of the note extracted from the *Northumberland Household Book*, but no alteration will be made in it without H.'s express command.—Suggestion for a continuation of the *Hist.* by [Sir John] Dalrymple or [James] Macpherson.—H. has raised Captain Brydon's expectations for his book too high.—A copy of Andrew Stuart's *Letters to Lord Mansfield* on the Douglas Cause has been sent to H.—*P.S.* Compliments from Benjamin Franklin, and to Robertson. S.'s son has become vicar of Islington.

VII, 66. [H.]. London, 30 Oct. 1775. *Signed.* [4 pp., 18·5 x 23 cm.; E.P., 100–2, inc.]

Has enclosed a letter to Dr Wilson on behalf of Dr Wight. H.'s advertisement [about the *Treatise*] will be affixed to the remaining copies of the *Essays.* A new edit. of the *Hist.* will be required next summer. S. congratulates H. on recommencing writing. Disagrees on American affairs. Supports coercive measures. England, as usual, has begun ill, but will recover. America seems lost, but will be gained back again by the fleet. The States must have trade. Conway and Grafton supported the minority. North acquitted himself well. North is an honest man, and means well.

VII, 67. Hume, Edinburgh. London, 12 Apr. 1776. *Signed.* [2 pp. + 2 pp., 18 x 22·5 cm.]

New edit. of the *Essays* needed. H. should therefore make his corrections at once. Gibbon's and Smith's books [*The Decline and Fall* and *The Wealth of Nations*]. Former more popular, but latter sells better than was expected. S. encourages H. to come to London.—Dangerous position in America. Delay means defeat, but the expense of an expedition is enormous. The result may overthrow the whole government of England. The chief hope is a voluntary return of the Colonies through lack of trade and weariness of anarchy.

VII, 68. Hume, Bath. London, 3 June 1776. *Signed.* [1 p. + 3 pp., 18 x 22·5 cm.; E.P., 102–3.]

Asks about the effect of the Bath waters on H. A paragraph from a letter

of Dr Beattie's enclosed, saying that he has considerably softened his *Essay* [*on Truth*].—*P.S.* *Essays* and *Hist.* both in the press. Makes a request about the new essay [*Of the Origin of Government*].

VII, 69. Hume, Edinburgh. London, 1 Aug. 1776. *Signed.* [2 pp. + 2 pp., 18 × 23 cm.; E.P., 103–4.]

Congratulates H. on his philosophic attitude towards his illness, and hopes he will recover, or at least enjoy periods of relief. Though the state of the roads round Edinburgh may prevent travelling, H. has his friends in Edinburgh.—An accurate edit. of the *Hist.* is being prepared. H. will have the satisfaction of a reputation among posterity.— Our fate in the American War will soon be known.

VII, 70. Hume, Edinburgh. London, 19 Aug. 1776. *Signed.* [1½ pp. + 2½ pp., 18 × 22·5 cm.; Burton, II, 512–3.]

Answering H.'s farewell letter, S. hopes this will find H. still alive. Is not the existence of the soul after death implied in our interest in futurity? Requests H. to give his last thoughts on this subject.

From Andrew Stuart of Torrance, W.S.

VII, 71. [H.]. Berkeley Square [London], 11 Mar. 1773. *Signed.* [7½ pp. + ½ p., 18 × 22·5 cm.; E.P., 324–9.]

Long silence is due to his being engaged in preparing the *Letters* [*to Lord Mansfield*]. Since their printing, he has been idle and dissipated.—Grateful to hear of a meeting of his friends in Baron Mure's house, and of their approbation. Has also roused much sympathy in England. This has had a great effect on his health. Proposes to share his happiness with his friends in Scotland. Not afraid of the threats of prosecution.—Recognises the justice of H.'s censure of occasional prolixity and of the use of the word *circumstantial*. Someone else is collecting the Scotticisms for him, on which he will ask H. for his judgment.

VII, 72. [H.]. Berkeley Square [London], 10 July 1775. *Signed.* [8 pp., 18·5 × 24 cm.; E.P., 329–34.]

Occupations that have prevented him from writing sooner: his attendance in Parliament, the contest about his election, and the Indian ambitions of his brother. The last have been at length attained, viz., the position of second-in-command of the Company's troops in India and the reversion of the chief command.—Asks H. not to desert him, now that his previous ill-fortune has ceased; at least, not till his prosperity is more firmly established.—News of Baron Mure. S. urges H. to come south, both to satisfy his friends and for the sake of his own health. Similar advice from Sir John Pringle.—Will remain in London till his brother sails; and then come to Scotland.

From Elizabeth Stuart.

VII, 73. Hume, Paris. Cringlety, 9 May 1765. [*Two documents as under :—*]

A. Letter to H. *Signed.* [3 pp. + 1 p., 18·5 × 23 cm.]

Begs a favour. Knows H. has in his train all the abbés of France, who will be more anxious to oblige him than to gain the Kingdom of Heaven. Has promised to help the nephew of some benefactors, Mesdemoiselles Grechi, and begs H. to try to secure him some Church preferment.

B. Enclosure addressed to Mlle Grechi, Charlimont, pres de Givet. *In French.* *Signed.* [3 pp. + 1 p., 18·5 × 23 cm.]

Telling her to apply to H. and to inform him more exactly of her intentions about the boy.

From Gilbert Stuart.

VII, 74. Hume, London. Edinburgh College, 2 May 1768. *Signed.* [1 p. + 3 pp., 18 × 23 cm.]

Sends, as a token of esteem, a small treatise he has published on the English Constitution, in which he has differed from H. on certain points, but only with the greatest deference and respect.

From Jean-Baptiste-Antoine Suard.

VII, 75. [H.]. Paris, 2 Nov. 1766. *Signed.* [4 pp., 16·5 × 21·5 cm.; Burton, II, 357, inc.; Greig, App. K, inc.]

Regrets his failure to write sooner about the Memoir [*Exposé succinct*], a copy of which was sent when printed.—S. has long considered Rousseau a dangerous charlatan, who has passed his life in receiving favours and doing the most harm he can to those who have done most good to him.—The pamphlet has had its expected influence on those not prejudiced.—S.'s own work as translator: certain modifications have been made with the consent of d'Alembert. These may be restored in the English edit., if necessary. H. would be well advised to let R. bay the moon.—Readiness to translate any work H. may be engaged on.

VII, 76. [H.]. London, 28 May 1776. *Signed.* [2½ pp. + 1½ pp., 15·5 × 20 cm.; E.P., 312–3.]

The fall of Turgot; to be ever regretted. T. aimed only at the good of the people and at truth.—Glad to hear that the Bath waters are improving H.'s health.

From [Adrienne-Catherine de] Noailles, Comtesse de Tessé.

VII, 77. Hume, ches M. l'Ambassadeur d'Angleterre, rue de l'Université, Paris. [Paris] 23 Aug. [1764]. *Signed.* [1½ pp. + 2½ pp., 15·5 × 19 cm.]

Thanks H. for the copy of Richardson's letters he sent, and regrets her own ignorance of English.

From Bernhard Tscharner.

VII, 78. [H.]. Berne, Switzerland, 7 Apr. 1762. *In French.* *Signed.* [4 pp., 18 x 24 cm.]

Asks H.'s support for a project, the exact nature of which is revealed in the enclosed pamphlet [lost].

From the Rev. Josiah Tucker, Dean of Gloucester.

VII, 79. [H.]. Paris, 27 July 1765. *Signed.* *Torn.* [3 pp. + 1 p., 19 x 24·5 cm.]

Thanks H. for his kindness at Compiègne. Thanks are mingled with ambitions excited by H.'s expected advancement in Ireland. Has already selected a text for the occasion. Proceeding to the miraculous, he imagines himself a bishop through having converted H. On reflection decides that H. is not unbeliever enough. H.'s infidelity fades to nothing beside that of Wilkes. Time of miracles not yet over. But conversation with men of letters forms T.'s highest ambition, and he is therefore specially grateful to H.

VII, 80. Hume, Brewer's Street, London. Gloucester, 16 June 1768. *Signed.* [2 pp. + 2 pp., 18·5 x 24 cm.]

Unacquainted with Turgot. Was unable to discover who had translated his "commercial queries." Much obliged both for Turgot's candid expression of his own opinion and for his general deference to that of T. H. is welcome to those of T.'s works which he has. T. is sending a copy of his *Elements of Commerce*, which H. is requested to send to Turgot, together with T.'s compliments and his latest work, *A Letter from a Merchant in London to his Nephew in America.* But for H.'s disbelief in prophecies, T. would appeal to him to judge whether T. had not been completely justified.

VII, 81. Hume, Brewer's Street, London. Gloucester, 25 June 1768. *Signed.* [2½ pp. + 1½ pp., 18·5 x 24 cm.]

H. has asked whether T. ever reflects on the uselessness of all his plans for mankind, when one scoundrel can upset them all. Such reflections have in fact caused T. to desist from his studies.—Sends some of his works to H. Suggestions by H.'s foreign friends will be welcome.—Account of a fray between some officers and the mob, through the magistrates' having omitted to celebrate the King's birthday. Fortunately no Scotsmen were concerned.—*P.S.* Asks whether the Parlements in France have cleared themselves of the charge of trumping up a forged edict of Henri IV against the Jesuits.

VII, 82. Hume, Brewer's Street, London. Gloucester, 16 Jan. 1769. *Signed.* [3 pp. + 1 p., 18·5 x 24 cm.; E.P., 176–7, inc.]

Trying to obtain copies of the tracts that Turgot wishes.—Explains his

financial system: "to render INDUSTRY very *cheap* and IDLENESS very *dear.*" Everything tending towards industry should be encouraged by freeing it from all restraints and especially by exempting it from taxation. Idleness and drunkenness should be heavily taxed. The value of a tax on spirituous liquors. Diligence alone makes for national wealth.—*P.S.* Asks H. to lend him a copy of Queen Elizabeth's General Warrant for suppressing riots, included in Rymer's *Fœdera.*

From Anne-Robert-Jacques Turgot.

VII, 83. [H.]. Paris, 23 July 1766. *Signed.* [5 pp. + 3 pp., 19·5 × 24 cm.; E.P., 130–6; *Œuvres de T.*, ed. by Schelle, II, 495–500; Greig, App. K, inc.]

Sends the Bp. of Lavaur's funeral sermon on the King of Poland.—T. is about to offer a prize on a question he and H. have often disputed about. The followers of Quesnay take up a position radically opposed to that of Pitt on the jealousy of trade.—The Rousseau affair. T. at first felt that H. ought not to print the correspondence, but then realised that, the letter to d'Holbach having been made public, H. must justify himself as R.'s accuser. The views of de Malesherbes. The seeming lack of reasonable motives in R. H.'s French friends are anxious to show him the impression the affair makes on men interested in both parties. H.'s reputation will not suffer. But he must let his French friends see the MS. before publication.

VII, 84. Hume, Lisle Street, Leicester Fields, London. Paris, 27 July 1766. *Signed.* [5 pp. + 1 p., 19 × 24 cm.; E.P., 136–43; Greig, App. K. inc.]

Rousseau did not refuse the pension. R. was probably aroused by the supposed letter from the King of Prussia, containing a joke of H.'s. H. need fear nothing for himself; except the accusation of being the dupe of his own kind heart. But H.'s unfortunate error in misrepresenting R.'s letter [to Conway] about the pension has confirmed R.'s suspicions. What H. has to justify himself against is not the accusations of R., but his own accusations of R. disseminated by R.'s enemies. The lines he ought to follow.—*P.S.* 30 July. T. has perhaps overestimated the affair. But Walpole should at least confess authorship of the King of Prussia letter.

VII, 85. [H.]. Paris, 7 Sept. 1766. *Unsigned.* [4 pp., 19 × 24 cm.; E.P., 144–8; *Œuvres de T.*, ed. by Schelle, II, 500–3: Greig, App. K, inc.]

Rousseau's letter was inspired by folly rather than by wickedness. H.'s precipitancy confirmed R. in his opinions. H. and Conway misunderstood R.'s letter about the pension. Walpole's joke was the cause of all R.'s fantastic suspicions, and W. is by no means innocent.—T., however, now advises H. to publish the correspondence; only the essential facts, without accusations. But there is no hurry.—Discusses indirect taxation, and its effects on the proprietors of land, on whom it would naturally fall.

VII, 86. [H.]. Paris, 25 Mar. 1767. *Signed.* [6½ pp. + 1¾ pp., 19 × 24 cm.; E.P., 149–57; *Œuvres de T.*, ed. by Schelle, II, 658–65.]

Compliments H. on governmental post.—His previous letters were not dictated by fondness for Rousseau, whom he has never seen; though he values R.'s works. R. is something of a charlatan, to be sure. But there are many truths scattered through the *Emile* and the *Contrat social.*—Personally R. is in many ways objectionable. T. did not mean to defend H., but only to save him from giving a weapon to R.'s partisans. H. need fear nothing. R.'s strongest supporters do not credit the wild statements in his letter [of 10 July 1766].—Continues the argument about taxation. The proprietor of land always pays in the end.

VII, 87. [H.]. Paris, 1 June 1767. *Signed.* [4 pp., 18 × 24 cm.; Burton, II, 381, inc.; E.P., 158–61.]

H.'s goodness in retaining an interest in Rousseau's welfare. The Duc de Choiseul, the Prince de Conti, the Princesse de Beauvau, etc., are co-operating to protect R. To procure a place of security in France is difficult. Besides, R. will regard as persecution all attempts to discover his whereabouts, even though made with the idea of aiding him.—H.'s attitude to public business is just the opposite of T.'s. - The embarrassing attention to detail demanded by the internal administration of France. *-P.S.* 3 June. The Parlement has no intention of seizing R., if he merely passes through France. His own aims must be learnt before any further step can be taken. Facts communicated by Mme de Verdelin have persuaded T. that R.'s suspicions against H. were roused by Mlle le Vasseur.

VII, 88. [H.]. Paris, 8 Mar. 1768. *Signed.* [2 pp., 16·5 × 20 cm.; E.P., 162–3.]

Recommends the Abbé Bon.—The prize essays on taxation have not answered expectations, but will serve the country, since truth is attained only through such discussions. Hopes H. will some day resume the argument on finance. Rejoices at H.'s return to literature, but his philosophy will keep him from unhappiness however he may be engaged.—Rousseau is still in the place of asylum granted to him by the Prince de Conti.

VII, 89. Hume. Paris, 11 May 1768. *Signed.* [2½ pp. + 1½ pp., 17·5 × 21·5 cm.]

Foundation of a new college at Parma, at which a chair of English literature is to be established. Knowledge of Italian and literary ability are necessary, and the professor should be English, not Irish. Social standing of professors. Uncertainty on question of religion. T. begs H. to look out for a suitable person.

VII, 90. [H.]. Paris, 3 July 1768. *Unsigned.* [3 pp. + 1 p., 15 × 20 cm.; E.P., 163–4, inc.]

Regrets that the college at Parma positively demands a Catholic, and so Mr

[Robert] Liston cannot be accepted.—On the perfectibility of the human race. At present no government is perfect. The perfect government will be attained only at the cost of some disorder. But this disorder will not be worse than tyranny and superstition. Progress of knowledge will render truths popular which are now attained only by labour.—Compliments from H.'s Paris friends.

From the Marquise de Verdelin.

VII, 91. [H.]. Soisy, 10 Nov. 1765. *Signed.* [2½ pp. + 1½ pp., 18·5 × 23 cm.]

H. will have seen by the letter she sent him that Rousseau has left for Berlin, and stopped near Strasbourg. Sends copies of letters between R. and the Bailly de Nidau, which H. is not to show to anyone else. R.'s letters are full of gratitude for H.'s good offices.

From John Vivian.

VII, 92. [H.]. Lisle Street [London], 24 Aug. 1767. *Signed.* [1½ pp. + 2½ pp., 17·5 × 22 cm.]

Hopes that H. enjoys his visit to Scotland, but that he is not tempted to stay.—V.'s ill-health, the nature of which is yet unknown to the doctors. Compliments from Mrs V.

From the Rev. Robert Wallace.

VII, 93. [H.]. Edinburgh, 3 Apr. 1764. *Signed.* [2 pp. + 2 pp., 18·5 × 23 cm.]

Rejoices at the respect shown to H. in France, which will increase on further acquaintance.—Requests some further information about the Genevan ministry and professors.—Recommends Hon. Alexander Lesly.—On Reid's *Enquiry into the Human Mind.* R. has knowledge of mathematics, and has carried out experiments in optics, but is bewildered when dealing with metaphysics.

From Horace Walpole.

VII, 94. [H.]. Strawberry Hill, 15 July 1758. *Signed.* [4½ pp. + 1½ pp., 18·5 × 23 cm.; E.P., 1–6; *Corr. of H. W.*, ed. Toynbee, IV, 158–62.]

Compliments H. as " the author of the best History of England." Answers some criticisms made by H. on W.'s *Catalogue of Noble Authors.* Surprised at the fame of Sir Philip Sidney. Falkland. Whitlocke. The King and Glamorgan. Faulty reference to the *General Dictionary.* Apologises for misinterpreting a passage in H. Further compliments.

VII, 95. [H.]. Arlington Street [London], 26 July 1766. *Signed.* [2 pp. + 2 pp., 16 × 20 cm.; *Corr. of H. W.*, ed. Toynbee, VII, 31–2; Greig, App. K, inc.]

H. should certainly not print unless Rousseau does. H.'s literary friends in Paris are thinking of their own injuries, not his.—Story of the writing of the

King of Prussia letter.—Would be sorry if H. were blamed on his account, but has a profound contempt for Rousseau.

VII, 96. [H.]. Arlington Street [London], 6 Nov. 1766. *Signed.* [3½ pp. + ½ p., 16 × 20 cm.; *Corr. of H. W.*, ed. Toynbee, VII, 66–8.]

Regrets the publication of the Rousseau correspondence. But H. showed a moderation his editors have not imitated.—D'Alembert and his dislike of W.'s King of Prussia letter.

VII, 97. [H.]. Arlington Street [London], 11 Nov. 1766. *Signed.* [4 pp., 19 × 24 cm.; *Corr. of H. W.*, ed. Toynbee, VII, 68–72.]

H. need not apologise for having used W.'s letter. But H. is certainly wrong in his judgment of d'Alembert. The character of philosophers, ancient and modern. D'Alembert's injustice towards Mme du Deffand. D'A.'s peevishness in omitting the part of H.'s letter which praised W.—W.'s reasons for writing the King of Prussia letter. D'A.'s insolence in reprehending W.'s conduct in a book attacking R.—*P.S.* Will not make use of H.'s remark about d'Alembert's hatred of Mme du Deffand.

From Alexander Wedderburn, afterwards Lord Loughborough.

VII, 98. [H.]. Paris, 28 Oct. 1764. *Signed.* [2½ pp. + 1½ pp., 19 × 23·5 cm.; E.P., 110–11.]

Regrets having to set out for England without seeing H.—The Douglas Cause: W. is at a loss to know what Mallet and Burnet have discovered. Their character. Feels an inclination to believe just the opposite of what Mallet says. Burnet's behaviour, on the other hand, has been more decent.

From Colonel David Wedderburn.

VII, 99. Hume, Edinburgh. Cav[endish] Square [London], 10 Mar. 1770. *Signed.* *Torn.* [1 p. + 3 pp., 18·5 × 23 cm.]

Thanks H. for his letter, and promises to do all in his power for Mr Callendar.—Compliments to Lady Erskine and the Wedderburns.

From Robert Wood.

VII, 100. [H.]. Cleveland Row [London], 30 Aug. 1762. *Signed.* [2 pp. + 2 pp., 18·5 × 23·5 cm.; E.P., 263–4.]

Regrets not having met Rousseau, whom he greatly respects. Uncertain what "our great folks" would think of R., Andrew Millar being the only "Protector of letters" that he knows.—Hopes H. will soon have two more 4to.'s ready, and that, when he reaches the present day, he will do justice to the Ministry that gave rise to the paper credit.

VII, 101. [H.]. Brompton, 28 July 1763. *Signed.* [1½ pp. + 2½ pp., 18·5 × 24 cm.]

Lord Hertford will recommend H. as Secretary to the Embassy when there is a vacancy, which is soon expected. W. advises H. to accept it.

VII, 102. Hume. Paris, 24 June 1767. *Signed.* [1 p. + 3 pp., 18·5 × 23 cm.; E.P., 264.]

Mme de B. has been kindly inquiring about H. Rousseau has appeared at Meudon. H.'s kindness to him has done him more harm than all his enemies.—*P.S.* Unanimity of the feeling in H.'s favour.

From ? .

VII, 103. Hume, chez M. l'Ambassadeur d'Angleterre, Hôtel de Brancas, rue de l'Université [Paris]. [?] 11 July 1765. *Unsigned.* [2 pp. + 2 pp., 13 × 18·5 cm.]

Asks H. to verify particulars about a young Mr Murray, 17½ years old, who has come to Soissons after having been in Paris and Reims, and who has given himself out to be of good family in Scotland, to be related to various members of the nobility, and to be a captain in some regiment.

MISCELLANEOUS LETTERS NEITHER BY NOR TO HUME.

From Jean le Rond d'Alembert to John Home of Ninewells.

VIII, 3. Home, St Andrew's Square, Edinburgh. Paris, 14 Sept. 1776. *Signed.* [1½ pp. + 2½ pp., 17 × 22 cm.; E.P., 218–9.]

Deep sorrow at H.'s death, and his pleasure at the mark of friendship given in H.'s will.

VIII, 4. Home, St Andrew's Square, Edinburgh. Paris, 28 Oct. 1776. *Signed.* [2 pp. + 2 pp., 17 × 21·5 cm.]

Concerning the legacy of £200 left him by H. His lasting admiration of H., and his sorrow at the loss which letters, philosophy, and d'A. more than anyone, have suffered.

From Jean le Rond d'Alembert to Jean-Jacques Rousseau.

VIII, 5. Rousseau. Paris, 3 Aug. 1766. *Signed.* [Enclosure to d'A.'s letter of 4 Aug. 1766 to H., **III, 6.**] [1½ pp. + ½ p., 13 × 19 cm.]

Denies any share in the King of Prussia letter, which was written by Walpole. Claims that he can produce witnesses to show that he strongly disapproved of it. Moreover, his respect for the King of Prussia would have prevented him from making use of the King's name. Has never, openly or secretly, been an enemy of R.

From Joseph Black to Adam Smith.

VIII, 6. Smith. [Edinburgh, Apr. 1776.] *Signed.* [2½ pp. + 1½ pp., 19 × 22 cm.; Burton, II, 489, inc.]

Praises *The Wealth of Nations.*—H.'s decline in health. Account of his illness: external heat, internal hæmorrhage.

VIII, 7. Smith, Kirkcaldy. Edinburgh, 14 Aug. 1776. *Signed.* [1 p. + 3 pp., 18·5 × 22·5 cm.]

Announces that H. is easier.

VIII, 8. Smith, Kirkcaldy. Edinburgh, 22 Aug. [1776]. *Signed.* [1 p. + 3 pp., 18·5 × 22 cm.; Smith to Strahan; Burton, II, 514.]

H. weaker, oppressed by conversation, but amuses himself by reading.

VIII, 9. Smith, Kirkcaldy. Edinburgh, 26 Aug. 1776. *Signed.* [1 p. + 3 pp., 18·5 × 23 cm.; Smith to Strahan; Burton, II, 515.]

H.'s death. The approach. Contented end.

From the Rev. Hugh Blair to David Hume the Younger.

VIII, 10. Hume, St David's Street [Edinburgh]. Argyle Square [Edinburgh], 20 Nov. 1797. *Signed.* [1¼ pp. + 2¾ pp., 18·5 × 23·5 cm.]

Sends a packet of H.'s letters.—Admiration for H. Marks of genius; sprightliness and real genius in the letters.

From the Comtesse de Boufflers to Lord Beauchamp.

VIII, 11. Beauchamp. [Paris] "ce vendredi." *In third person.* [1 p. + 1 p., 11·5 × 17·5 cm.]

Willing to receive Mr Strange on B.'s recommendation whenever S. cares to come, but she is not accustomed to take any steps to attract him towards her.

From Mme de Hume Chevizy Chastenay to Mme de Buffon.

VIII, 12. Mme de Buffon, Intendante des Jardins du Roy, Jardins du Roy, Paris. Chatillon, 26 Jan. 1764. *Signed.* [2 pp. + 2 pp., 16 × 20 cm.]

Has heard nothing of the effect of the genealogy she sent. Has H. traced an alliance between her family and his? Begs Mme de B. to interest H. in her son's behalf.

VIII, 13. Mme de Buffon, Intendante des Jardins du Roy, Jardins du Roy, Paris. Chatillon, 30 Dec. 1764. *Signed.* [2 pp. + 2 pp., 16 × 20 cm.]

Thanks for kindness to her son and daughter. Sends extract from genealogy of De Hume, hoping this will induce H. to take an interest in her son.

From Patrick Clason to Adam Smith.

VIII, 14. Smith, London. Geneva, 25 Feb. 1775. *Signed.* [3 pp. + 1 p., 18·5 × 23 cm.]

M. Bonnat was at a loss how to send his *Recherches* and *Palingénésie* to H. Extract from a letter of Bonnat's in which he expresses lack of confidence to write to H. personally, and suggests Smith as a suitable intermediary. C. therefore begs S. to send on the volumes to H.

From Sam Dexter to Dennys de Berdt.

VIII, 15. De Berdt. Dedham, Massachusetts Bay, 3 Aug. 1767. *Signed.* [9½ pp. + 2½ pp., 18·5 × 23 cm.]

The Colonists in Massachusetts are in a state of doubtful expectation, desiring to preserve their privileges and harmony with the Mother Country, and apprehensive of being deprived of their rights. Actions of the General Assembly not ill meant. May have been sometimes ill-advised and precipitate, but mistakes of that kind should not be exaggerated in England. Colonists need the protection of the Mother Country.—Massachusetts is far from desiring permission to issue paper currency. Ever since 31 Mar. 1750 it has had a good and stable currency. —An appeal to Lord Shelburne.—Praise for de B.'s efforts in connection with fisheries. Grants made to him.—D. has been appointed for the second time by the two Houses of Representatives to H.M. Council for the Province, but the Governor has again disapproved of the choice. D. does not blame the Governor. Will do all in his power to serve his King and Country by promoting peace and harmony. "I am loyal from Principle, from a deep Conviction that this People can never be happy but in the paternal Smiles of their King, and their Connection with and Dependance upon the Mother Country." Trusts he will always promote union with Great Britain.—Jealousies between the Governor and the people.

From Francis Home to Andrew Millar.

VIII, 16. [M.]. Edinburgh, 20 June 1758. *Signed.* [1 p. + 3 pp., 15·5 × 18·5 cm.]

Asks M. what he will offer for a new edit. of Home's *Principles of Agriculture*, Hamilton's edit. being sold out, while there is still a demand for copies.—Home has also a collection of papers called *Medicall Facts and Experiments*, containing among other things a paper on inoculation for measles, which should make it sell.

From John Home of Ninewells to Adam Smith.

VIII, 17. Smith, Dalkeith House. Edinburgh, 2 Sept. 1776. *Signed.* [1 p. + 3 pp., 18·5 × 23 cm.]

Asks S. to accept the legacy from H. *Dissertations* and *Life* [of H.] are being printed. S. can correct them at his leisure, and will receive a copy of the collected works.

VIII, 18. Smith, Kirkcaldy. Ninewells, 14 Oct. 1776. *Signed.* [1 p. + 1 p., 23 × 37 cm.]

Concerning S.'s additions to H.'s *Life*. Home would have preferred a little less detail, particularly about the journey. But as any alteration would require a complete remodelling, it had better be printed as it is. Two suggested verbal alterations.—Strahan will publish the *Dialogues* along with the new edit. of the works.—Home considers that the legacy was in every way due to S. [Follows a copy of a letter from H. to Home dated 13 Aug. 1776; Greig, No. 535.]

From Earl Marischal Keith to d'Alembert.

VIII, 19. D'Alembert. [Potsdam] Nov. 1764. *Copy only.* [2 pp. + 2 pp., 19 × 23·5 cm.]

Pleasure in reading d'Alembert's works. Finds the *Dictionnaire philosophique* "plus plaisant que sage."—Respects to Mme Geoffrin and to H.

From M. de Landivisiau to M. Caillaud.

VIII, 20. Extract from a letter dated, Madras, 21 Aug. 1761. [2½ pp. + 1½ pp., 19 × 23 cm.]

Stranded and a prisoner. Asks help from his honourable enemy.

Extract from another letter dated 5 Oct. 1761.

Thanks for his goodness. On the point of embarking. [See also **VI, 80.**]

From Charles Lowell to Baron David Hume.

VIII, 21. Hume, St Andrew's Square, Edinburgh. Glasgow, 16 Aug. 1837. *Signed.* [2½ pp. + 1½ pp., 18·5 × 23 cm.]

Requests the originals or copies of letters from Benjamin Franklin to H., corresponding to those from H. to F. found by Sparks in preparing a new edit. of F.'s works.

From Mme Roux de Berard de Montalet to Jean-Jacques Rousseau.

VIII, 22. Rousseau. Nîmes, 25 Apr. 1766. *Signed.* [4 pp., 17 × 23.cm.]

Extreme admiration for R. Has hesitated to write. Will he deign to reply? She is his absolute convert, and desires wealth only to provide him with a shelter. —Her life story. Has brought her daughter up in accordance with R.'s ideas. In everything she attempts to follow nature. Begs R. to aid her in completing her child's education.

From the Abbé de Mably to Jean-Jacques Rousseau.

VIII, 23. Rousseau. *Copy only.* *Undated.* [1¼ pp. + ¾ p., 13 × 18·5 cm.]

M. was himself the author of the letter. Pities R. in his distress, as he pities Socrates. But Socrates, to revenge himself on his judges, did not try to raise

an insurrection in Athens. It would be imprudent to project a democracy by arming the citizens against the magistrate. Recognises the uprightness of R.'s heart and the purity of his intentions.

From James Oswald the Younger to David Hume the Younger.

VIII, 24. [Hume]. Dunnikier House, Kirkcaldy, 31 Mar. 1817. *Signed.* [4 pp., 18·5 × 23 cm.]

Sends copies of H.'s letters to his grandfather, James Oswald of Dunnikier [see **II, 16**]. Praise of latter for his patronage of H. and Adam Smith.

From Sir John Pringle to ? .

VIII, 25. Undated fragment of a letter. *Unsigned.* [2½ pp. + 1½ pp., 19 × 23 cm.]

Captain Furneaux, circumnavigator, who went out as second in command to Captain Cook, has returned. Cook has not yet returned. Account of the voyage.

From the Chevalier Ramsay to [Michael] Ramsay.

VIII, 26. Ramsay, Hôtel de Provence, rue de Condé, Paris. [Paris] 1 Sept. 1742. *Signed.* [1½ pp. + 2½ pp., 17 × 22·5 cm.]

Has written to various people to obtain letters of introduction for R.—Has read first book of *Joseph Andrews*, but does not believe he will be able to finish the first vol. Finds it dull "burlesk." "I am affrayd your Brittanick wit is at as Low an Ebb as the french." Hopes to find more amusement in the Duchess of Marlborough's adventures. Longs to be in a condition to travel, so that he can see R. and meet his amiable young lord.

From Michael Ramsay the Younger to David Hume the Younger.

VIII, 27. Hume. Edinburgh, 4 May 1779. *Signed.* [1 p. + 1 p., 20 × 32 cm.]

Sending H.'s letters which had been addressed to Michael Ramsay the Elder.

From [? Abbé] Roubaude to ? .

VIII, 28. [?]. London, 31 Jan. 1766. *In French.* *Signed.* [4 pp., 15 × 18·5 cm.]

Has only just learned of Rousseau's arrival in London. Wished to meet R. in France, but belongs to a society [? Jesuits] in which freedom of thought beyond a certain point is not permitted. Has been sent to the forests of Canada for thinking too freely. Complete freedom came to him only with the conquest of Canada by the British. Was sent to London by General Murray, and is now stranded there, without friends or means of support.—Admiration for H.'s works. Wishes to meet H. Can the addressee arrange it for him ?

From Jean-Jacques Rousseau to various correspondents.

VIII, 29. [Mme de Boufflers]. Motiers-[Travers], 28 Dec. 1763. *Copy only.* [2½ pp. + 1½ pp., 18·5 × 23·5 cm.; *Corr. de R.*, 1826, III, 285–7; *Corr. gén. de R.*, ed. Dufour, X, 278–80.]

Rejoices not to have lost her friendship, as he had feared.—His own forlorn state. Sometimes thinks of joining Lord Marischal [Keith]. Otherwise may just as well suffer and die where he is.—News of Mlle le Vasseur. She alone persuades him to stay where he is.

VIII, 30. Mme de Boufflers. Motiers-[Travers], 26 Aug. 1764. *Copy only.* [3½ pp. + ½ p., 18·5 × 23·5 cm.; *Corr. de R.*, 1826, III, 282–6, inc.; *Corr. gén. de R.*, ed. Dufour, XI, 239–42.]

Sorrow at the death of M. de Luxembourg. Mme de B.'s plan to send her son to Leyden. Finds much against sending young people to the universities. But probably less danger in Holland than elsewhere.—His enemies have made him proud.—His life is uniform, idle, and indolent. Wishes he knew botany. Has every necessary.—Pleased that the Chevalier [Lorenzi] diverts her. Appreciates the continued kindness of the Prince de Conti.

VIII, 31. Abbé de Mably. Motiers-Travers, 5 Feb. 1765. *Copy only.* [1 p. + 1 p., 13 × 18·5 cm.; *Corr. gén. de R.*, ed. Dufour, XII, 317–8.]

Begs M. to inform him how he is to speak of a letter circulated in Geneva under M.'s name, and to remember their former friendship.

VIII, 32. Clairaut, de l'Académie des Sciences, Paris. Motiers-Travers, 3 Mar. 1765. *Signed.* [1 p. + 3 pp., 16·5 × 22 cm.; *Corr. de R.*, 1826, IV, 86–7; *Corr. gén. de R.*, ed. Dufour, XIII, 70–1.]

Begs him to look through and correct his *Dictionnaire de musique.*

VIII, 33. Mme [de Boufflers]. L'Isle St Pierre, 24 Oct. 1765. *Unsigned.* [1 p. + 1 p., 16·5 × 22 cm.; *Corr. gén. de R.*, XIV, 221.]

Is driven away from l'Isle St Pierre, and intends to go first to Berlin, and then, if he lasts out the winter, to H. in England.

VIII, 34. [M. de Graffenried]. [Oct. 1765.] *Copy only.* [2½ pp. + 1½ pp., 15·5 × 20 cm.; *Corr. de R.*, 1826, IV, 209–11; *Corr. gén. de R.*, XIV, 206–8.]

Considering the time of year and the state of his health he cannot venture into a strange country. Nor can Neuchâtel protect him. Begs for some castle in which to end his days.—Life he has led terrible.—Appreciates the kindness of H.

VIII, 35. Davenport. Wootton, 2 July 1766. *Signed.* [½ p. + 3¼ pp., 16·5 × 20 cm.; *Corr. de R.*, 1826, IV, 358.]

Has determined to satisfy H. by offering him an explanation.

VIII, 36. Two documents in the handwriting of Lady Stanhope. [3 pp. + 1 p., 15 × 20 cm.]

A. Copy of part of a letter from R. to M. D'Ivernois, 30 Aug. 1766. [*Corr. de R.*, 1826, V, 26 7.]

Has been advised how to defend himself against H.'s accusations, but the only one that affects him is that of having brutally refused the pension. Encloses the letter he wrote to Conway.

B. Copy of letter from R. to Gen. Conway, 12 May 1766. [*Corr. de R.*, 1826, IV, 334-5.]

The letter about the pension.

VIII, 37. Guy, Paris. [Wootton, 2 Aug. 1766.] *Copy in Lady Stanhope's hand; incomplete.* [2 pp., 18 × 23·5 cm.; *Corr. de R.*, 1826, IV, 405–7.]

Has determined to have no communication with the outer world. Insulting letters do not affect his honour. To answer insulting imputations would be to show himself worthy of them.—H. brought him to England to insult him. Has no fears of the publication of the correspondence.

From Adam Smith to John Home of Ninewells.

VIII, 38. Home. Dalkeith House, 31 Aug. 1776. *Signed.* [1 p. + 3 pp., 18·5 × 23 cm.]

Encloses what he proposes to add to H.'s *Life*.—Discharges Home of the legacy of £200.

VIII, 39. [Home]. [7 Oct. 1776.] *Unsigned draft letter.* [1 p. + 1 p., 20 × 24 cm.]

Sends draft of letter he proposes to add to H.'s *Life*.—Has added to his will a note discharging H.'s legacy of £200. Satisfied that in justice it is not due to him, and he cannot in honour accept it.

VIII, 40. [Home]. Kirkcaldy, Fifeshire, 7 Oct. 1776. *Signed.* [1¼ pp. + 2¾ pp., 16·5 × 20·5 cm.]

The actual letter of which **VIII, 39,** above is the draft.

From Adam Smith to William Strahan.

VIII, 41. Strahan. [Sept. 1776.] *Unsigned draft letter.* [1½ pp. + ½ p., 20 × 32 cm.]

Care of H.'s MSS. has been entrusted to Strahan. H. appeared only to

desire the publication of the short autobiography and of the *Dialogues*. Property of latter has been left to his nephew David, in case they are not published within three years. Had the office been entrusted to S., he would have kept them in MS. S. will add to the *Life* an account of H.'s behaviour in his last few days, provided this is not published with the *Dialogues*. Offers to superintend the edit. of H.'s works, as he had promised.

VIII, 42. [Strahan.] [Oct. 1776.] *Unsigned draft letter.* [2 pp., 20 × 32 cm.]

Addition to H.'s *Life* has been read over by John Home of Ninewells and Dr Joseph Black. Now waiting for the arrival of John Home the poet.—The printing of the *Dialogues*, if postponed, may not only not hurt the sales of the new edit. of H.'s works, but even contribute to sell another edit., when the storm has blown over.—Asks for information about the sales of his own book.

From William Strahan to John Home of Ninewells.

VIII, 43. Home, Ninewells, near Berwick. Wincklo, near Ringwood, Hampshire, 9 Sept. 1776. *Signed.* [1 p. + 3 pp., 18 × 22·5 cm.]

Death of H. S. will exactly fulfil H.'s wishes, and will provide Home with as many copies of any future edit. of H.'s works as he may desire. The new edit. will not be printed till Smith's additions to the *Life* are ready. S. will discharge the duties entrusted to him in H.'s will.

VIII, 44. [Home]. London, 3 Mar. 1777. *Signed.* [2 pp. + 2 pp., 19 × 23 cm.]

Declines to publish the *Dialogues*. David Hume [the Younger] could publish them under the shield of fulfilling his uncle's wishes, but S. himself would be accused of interested motives.—All H.'s friends agree that the two suppressed essays [*Of Suicide* and *Of Immortality*] should not be reprinted.—S. does not think that the printing of the *Life* separately is against the intention expressed in H.'s will. Is sending some copies to Home. The idea of including some of H.'s letters was dropped on the advice of Adam Smith.

VIII, 45. Home, St Andrew's Square, Edinburgh. [London, ? 1778.] *Unsigned; incomplete.* [2 pp. + 2 pp., 18 × 23 cm.]

Explains David Hume the Younger's reasons for desiring to print the *Dialogues* immediately. Moreover, the public is now expecting the work.

From William Strahan to David Hume the Younger.

VIII, 46. Hume, at Professor Millar's, Glasgow College. London, 13 Feb. 1777. *Signed.* [1 p. + 3 pp., 18·5 × 23 cm.]

H.'s *Life*, with Smith's letter, will appear in a week or two. S. hesitates to publish the *Dialogues*. Perhaps it had better be done by D. H. the Younger.

From William Strahan to Sir Andrew Mitchell.

VIII, 47. Mitchell. London, 1 Apr. 1768. *Copy only, but in S.'s handwriting.* [5 pp. + 3 pp., 18 × 23 cm.; E.P., 86–91.]

Account of the Middlesex Election. Wilkes declares his intention to surrender himself at the Bar, but none the less offers himself as a candidate. Poll going against him in the City, he decides to stand for Middlesex, where he easily wins. Excitement of the mob. Strong and spirited ministry is needed. Wilkes will probably be imprisoned, but the overthrow of the Bute Ministry may result. Peculiar position of Bute: a scapegoat; no real power.—Chatham is in retirement; probably waiting till he is indispensable. Lord Townshend in Ireland.—S. has given an impartial account of affairs, all parties being equally contemptible. H.'s reputation increasing. H. is collecting materials to continue his *Hist.*

From William Strahan to Adam Smith.

VIII, 48. Smith, Kirkcaldy. London, 10 June 1776. *Signed.* [2 pp. + 2 pp., 18 × 23 cm.]

H.'s health is not improved by the Bath waters, and he proposes to try Buxton. Has left Strahan informed of his wishes, should he die.—A success gained by Gen. Carlton in a sally from Quebec. This, with Gen. Burgoyne's presence, has strengthened our position there. Moreover, above 70 American ships have fallen into the hands of the English Navy.

VIII, 49. Smith, Kirkcaldy. Southampton, 16 Sept. 1776. *Signed.* [2 pp. + 2 pp., 18 × 22·5 cm.]

Acknowledges receipt of Smith's letter and the parcel of MSS. H. declared there was nothing in the *Dialogues* worse than what he had already printed. In any case, they will be published apart from the *Life*. This should appear in the winter, and then be prefixed to the new edit. of the *Hist.* Eagerly awaiting Smith's contribution. H.'s own care that the *Dialogues* should not be suppressed. —Compliments to Smith's mother. The great loss sustained through H.'s death.

VIII, 50. Smith, Kirkcaldy. London, 26 Nov. 1776. *Signed.* [1 p. + 3 pp., 18 × 22·5 cm.]

Even with Smith's addition, H.'s *Life* will not fill a vol. If Smith agrees, S. will include also some of H.'s letters. Gibbon has approved of this plan.—S. and Cadell are willing to accept Smith's proposal, of printing his book at their cost and dividing the profits.

MISCELLANEOUS PAPERS OTHER THAN LETTERS.

IX, 3. Cheques drawn by Hume.

London, 3 May 1776. [20 × 8 cm.] On Royal Bank of Scotland. Pay £200 to Thomas Coutts & Co.

Buxton, 26 June 1776. [19 × 12 cm.] On Thomas Coutts & Co. Pay £30 to John Home.

St Andrew's Square [Edinburgh], 4 July 1776. [17 × 19 cm.] On Royal Bank of Scotland. Pay £20 to Bearer.

St Andrew's Square [Edinburgh], 20 Aug. 1776. [18 × 11·5 cm.] On Royal Bank of Scotland. Pay £20 to Bearer.

IX, 4. MS. in H.'s handwriting (youthful). *Incomplete.* [8 pp., 18·5 × 30 cm.] "An Historical Essay on Chivalry and modern Honour."

IX, 5. Articles of Agreement, 26 Sept. 1738, between David Hume of Lancaster Court [London], and John Noon of Cheapside, London, Bookseller. [1 p. + 1 p., 33 × 43 cm.]

For the publishing of one edit. of Bks. I and II of *Treatise of Human Nature.*

IX, 6. Warrant, dated on board H.M.S. "Superbe," 3 Aug. 1746. *Signed* "J. St Clair." [1 p. + 1 p., 40 × 17 cm., vellum.]

Appoints H. Judge Advocate of all the forces under the General's command.

VERSES.

IX, 7. MS. in H.'s handwriting, dated 4 Nov. 1747. [1 p. + 1 p., 19 × 31 cm.; 8 stanzas; Burton, I, 229.]

"Go! plaintive Sounds, and to the Fair
My secret Wounds impart.
Tell all I hope; tell all I fear
Each Motion in my Heart."

IX, 8. MS. in H.'s handwriting, "To Oberon." [3 pp. + 1 p., 16·5 × 23 cm.; 16 stanzas.]

"I've oft implor'd the Gods in vain,
And pray'd till I've been weary:
For once I'll seek my wish to gain
Of Oberon the Fairy."

IX, 9. MS. in H.'s handwriting. [1 p. + 1 p., 18 × 22 cm., 5 stanzas; Burton, I, 230.]

"Tell me, Clorinda, why this Scorn?
Why Hatred give for Love?
Why, for a gentler Purpose born,
Wou'd'st thou a Tyrant prove?"

IX, 10. MS. in H.'s handwriting, "The Debate." [1½ pp. + ½ p., 17 × 28·5 cm.; 8 stanzas.]

A mildly improper poem on the relations of I——y [? Lord Islay, afterwards

Duke of Argyll] with the wives of two of his adherents, M——n and T——m C——n.

IX, 11. MS. in H.'s handwriting, "An Epistle to Mr John Medina." *Torn.* [2½ pp. + ½ p., 12 × 19 cm.; 48 rhyming lines; Burton, I, 234–5.]

IX, 12. MS. Account of Gen. St Clair's expedition to the Coast of France in 1746. [31½ pp. + 2½ pp.: 1–28, 22·5 × 36·5 cm.; 29–32, 16·5 × 21 cm.; 33–34, 19 × 21 cm.; Burton, I, App. A.]

IX, 14. Miscellaneous memoranda from books read. [22 pp. + 2 pp., 18 × 29 cm.; Burton, I, 126–35, inc.]

Sect. I: Politics and Economics, with many references to Greek and Roman authors.—Sect. II: Natural Philosophy.—Sect. III: Philosophy.

IX, 15. MS. of Conclusion of *Enquiry concerning the Principles of Morals.* [3½ pp. + ½ p., 18·5 × 22 cm.]

IX, 16. Memorandum, Edinburgh, 27 June 1754. [1 p. + 3 pp., 17 × 21 cm.]

Draft minute of Curators, Advocates' Library, laying down that in future no books are to be purchased without their authority.

IX, 17. MS. in H.'s handwriting, "Of the Poems of Ossian." [20 pp., 18 × 23 cm.; Burton, I, 471–80.]

An examination of the authenticity of Ossian.

IX, 18. Notes of reading. [2½ pp. + 1½ pp., 11 × 18·5 cm.]

To prove "that the Earl of Bothwel who payd Court to Margaret of Guise was Patrick and Father to James who marryd Q. Mary."

IX, 19. Memorandum in H.'s handwriting about his house in James's Court. Undated. A draft for some lawsuit. [1½ pp. + ½ p., 20 × 32 cm.]

Mr [James] Boswell, advocate, vacated the house at Whitsuntide, and Lady Wallace, dowager, took it over. The attempts of one Adam Gilles, mason, to impose on Lady W. and on H. by representing that many repairs had been necessary in the kitchen. "Mr Hume prays, that he may be acquitted with costs from every article, except the Plaister and whitening, which he offer'd at first to pay."

IX, 20. Draft in H.'s handwriting for a revision of some passage in the *Hist.* [1 p. + 1 p., 19 × 24 cm.]

On "the Clamour of the People." "And in opposing the Madness of the Vulgar often consists the most noble Efforts of Virtue and Courage."

Buxton, 26 June 1776. [19 x 12 cm.] On Thomas Coutts & Co. Pay £30 to John Home.

St Andrew's Square [Edinburgh], 4 July 1776. [17 x 19 cm.] On Royal Bank of Scotland. Pay £20 to Bearer.

St Andrew's Square [Edinburgh], 20 Aug. 1776. [18 x 11·5 cm.] On Royal Bank of Scotland. Pay £20 to Bearer.

IX, 4. MS. in H.'s handwriting (youthful). *Incomplete.* [8 pp., 18·5 x 30 cm.] "An Historical Essay on Chivalry and modern Honour."

IX, 5. Articles of Agreement, 26 Sept. 1738, between David Hume of Lancaster Court [London], and John Noon of Cheapside, London, Bookseller. [1 p. + 1 p., 33 x 43 cm.]

For the publishing of one edit. of Bks. I and II of *Treatise of Human Nature.*

IX, 6. Warrant, dated on board H.M.S. "Superbe," 3 Aug. 1746. *Signed* "J. St Clair." [1 p. + 1 p., 40 x 17 cm., vellum.]

Appoints H. Judge Advocate of all the forces under the General's command.

VERSES.

IX, 7. MS. in H.'s handwriting, dated 4 Nov. 1747. [1 p. + 1 p., 19 x 31 cm.; 8 stanzas; Burton, I, 229.]

"Go! plaintive Sounds, and to the Fair
My secret Wounds impart.
Tell all I hope; tell all I fear
Each Motion in my Heart."

IX, 8. MS. in H.'s handwriting, "To Oberon." [3 pp. + 1 p., 16·5 x 23 cm.; 16 stanzas.]

"I've oft implor'd the Gods in vain,
And pray'd till I've been weary:
For once I'll seek my wish to gain
Of Oberon the Fairy."

IX, 9. MS. in H.'s handwriting. [1 p. + 1 p., 18 x 22 cm., 5 stanzas; Burton, I, 230.]

"Tell me, Clorinda, why this Scorn?
Why Hatred give for Love?
Why, for a gentler Purpose born,
Wou'd'st thou a Tyrant prove?"

IX, 10. MS. in H.'s handwriting, "The Debate." [1½ pp. + ½ p., 17 x 28·5 cm.; 8 stanzas.]

A mildly improper poem on the relations of I——y [? Lord Islay, afterwards

Duke of Argyll] with the wives of two of his adherents, M——n and T—— —m C——n.

IX, 11. MS. in H.'s handwriting, "An Epistle to Mr John Medina." *Torn.* [2½ pp. + ½ p., 12 × 19 cm.; 48 rhyming lines; Burton, I, 234–5.]

IX, 12. MS. Account of Gen. St Clair's expedition to the Coast of France in 1746. [31½ pp. + 2½ pp.: 1–28, 22·5 × 36·5 cm.; 29–32, 16·5 × 21 cm.; 33–34, 19 × 21 cm.; Burton, I, App. A.]

IX, 14. Miscellaneous memoranda from books read. [22 pp. + 2 pp., 18 × 29 cm.; Burton, I, 126–35, inc.]

Sect. I: Politics and Economics, with many references to Greek and Roman authors.—Sect. II: Natural Philosophy.—Sect. III: Philosophy.

IX, 15. MS. of Conclusion of *Enquiry concerning the Principles of Morals.* [3½ pp. + ½ p., 18·5 × 22 cm.]

IX, 16. Memorandum, Edinburgh, 27 June 1754. [1 p. + 3 pp., 17 × 21 cm.]

Draft minute of Curators, Advocates' Library, laying down that in future no books are to be purchased without their authority.

IX, 17. MS. in H.'s handwriting, "Of the Poems of Ossian." [20 pp., 18 × 23 cm.; Burton, I, 471–80.]

An examination of the authenticity of Ossian.

IX, 18. Notes of reading. [2½ pp. + 1½ pp., 11 × 18·5 cm.]

To prove "that the Earl of Bothwel who payd Court to Margaret of Guise was Patrick and Father to James who marryd Q. Mary."

IX, 19. Memorandum in H.'s handwriting about his house in James's Court. Undated. A draft for some lawsuit. [1½ pp. + ½ p., 20 × 32 cm.]

Mr [James] Boswell, advocate, vacated the house at Whitsuntide, and Lady Wallace, dowager, took it over. The attempts of one Adam Gilles, mason, to impose on Lady W. and on H. by representing that many repairs had been necessary in the kitchen. "Mr Hume prays, that he may be acquitted with costs from every article, except the Plaister and whitening, which he offer'd at first to pay."

IX, 20. Draft in H.'s handwriting for a revision of some passage in the *Hist.* [1 p. + 1 p., 19 × 24 cm.]

On "the Clamour of the People." "And in opposing the Madness of the Vulgar often consists the most noble Efforts of Virtue and Courage."

IX, 21. Extracts in H.'s handwriting from the *Northumberland Household Book.* [4 pp., 11 × 16 cm.]

IX, 22. Notes, with page references, in H.'s handwriting, for proposed corrections in some historical work [? Barrington's *Observations on the Statutes*]; with comments in another hand, probably the author's. [1½ pp. + 2½ pp., 18·5 × 23 cm.]

IX, 23. MS. of *My Own Life*, dated 18 Apr. 1776. [12¼ pp. + 3¾ pp., 18·5 × 22·5 cm.]

IX, 24. Codicil to H.'s will, dated 7 Aug. 1776. [1 p. + 1 p., 18·5 × 32 cm.]

X–XI. MS. of *Hist. of England from the Conquest of Julius Cæsar to the Accession of the Tudors.* [334 pp., 22 × 33·5 cm.; 298 pp., 22 × 33·5 cm.]

XII. MS. of *Dialogues concerning Natural Religion.* [84 pp., 20·5 × 31 cm.]

MISCELLANEOUS PAPERS NOT IN HUME'S HANDWRITING.

VERSES.

XIII, 34. "Miss A. B. to Mrs H. by her Black Boy." [1 p. + 3 pp., 18 × 22·5 cm.; Burton, I, 296.]

XIII, 35. "To a Lady suspecting that the Friendship of Men to her Sex always conceal'd a more dangerous Passion." With "Laura's Answer." [1½ pp. + 2½ pp., 14·5 × 20 cm.; 5 stanzas and 2 stanzas; Burton, I, 230–1.]

XIII, 36. Epitaph for a young man. [1½ pp. + 2½ pp., 18·5 × 23·5 cm.; 16 lines, with 8 additional lines in an alternative version.]

XIII, 37. "Anecdotes sur le P[ain] Mollet." *In French.* [4 pp., 15·5 × 19·5 cm.; 75 lines.]

"On connoissoit le pain mollet
Longtems avant l'Abbé Nollet. . . ."

XIII, 38. "Charackter of —— [H.] written by himself." 16 numbered paragraphs. Occasional corrections and additions in H.'s handwriting. [1½ pp. + ½ p., 16 × 19·5 cm.; Burton, I, 226, inc.; Greig, *Life*, 411 f.]

XIII, 39. Comments, with page references, in the handwriting of [?] Sir David Dalrymple of Newhailes, on H.'s *Essays.* [4 pp. + 4 pp., 20 × 31·5 cm.]

Mainly criticisms of the style.

XIII, 40. Short paper in the handwriting of [?] the Rev. Robert Wallace, on mathematical principles. [3¾ pp. + ¼ p., 14·5 × 18 cm.]

Agrees that there are no abstract general or universal ideas. But we may abstract certain properties from things, and reason concerning these properties. "My reasonings in the mathematics are wholly about my suppositions [*i.e.* the abstractions deliberately made], and in one sense about nothing real." "In another sense my suppositions and the conclusions drawn from them are all about things reall in nature, for my ideas or the objects of them (which you will) are all real." As a mathematician he does not consider what is real in nature, but rather what he conceives or supposes.

XIII, 41. Short paper by [?] on the principles of mathematical reasoning. [9½ pp. + ½ p., 15 × 19 cm.]

"To reason abstractly is to reason from certain properties; affections, or relations, of objects, without regard to others, of the same objects; whether these, we neglect, be, or be not, separable from those we regard." Examples: carpenter, physician, and geometrician. Last regards *figure*, *surface*, *length* in abstraction from *colour*, *taste*, and other properties of sensible objects. So we may abstract from *length* and reach a mathematical *point*. Practical geometry is imperfect, because our powers are imperfect; but pure or theoretical geometry is not thereby affected.

XIII, 42. Memorial to H. by Professor Gaudio, dated the Hague, 3 Jan. 1764. *In French.* [8 pp., 20 × 33 cm.]

"Deux Questions proposées par un Philosophe sur le Bien de chaque Particulier, et de chaque Société, et de chaque Gouvernment, et de tout le Genre Humain." With an outline of G.'s own spiritual history.

XIII, 43. Account of the education and opinions of the Dauphin. *In French.* [Enclosure to letter of Mlle de Lespinasse to H., 23 Feb. 1766. 3 pp. + 1 p., 15·5 × 19·5 cm.; E.P., 180–2.]

Fragments of autobiography by Earl Marischal Keith.

XIII, 44. Statement dated Aix-la-Chapelle, 30 May 1746. *In French. Unsigned.* [3½ pp. + ½ p., 19 × 23 cm.]

Reasons for withdrawing from Jacobite affairs.

XIII, 45. Account, undated, unsigned, in French, of the same. [16 pp. + 4 pp., 20 × 31·5 cm.]

The attempt of Spain in 1740 to use the Jacobites as a stalking-horse in Spain's war with England.—The project, some time later, of an expedition from Spain to Scotland in conjunction with another from France to England. The Earl M.'s doubts of French good faith.—A story from his own life and that of

his brother to illustrate the intrigues and rascality of the French ministers.—The mustering of troops at Dunkirk in 1743, with Prince Charles Edward.—The Earl M.'s resignation from the Spanish service; his attempt to enter Russia, and subsequent retirement to Vienna. His entry into the King of Prussia's service, and his Prussian ambassadorship at Paris. His relations with the Young Chevalier there.—The negotiations between the English Jacobites and the Young Chevalier. The Prince's refusal to comply with their wishes, particularly with reference to his mistress. The breaking off of all negotiations with his friends in England.—The Earl M.'s final breach with the Prince.—The Earl M.'s meeting and conversation with Helvétius in Paris.—The King of Prussia's efforts to procure the Earl M. a pardon in England. *Appendix.* Account of Jacobite affairs in France in 1745, exhibiting the continued perfidy of the French and the folly of the Jacobite Duke of York.

XIII, 46. Note in [?] Baron David Hume's handwriting relating the story of Col. Edmonstoune's last visit to the philosopher. [2 pp., 11 × 17 cm.]

(*Issued separately January 20, 1932.*)

II.—The Employment of Intracardiac Injection of Adrenaline in Asphyxia.

By **Sir E. Sharpey-Schafer**, F.R.S., President, R.S.E., and **William A. Bain**, B.Sc. (From the Department of Physiology, University of Edinburgh.) (With Six Plates.)

(MS. received November 26, 1931. Read January 11, 1932.)

Asphyxia from Occlusion of the Trachea.

The efficacy of intracardiac injection of adrenaline to assist recovery from asphyxia having been recently disputed,* we have instituted a number of experiments with the object of determining what effect, if any, is produced by such injection in asphyxiated animals in which the respirations and/or the heart had entirely stopped, and the blood-pressure had become reduced almost or quite to zero.

Since the differences in the mode of causation of asphyxia might influence the result, it seemed desirable to investigate the effect of intracardiac injection in asphyxia produced under different conditions. In the present paper we deal with its effect in asphyxia produced by occlusion of the trachea; in future communications we shall deal with its effect in asphyxia produced by drowning, by electrocution, and by inhalation of various gases and vapours. Lastly, we propose to consider its application to asphyxia neonatorum, which is not amenable to ordinary methods of promoting artificial respiration.

Methods.—In the present experiments we have used cats exclusively. The animals have been anæsthetised with urethane, administered hypodermically in a dose of 1·5 grm. per kilogram. When completely anæsthetised the animal is placed on a warm table, a Y-shaped trachea tube inserted, and the blood-pressure and respirations recorded on a smoked surface driven at a moderate rate. The blood-pressure is recorded by a mercury manometer connected with the femoral artery; the respirations by a double tambour applied to the lower part of the chest and connected with a recording tambour. Asphyxia has been produced by closing the trachea tube, which was so arranged that at any given moment artificial respiration by intermittent admission of compressed air could be commenced. Or, alternatively, artificial respiration by

* *Journ. Amer. Med. Assoc.*, Jan. 11, 1930. See also Yandell Henderson, *Brit. Med. Journ.*, Oct. 17, 1931.

intermittent manual compression of the thorax could be substituted. In the cat such manual compression also involves intermittent compression of the heart, and produces a pumping action which mechanically promotes an artificial circulation and even, if the condition of asphyxia is not too advanced, provokes cardiac contractions which may be efficient enough to cause the blood to pass from the venous to the arterial system.

Results.

The result of causing complete asphyxia by tracheal occlusion without any attempt at resuscitation by artificial respiration and without administration of adrenaline may first be described (see fig. 1).

The immediate effect upon the respirations of closing the trachea (in this instance it was closed shortly after the beginning of an expiration) is to slow them. The extent of each respiration is at first diminished, but as asphyxia becomes more pronounced the movements of respiration become gradually more extensive and ultimately exhibit well-marked inspiratory spasms. After a little more than two minutes the respirations suddenly and completely stop: the arrest occurring in the expiratory phase. Although the respiratory movements cease, the tracing does not show a horizontal line, for after about a quarter of a minute an inspiratory tone slowly develops, persists for a few seconds, and then gradually subsides. About a minute after the cessation of respiration the so-called "agony respirations" begin. They are from three to ten in number, and succeed one another at intervals of 15 to 30 seconds. If the heart has stopped before they begin they are usually ineffective in restoring the circulation and in reviving the animal. But if the heart is still beating and the trachea is no longer occluded, they may admit enough air for spontaneous recovery to occur (see p. 144 and fig. 6). Sometimes the respirations become much deeper as asphyxia proceeds, and before it is complete assume a convulsive character; but in the tracing shown they are at no time excessive.

The immediate effect on the arterial pressure of closing the trachea is to cause a slight fall. The respiratory waves are accentuated upon the tracing and are of course at first slower than before. After a short time the pressure begins to rise; the rise is gradual, and in the case illustrated, not marked. The heart-beats are slowed, and therefore the pulse-waves are accentuated. Shortly before the respirations cease, the heart-beats again become fast; the blood-pressure becomes more rapidly depressed on stoppage of the respiratory movements and gradually descends to zero, a position attained about two minutes after the

cessation of the respiratory movements, by which time the heart-beats are no longer visible on the tracing.

If the occlusion of the trachea is now removed, it is still possible to revive the animal by intermittent manual pressure on the thorax, for, as already explained, this action not only renews air in the lungs, but also pumps blood through the heart and into the arteries, and starts an artificial circulation. The blood driven through the heart by this pumping action is oxygenated in the lungs, the air of which is being renewed by the artificial respiration, and passing to the respiratory centre, which had become paralysed from want of oxygen, restores its activity: always provided the cessation of respiration and circulation has not been too prolonged.

It is not easy to estimate the time which may elapse if such recovery is to be possible, for it appears to vary in different individuals and under different circumstances. Instances from the human subject have been cited in which recovery has been obtained by artificial respiration even half an hour after natural respiration has ceased; but this is exceptional. Ten minutes is probably the limit under ordinary circumstances.

The combination of artificial respiration with heart massage may succeed when the former alone is inadequate. In the human subject heart massage is difficult; it involves the insertion of the fingers through an incision in the epigastrium and compression of the heart through the diaphragm. In performing such massage the heart is liable to be bruised by the pressure of the fingers. In the smaller animals, such as the cat and rabbit, and in young dogs, the thorax is sufficiently yielding for it to be possible to compress the heart through the chest wall. Mere renewal of the air in the lungs is not effectual by itself. This is illustrated in fig. 2, which shows the result of an experiment on resuscitation of a cat from asphyxia, in which respiration is practised not by manual pressure on the thorax but by driving air intermittently into the lungs through the trachea tube. The procedure was started four minutes after natural respirations had ceased, and after the blood-pressure had descended to zero, with disappearence of any sign of heart-beats, the animal being to all intents and purposes dead.

The respiration curve shows nine "agony respirations," which began one minute after cessation of the ordinary respirations, and occurred during the next two minutes, but were ineffectual for recovery. One minute later, *i.e.* four minutes after cessation of the ordinary respirations, the trachea was connected with the artificial respiration apparatus usually employed in laboratories, and for rather more than one minute air was

pumped intermittently into the lungs. This produced no sign of recovery either of natural respiration or of the heart or of blood-pressure. A dose of adrenaline (0·5 c.c. of a 1/10,000 solution of adrenaline chloride) was then injected into the myocardium. The heart immediately resumed its activity. The blood-pressure rapidly rose from zero to 180 mm. Hg, and continued at a level not much below this; and then gradually fell to the level it occupied before asphyxia commenced. Following the rise of blood-pressure, natural respirations were resumed. They were at first slow and shallow, but gradually became faster and deeper, so that in two or three minutes they had acquired their normal character. There can be no doubt that the recovery was due to the action of adrenaline upon the arrested heart, which was thereby caused to resume its activity, so that blood was pumped through the now aerated lungs, and the asphyxiated centres in the bulb were restored to their normal condition.

Although in such a case as this the commencement of recovery is actually due to the action of adrenaline in stimulating the heart to increased contraction, there would be no permanent recovery unless the pressure in the arteries were not only raised, but maintained at a sufficient level. This could only be brought about by contraction of the arterioles, owing to the action of the adrenaline upon them. It is their contraction that has caused the great rise in blood-pressure observed in the tracing, and which is maintaining the arterial pressure at a high level. In this way a steady flow of blood is kept up in the bulb; its centres (vasomoter, respiratory, etc.) resume their lost activities, and the carotid sinus reflexes, which had disappeared, reappear.

If the heart is still beating, although the respirations may have completely ceased and the blood-pressure may have fallen almost to zero, the administration of adrenaline is obviously more likely to be effective than if there is no circulation at all. In the experiment illustrated by fig. 3, 0·5 c.c. of 1/5000 adrenaline chloride was injected into the heart. The effect on the heart itself was not immediate: from this we infer that the injection was not into the muscular substance—in which case it would not fail to produce an immediate effect—but into one of the heart cavities, probably the left ventricle. About a quarter of a minute elapsed before the rise in blood-pressure began. This rise is caused both by increased force and frequency of the heart and by contraction of the systemic arterioles. The adrenaline-containing blood has passed out of the ventricle, and is now reaching the capillaries of the coronary and aortic systems. Simultaneously with the rise of blood-pressure and resumption of the circulation through the bulb, natural respirations are resumed. They at

first show apnœic intervals; but after certain fluctuations of blood-pressure and heart-rate, due probably to carotid sinus reflexes, both circulation and respirations settle down into normal conditions.

Fig. 4 illustrates the advantage of artificial respiration together with heart massage (by chest compression) in facilitating the passage into the blood-vessels of adrenaline which has been injected into one of the cavities of the heart. In spite of the injection of 0·55 c.c. of 1/1000 adrenaline chloride into the beating heart, the latter showed no increase in rate or force; indeed its beats were diminished. Presumably the injection was into one of the cavities, probably into the right ventricle. Half a minute after the injection, the chest was compressed intermittently so as to permit artificial respiration, and at the same time to massage the heart. This had no immediate effect on the blood-pressure. The tracing of the compression is interspersed with a few deep respirations at intervals of about fifteen seconds: they are probably "agony respirations." Half a minute later the blood-pressure suddenly rose, the heart-beats becoming stronger, and the arterioles contracting. The massage was then stopped, natural respiratory movements being resumed. The rise in blood-pressure was very sharp, and rapidly attained the very considerable height of 184 mm. Hg. The natural respiratory movements, at first very deep and marked by apnœic intervals, became in a minute or two regular and of normal character. The blood-pressure continued very high for some minutes, and showed strongly marked irregularities. These were probably the effect of the large dose of adrenaline, the solution employed being ten times as strong as those used in most of our experiments.

An adrenaline effect may be obtained in cases of recovery without the artificial administration of adrenaline. Such a case is illustrated in fig. 5. Asphyxia was, as before, induced by occlusion of the trachea tube. Two minutes after closure the natural respirations, which were at first slow but later more rapid and convulsive, stopped. One minute later the "agony respirations," five in number, began, and succeeded one another at intervals of about twenty seconds, showing, as is usual, a well-marked "staircase." Two minutes after stoppage of the natural respirations the trachea tube was reopened, and half a minute later the chest was intermittently compressed at a rapid rate in order to promote artificial circulation by heart massage. No adrenaline was administered. The blood-pressure had fallen to zero, and showed no heart-beats: it seemed as if there were no chance of effecting recovery. But three minutes after the respirations had ceased and after the heart massage had been practised for half a minute, the heart-beats were again visible. The blood-pressure began to rise, at

first gradually and then very rapidly, and soon attained a height of 160 mm. Hg. The effect was exactly like that seen when a dose of adrenaline is administered by intravascular injection (*cf.* figs. 2, 3, 4). The pressure soon fell somewhat, and gradually settled down to its normal level. Natural respirations were resumed, being at first slow and deep, then showing marked alternations in depth and extent, but gradually becoming more regular. Four and a half minutes after cessation of the massage and commencement of recovery, a dose of 0·5 c.c. of 1/10,000 adrenaline chloride was injected into the heart. The effect of this was gradually to produce a steady and maintained rise of blood-pressure, accompanied by a resumption of regularity of respirations, followed by complete recovery.

The explanation we offer of the result of this experiment is as follows: During asphyxia the centre for secretion of adrenaline—along with the other centres in the bulb—is stimulated, and an unusual amount of adrenaline passes into the suprarenal vein. With the failure of the circulation which ultimately results from the asphyxia, the blood of the suprarenal vein does not pass into the inferior vena cava and through the heart; the secreted adrenaline accumulates in the vein. But the intermittent pressure on chest and abdomen has in this case caused it to pass into the inferior vena cava and right auricle, and through the pulmonary circulation into the left side of the heart and aorta, producing as it reached the coronary and systemic arterioles the rapid and great rise of pressure characteristic of intravascular administration of adrenaline. In this instance the respirations were slow in attaining regularity, although there was no question that the animal had recovered from asphyxia. It was to ascertain whether the recovery would be accelerated by intracardiac administration of adrenaline that we subsequently injected the 0·5 c.c. of a 1/10,000 solution into the myocardium. This had the effect of steadying both respiration and blood-pressure, and evidently proved helpful towards complete recovery.

In fig. 6 we give an illustration of spontaneous recovery without the assistance of massage of the heart, or artificial respiration, or adrenaline. Natural respirations ceased about two and a half minutes after occlusion of the trachea. The cessation was followed by the usual gradual fall of blood-pressure almost to zero, the heart-beats being, however, just visible on the tracing. The trachea tube was opened about half a minute after respirations had stopped. Less than a minute later the "agony respirations" began. In this case they were effective not only in drawing air into the lungs, but apparently also in causing the adrenaline-containing blood of the suprarenal vein to pass towards the heart, thus promoting

its passage through the pulmonary circulation into the coronary and systemic vessels. In consequence the blood-pressure rose significantly (from zero to 50 mm. Hg), although not in so striking a manner as in the last experiment. Simultaneously natural respirations were resumed. At first slow and deep, and somewhat irregular, they soon became regular and normal in extent. Although an adrenaline effect is less obvious in this case than in the experiment illustrated in fig. 5, the difference is probably due to the fact that the passage of the adrenaline-containing blood through the heart must have been less rapid than when the circulation was promoted by heart massage. Unfortunately the arterial cannula was partly obstructed by a clot, which prevented the heart-beats from showing themselves on the rising blood-pressure curve.

Literature.

The literature of intracardiac administration of adrenaline has of late years become extensive. It relates mainly to the recovery of a heart which has been arrested, whether as the result of shock, or from the administration of anæsthetics, or from indeterminate causes. The effect in simple asphyxia does not seem to have been previously investigated, although it has been employed as an auxiliary method in cases of failure of heart and respiration such as occurs in electrocution and drowning. Other drugs (camphor, ether, strophanthine, strychnine) have also been recommended for intracardiac injection, but none approach adrenaline in effectiveness and rapidity of action. It is, moreover, *the* physiological stimulant of the heart, as well as of the whole vascular system.

Most of the published papers on the subject merely relate one or two cases; but some authors have taken the opportunity to collect all the known clinical evidence up to the date of their article, and thus furnish a useful bibliography. The latest writer, Hyman (3), collected 250 cases of intracardiac injection, with 25 per cent. successes. This proportion of successes is the more remarkable when we consider that an intracardiac injection is usually administered when the patient is moribund; sometimes when he is to all intents and purposes dead, and could not otherwise recover. Hyman rightly recommends that the injections should be made into the substance of the myocardium* and not into the cavities, nor into the pericardium. The last is no more effective than a hypodermic injection. Even intravenous injection is not to be recommended, being both difficult and uncertain. Hyman's own cases are nine in number. In one—a man

* By injecting into the substance of the heart, the drug at once gets to every part through the extensive lymphatic system which pervades the myocardium.

of thirty-six—there was complete cardiac arrest on the operating table, and all the usual remedies had been ineffectually essayed. Adrenaline was injected intracardially eleven minutes after the heart had stopped; it produced complete recovery. Another case of a child injected fourteen minutes after heart stoppage gave a similar result. He cites another case in which the heart had completely stopped for thirty minutes; another of twenty minutes' arrest; and others of twelve and ten minutes.

Hyman comes to the curious conclusion that it is the mechanical prick of the hypodermic needle which is effective, and recommends a succession of pricks of the right auricle (the *primum movens*). But in view of the unquestionable benefit of adrenaline both on cardiac activity and in promoting contraction of the systemic arterioles, as well as its beneficial effect on the respiratory system—effects which are clearly demonstrated in the tracings given in this paper—Hyman's conclusion is unacceptable.

Frenzel (2) mentions a case of heart arrest under anæsthesia, treated successfully by intracardiac injection of 1 c.c. of 1/1000 adrenaline solution. He also cites a number of other cases in which heart massage had been tried unsuccessfully, but in which adrenaline produced rapid recovery. He expressly states that in the event of recovery there are no after-consequences, and recommends that adrenaline should always be kept ready for injection at an operation.

Vogt (4) also strongly recommends intracardiac administration in cases of heart failure. He describes it as the simplest and best method, far more effective than heart massage; which is, moreover, liable to bruise the heart.

In an interesting paper in the *Lancet* of 1923, Bodon (1) describes the case of a man, aged fifty-six, with syphilitic disease of pulmonary arteries, who became moribund and completely unconscious. The respirations and heart stopped, the corneal reflexes disappeared, the eyes became glazed, the pupils widely dilated, the face cyanosed. Urine and fæces were passed involuntarily. The patient was to all intents and purposes dead. A few seconds after 1 c.c. of 1/1000 adrenaline solution had been injected into the heart, its action recommenced, the radial pulse could be felt, and respirations were resumed. About two hours later the man recovered consciousness; he remembered nothing about his collapse. Half an hour later he could walk across the room with assistance; he eventually recovered completely. Bodon gives results in 90 cases which he had collected. Of these, 24 were permanently successful. He recommends the employment of the method in all cases of heart failure, whether from surgical shock, anæsthesia, electrocution, obstetrical hæmorrhage and

shock, acute diseases of the circulatory system, effect of drugs; in anaphylaxis, and in asphyxia neonatorum. He is insistent that the injection should be made with as little delay as possible. All authorities are agreed that no deleterious after-effects are to be anticipated.

The method recommended by Vogt and Bodon for man is to make the puncture with a fine hypodermic needle, 8 cm. (about 3 inches) long, in the fourth left intercostal space, just above the fifth rib and about a finger-breadth from the edge of the sternum. This avoids the internal mammary artery, and pierces the heart where it is close to the chest wall. The needle can be felt to penetrate the myocardium, and if the heart is still acting, it will be moved to and fro.

Summary and Conclusions.

1. The effect of intracardiac injection of adrenaline in assisting recovery from asphyxia produced by occlusion of the trachea has been investigated in cats anæsthetised with urethane.

2. It is shown that such injection has a striking effect in promoting recovery of the circulation. Even if the heart has ceased to beat, it may start contracting rapidly and strongly as the result of such injection, the blood-pressure being rapidly raised. The rise is due not only to the action of adrenaline on the heart, but also to its causing contraction of the systemic arteries.

3. The effect in promoting restoration of respiration is no less striking. Breathing recommences at the same time that the circulation is restored, and although at first slow and deep, the respiratory movements gradually resume their normal character and rate.

4. If the injection is into one of the heart-cavities instead of into the myocardium the effect is slower in manifesting itself, because the adrenaline has in the case of injection into the left ventricle to pass into the capillaries of the aortic and coronary systems, and in the case of injection into the right ventricle, also to traverse the capillaries of the pulmonary system. The passage is assisted by heart massage and artificial respiration such as can be produced in animals by intermittent pressure on the abdomen and lower part of the thorax.

5. Rarely an adrenaline effect is obtained with heart massage and artificial respiration without an actual injection of adrenaline. This appears to be due to the passage towards the heart of adrenaline-containing blood which has stagnated in the suprarenal veins, in consequence of the failure of the circulation, and, as a result of the intermittent pressure

applied to the thorax and abdomen, becomes driven towards and through the heart and pulmonary circulation, and thus eventually into the aortic and coronary capillaries.

6. Still more rarely spontaneous recovery may occur, without either heart-massage or artificial respiration or adrenaline administration. In such a case the passage of the adrenaline-containing blood of the suprarenal veins towards the heart seems to be facilitated by the "agony respirations," which begin, in all cases of asphyxia, about a minute after the natural respirations have ceased. If the occlusion of the trachea has been released, these respirations introduce air into the lungs, and at the same time draw and press the adrenaline-containing blood of the suprarenal veins towards the heart, and thus facilitate its passage into the circulation.

PAPERS REFERRED TO.

(1) Bodon, C., *Lancet*, 1923 (1), p. 586. (This paper gives the literature between 1905 and 1921.)

(2) Frenzel, H., *Münch. med. Wochenschr.*, 1921, lxviii, p. 730.

(3) Hyman, A. S., *Arch. Int. Med.*, 1930, xlvi, p. 553. (This paper has references to the literature between 1921 and 1930.)

(4) Vogt, E., *Münch. med. Wochenschr.*, 1921, lxviii, p. 732.

DESCRIPTION OF FIGURES.

Fig. 1. Effect of occluding the trachea without any attempt at resuscitation.

Cat, ♂. 3200 grms.*

The trachea was occluded at the beginning of expiration: the occlusion lasts for rather more than three minutes. Respirations cease in a little more than two minutes. The "agony respirations" begin about one and a half minutes after cessation of the ordinary respirations, but are ineffectual in promoting recovery. After cessation of respirations the blood-pressure gradually falls to zero, and the heart-beats, which are at first increased as the asphyxial condition develops, become gradually less obvious, until they are no longer visible on the tracing.

---

* In this and every other case the anæsthetic employed was 1·5 grms. of urethane per kg., given subcutaneously.

In all the tracings the uppermost line shows the respirations as recorded by a tambour connected with a pneumograph attached to the thorax-abdomen, the upstroke being inspiratory; the second line shows in millimetres of Hg the blood-pressure in the femoral artery; the third line, the period of occlusion of the trachea and any other recorded events; and the lowest line the time in minutes.

Fig. 2. Effect of intracardiac administration of adrenaline in promoting the recovery of heart, blood-pressure, and respirations after complete cessation of heart and respirations had occurred several minutes previously; artificial respiration by intermittent inflation having failed to effect any recovery.

Cat, ♂. 2650 grms.

The trachea is occluded at the end of an expiration: the occlusion lasts for five minutes. The respirations stop after two minutes after having been much increased in extent, although slower in rate. Blood-pressure, which had slowly risen, falls, at first more abruptly, afterwards gradually, and is at zero four minutes after respirations had ceased. (This record is interrupted by the formation of a clot in the cannula, which was removed.) The heart-beats become gradually weaker, and at last can no longer be detected. The "agony respirations" (nine in number) begin one minute after the natural respirations have ceased and continue for two minutes. They exhibit a well-marked "staircase," but have no influence in promoting recovery; nor could this be expected, since the trachea is still occluded. The animal is to all intents and purposes dead. The trachea having been released, an injection of 0·5 c.c. of 1/10,000 adrenaline chloride was then given, but produced no effect; probably it did not enter the heart. Artificial respiration was now started by a pump, the strokes of which are recorded on the respiration curve. This had no result. Five minutes after the natural respirations had ceased, 0·5 c.c. of 1/10,000 adrenaline chloride solution was injected into the heart, and artificial respiration was stopped. Recovery of the heart and blood-pressure rapidly followed; the effect was magical. The blood-pressure rose to 180 mm. Hg, was maintained for a short time at a high level, and gradually fell to normal. In less than a minute natural respirations were resumed. At first slow and shallow, they gradually increased in rate and regularity. Complete recovery of respirations and blood-pressure was soon obtained, but is not shown in the part of the tracing reproduced.

Fig. 3. Rapid recovery of respiration and blood-pressure as the result of an early intracardiac injection of adrenaline, without the assistance of artificial respiration or heart massage.

Cat, ♂. 3200 grms.

In this animal the trachea was occluded for rather more than three minutes. Cessation of respiration is seen to occur in about two minutes. The blood-pressure, after rising somewhat towards the end of asphyxia, gradually falls almost to zero, but heart-beats are continued strongly. One and a half minutes after cessation of respiration 0·5 c.c. of 1/5000 adrenaline chloride solution is injected into the heart. No effect is observed for about a quarter of a minute; there is then seen a rapid rise of blood-pressure and increase in force of the heart's action, with the vagal effects characteristic of adrenaline. The increase in the heart's action combined with arterial constriction has produced a supernormal blood-pressure; but, after some fluctuations, this gradually subsides to the normal pre-asphyxial level. Accompanying the rise in blood-pressure, the respirations recommence: they are at first deep and slow and irregular, with apnœic intervals, but soon become regular and of normal rate and depth.

Fig. 4. This is another illustration from the same animal of the effect of injecting adrenaline intracardially in promoting recovery from asphyxia.

One and a half minutes after occlusion of the trachea the respirations, after becoming greatly exaggerated, stop abruptly. The injection of adrenaline, which in this case was 0·5 c.c. of 1/1000, was administered about three-quarters of a minute after the respirations had stopped. Blood-pressure had fallen considerably, but the heart was still beating strongly, although slowly. The adrenaline at first produced no effect, having probably not been injected into the substance of the myocardium, but into the right ventricle. Since after half a minute more no recovery was apparent, massage of the heart by compression of the thorax was practised. At the end of another half minute or so an adrenaline effect on the heart and blood-vessels shows itself by the production of an enormous rise of blood-pressure accompanied by resumption of natural respirations; these are at first slow, deep, and irregular, but soon become regular. The dose of adrenaline was considerably larger—ten times as large—as in most of the other experiments. Apparently in consequence of this there is produced not only a very great excitatory effect on the heart and arteries, but considerable irregularities both of respiration and blood-pressure. These are no doubt due to the intense action of the large dose of adrenaline upon the bulbar centres, which would be affected both directly and reflexly (through the carotid sinus).

Fig. 5. Recovery from asphyxia produced by manual massage of the thorax, showing an adrenaline effect without artificial administration of the autacoid.

Cat, ♂. 3350 grms.

In this experiment the trachea was occluded for four minutes. The respirations, at first slow, gradually increase in amplitude and in rate and cease abruptly in two minutes. One minute afterwards five "agony respirations" begin, and succeed one another at intervals of about twenty seconds. Although the last two occur after the trachea is opened, they fail to effect recovery. Blood-pressure is at zero, and no heart-beats are visible (but there is some evidence of the presence of a clot in the arterial cannula). Manual massage of the thorax, and through this of the heart, is then practised. This soon has the effect of causing the heart to resume beating, and in about three-quarters of a minute the blood-pressure rapidly rises to 150 mm. Hg, exactly as if adrenaline had been injected into the vessels, although none has been administered. After falling again somewhat, an average pressure of about 130 mm. Hg is assumed. With the rise of blood-pressure the respirations recommence: at first they are infrequent and deep, soon to be alternated by shallower movements; but eventually they become more regular, although for a time irregular and exaggerated. A dose of 0·5 c.c. of adrenaline chloride 1/10,000 is now injected into the heart: this rapidly steadies both the blood-pressure and the respirations, and rapidly leads to complete recovery.

Fig. 6. Asphyxia from closure of trachea. Spontaneous recovery without either artificial respiration or adrenaline injection.

Cat, ♂. 3200 grms.

In this case respiration stops two and a half minutes after occlusion. The trachea is then released, and the animal left undisturbed. The blood-pressure

falls gradually almost to zero; perhaps owing to a partial clot in the arterial cannula, the heart-beats are only shown indistinctly upon the tracing. The "agony respirations" begin about a minute after the natural respirations have ceased: the trachea being now open, they prove effective in causing recovery; ordinary respirations are quickly resumed, at first deep and somewhat irregular, but soon becoming normal. Resumption of respiration is accompanied by recovery of blood-pressure; but the recovery is less abrupt and the rise less marked than in the experiment illustrated in fig. 5, where the circulation was assisted by manual massage of the thorax.

The spontaneous recovery in this case was probably brought about by the mechanical effect of the "agony respirations" in forwarding towards the heart adrenaline which had stagnated in the suprarenal veins. Further, since the trachea tube had been opened, the "agony respirations" would introduce air into the lungs and re-oxygenate the blood in the pulmonary capillaries: this, being forwarded to the systemic circulation, would revive the heart and blood-vessels as well as the reflex centres of the bulb.

(*Issued separately February* 5, 1932.)

III.—The Absorption Spectra of Cyanogen and the Cyanogen Halides.

By **R. B. Mooney**, M.A., B.Sc., Ph.D., Carnegie Research Fellow, and **H. G. Reid**, B.Sc. *Communicated by* Dr E. B. Ludlam.

(MS. received October 27, 1931. Read November 2, 1931.)

As a preliminary to photochemical work with cyanogen and the cyanogen halides, we thought it desirable to investigate the ultra-violet absorption spectra of these substances. For wave-lengths greater than 2000 A a Bellingham and Stanley quartz spectrograph was used. This gave the region 8000–2000 A on a 10-inch plate. A small Hilger spectrograph was used to study the region 2000–1850 A. The light source was a water-cooled hydrogen tube taking 0·5 amp. at 10,000 volts.

Gaseous cyanogen was found to give a complicated system of absorption bands between 2380 and 1850 A. The vapours of cyanogen chloride, bromide, and iodide showed regions of continuous absorption, of which the long wave-length limits have been approximately measured.*

Cyanogen.

Cyanogen, C_2N_2, was at first prepared by heating mercuric cyanide, or a mixture of mercuric cyanide and mercuric chloride, and purified by fractional distillation. Later, it was found more convenient to introduce the calculated quantity of silver cyanide (free from nitrate) into a side limb of the silica absorption cell, evacuate with a mercury diffusion pump, and seal off. By heating the side limb to redness, the cell was filled with cyanogen at a known pressure. Silver cyanide was preferred to mercuric cyanide because the latter tends to sublime without decomposing. Four different absorption cells, varying in length from 1 cm. to 120 cm., were used. The pressure of cyanogen could be varied by cooling a side limb of the absorption cell with ether and carbon dioxide snow. The 10-cm. cell was wound with nichrome ribbon and asbestos string, so that the influence of temperature on the relative intensities of the bands could be observed.

Several photographs were taken with both spectrographs on Wellington Antiscreen plates. For wave-length standards we used spark lines of copper, zinc, and aluminium. Measurements of the observed band edges

* Mooney and Reid, *Nature*, **128**, p. 271, 1931.

and rough estimates of the intensities are given in Table I. The bands, which are fairly narrow and shaded towards the red, do not show any apparent regularity.

TABLE I.

cm.$^{-1}$.	Intensity.	cm.$^{-1}$.	Intensity.	cm.$^{-1}$.	Intensity.
41959	0	45868	1	48621	2
(42157)	0	45889	1	48673	1
(42893)	0	45933	2	48773	1
(43526)	0	46085	0	48821	1
(43876)	0	46121	1	49150	5
(43920)	0	46150	2	49436	2
(44146)	0	46180	3	49478	2
(44153)	0	46240	0	49548	2
(44292)	0	46269	0	49785	3
44398	0	46307	0	49831	5
44426	0	46336	1	49882	1
44671	1	46394	0	49935	1
44804	0	46418	0	50005	2
44834	0	46452	0	50177	2
44895	3	46488	0	50244	2
44982	0	46593	3	50514	2
45045	0	46759	4	50682	4
45094	0	46809	1	70744	4
45143	1	47043	10	51222	1
45186	1	47089	2	51319	0
45267	0	47148	2	51452	2
45325	2	47190	5	51557	1
45372	4	47233	1	51648	0
45434	6	48455	2	51769	3
45501	1	47773	10	51835	3
45555	2	47849	1	(51927)	2
45583	10	47936	2	(51960)	2
45724	3	48183	2	52134	5
45752	2	48206	2	52216	2
45785	1	48352	2	52635	7
45811	1	(48452)	1	52825	10

The following bands showed a marked increase in intensity when the temperature of the cyanogen was raised to about 300° C.:—

44153, 44398, 44426, 44671, 45186, (46420), 46759 cm.$^{-1}$.

Many other bands gave increased absorption, but a closer examination showed that this was due to a spreading out of the bands towards the long wave-lengths.

Mecke * has discussed the probable vibration frequencies of the ground state of the cyanogen molecule. He concludes that, if cyanogen has a straight-line molecule like acetylene, it should possess three valency vibrations ν_1, ν_2, ν_3, and two transverse vibrations δ_1, δ_2. From Burmeister's

* *Zeit. für Physik*, **64**, 182.

measurements of the infra-red absorption of cyanogen,* he assigns the following values to the various frequencies:—

$$\nu_1 = 440 \text{ cm.}^{-1} \text{ (approx.)}, \quad \nu_2 = 2150 \text{ cm.}^{-1},$$
$$\delta_1 = 623 \text{ cm.}^{-1}, \quad \delta_2 = 740 \text{ cm.}^{-1}.$$

The Raman frequency, $\Delta\nu = 2330$ cm.$^{-1}$,† must be identified with the symmetrical vibration ν_3, which does not appear in the infra-red absorption.

The bands, of which the intensity increases as the temperature rises, must be due to absorption by molecules possessing vibration energy. The frequencies ν_1, ν_2, ν_3, δ_1 and δ_2 should therefore be found among the differences between these bands and the bands due to absorption of light by non-vibrating molecules. In Table II some such differences are listed, which may be vibration quanta of the normal cyanogen molecule.

TABLE II.

Bands.	Difference cm.$^{-1}$.	Identification.
44895–44426	469	ν_1
47233–46759	474	ν_1
46809–44671	2138	ν_2
46488–44153	2335	ν_3
47190–44398	2792	$\nu_3 + \nu_1$
47455–44671	2784	$\nu_3 + \nu_1$
45811–45186	625	δ_1
47043–46420	623	δ_1
44895–44153	742	δ_2
45933–45186	747	δ_2
49831–46759	3072	$\nu_3 + \delta_2$

These identifications are only tentative. Some of them must be incorrect, because the bands 44153, 44671, 45186, and 46759 appear twice in the first column.

THE CYANOGEN HALIDES: CNCl, CNBr, CNI.

Cyanogen chloride was prepared by bubbling chlorine into a solution of 26 grams potassium cyanide and 9 grams zinc sulphate in 1 litre of water at 0° C.‡ When all the precipitated zinc cyanide had redissolved, just sufficient potassium cyanide was added to remove excess chlorine. The mixture was then warmed to room temperature and the cyanogen chloride distilled over in a current of air, passing through a phosphorus

* Burmeister, *Verh. d. D. Phys. Gesell.*, p. 597, 1913.

† Daure and Kastler, *Comptes Rendus*, **192**, p. 1722.

‡ A. Held, *Bull. Soc. Chim.*, (3), **17**, p. 287, 1897.

pentoxide drying-tube to a receiver kept at $-60°$ C. Cyanogen bromide was prepared by dropping bromine on to a saturated solution of potassium cyanide cooled in ice. The cyanogen bromide was separated into a cooled receiver by a current cf warm air. Cyanogen iodide was obtained from laboratory stock. It was purified by recrystallisation from carbon tetrachloride, and then sublimed.

Cyanogen chloride at atmospheric pressure in a quartz absorption tube 25 cm. long showed continuous absorption, beginning about 2240 A and extending towards shorter wave-lengths as far as we could observe with the small Hilger spectrograph (1840 A). The absorption edge at 2240 A was very indefinite and the figure is only approximate. The cyanogen chloride contained as impurity a trace of cyanogen, which produced narrow bands at 2193 and 2135 A, crossing the continuous absorption due to cyanogen chloride.

Cyanogen bromide was investigated at a pressure of 76 mm. (the vapour pressure of the crystals at 18° C.) in a 25-cm. absorption tube. The absorption spectrum was continuous, like that of the chloride, but the long wave-length limit was displaced to approximately 2540 A.

For cyanogen iodide two absorption tubes were used—one, 10 cm. long, of quartz, and the other, 40 cm. long, of glass, with quartz ends fixed on with picein. Pure crystals of the iodide were introduced into the tubes, which were then evacuated with an oil pump and sealed off. The vapour pressure of cyanogen iodide does not appear to have been measured, but it is probably of the same order as the vapour pressure of iodine. In order to obtain a greater pressure of cyanogen iodide, the quartz tube was electrically heated. Sublimation of the crystals was prevented by winding the tube with stout copper wire so that ¾ inch of the winding protruded over each end of the tube. The copper wire was covered with a layer of asbestos paper, which was in turn wound with nichrome ribbon.

Cyanogen iodide was found to have two regions of continuous absorption. The first, which was only observed with high pressures of the iodide (temperature $>$ 100° C.), began about 3100 A, reached a maximum about 2500 A, and then decreased towards shorter wave-lengths. The second region, which showed up at room temperature and more intensely at 100° C., had its long wave-length limit at approximately 2150 A.

Since the above work was done, a paper has been published by R. M. Badger and Sho Chow Woo * on the absorption spectra of the cyanogen halides. Their observations agree well with ours. The values they give

* *J.A.C.S.*, 53, No. 7, p. 2572, 1931.

for the long wave-length limits of absorption are compared with ours in the following table (III):—

TABLE III.

Halide.	Long Wave-length Limit.	
	Badger and Woo.	Mooney and Reid.
Cyanogen chloride	2270 A	2240 A
Cyanogen bromide	2450 A	2540 A
Cyanogen iodide	2900 A	3100 A
" "	2100 A	2150 A

Products of Photodissociation

The continuous nature of the absorption spectra of the cyanogen halides indicates that absorption results in dissociation of the molecule. The products are almost certainly a halogen atom and a CN radical. The energy $h\nu$ of a quantum of light absorbed is equal to the dissociation energy + the excitation energy of the resultants + the kinetic energy of the resultants, thus:

$$h\nu = \mathrm{D} + \mathrm{E} + \mathrm{k.e.}$$

It has been found for some continuous spectra (*e.g.* the absorption of the vapours of NaCl and NaI) that the long wave-length edge corresponds approximately to dissociation of the molecule into parts with no relative kinetic energy, so that

$$h\nu = \mathrm{D} + \mathrm{E}.$$

In the general case, however, one can only say

$$h\nu \geqslant \mathrm{D} + \mathrm{E}.$$

It is thus possible to calculate upper limits for D, because the values of E, the excitation energies which the halogen atom* or the CN radical† may acquire, are accurately known. These values are Cl = 2·3 k.cal., Br = 10·2 k.cal., I = 21·6 k.cal., CN = 41·5 and 73·4 k.cal.

In Table IV, column 6, are given upper limits for the heat of dissociation of the cyanogen halides into a halogen atom and CN radical, both in the ground state. For each halide, three values of D are tabulated, corresponding to the three different values which may be given to E, the excitation energy of a halogen atom or a cyanogen radical. The mean

* Turner, *Phys. Rev.*, **27**, p. 397, 1926.
† Mulliken, *Phys. Rev.*, **29**, p. 923, 1927.

values given by Eucken * for the energy required to break a carbon-halogen linkage in aliphatic chlorides, bromides, and iodides are tabulated in column 7.†

TABLE IV.

Molecule.	Absorption Limit A.	$h\nu$ k.cal.	E k.cal.	Excited Radical.	D k.cal.	D′ k.cal.
CNCl	2240	127	2·3	Cl	123·7	
			41·5	CN	85·5	73
			73·4	CN	53·6	
CNBr	2540	112	10·2	Br	111·8	
			41·5	CN	70·5	59
			73·4	CN	38·6	
CNI	3100	92	21·6	I	70·4	
			41·5	CN	50·5	44
			73·4	CN	18·6	
CNI	2150	132	21·6	I	110·4	
			41·5	CN	90·5	
			73·4	CN	58·6	44

Comparison of columns 6 and 7 shows that the most probable result of light absorption is dissociation into an unexcited halogen atom and an excited cyanogen radical. The energy of excitation is 41·5 k.cal., except in the short wave-length absorption band of cyanogen iodide where the CN radical has 73·4 k.cal. excitation energy.

If this is the true mechanism of the photodissociation, the excited CN radical ought to give rise at low pressures to fluorescence in the visible region: the CN ($^2\Sigma$) with 73·4 k.cal. being the upper level of the well-known violet CN bands and the CN ($^2\Pi$) with 41·5 k.cal. the upper level of the red CN bands. We were unable to detect visually any such fluorescence of cyanogen chloride or iodide, exposed to an aluminium spark. Similar attempts by Badger and Woo also proved unsuccessful. This is probably due, as these authors point out, to insufficient absorption at the low pressures necessary for fluorescence.

SUMMARY.

The ultra-violet absorption spectra of C_2N_2, CNCl, CNBr and CNI have been photographed.

* *Lehrbuch d. chemischen Physik*, p. 882, 1930.

† Values calculated by Badger and Woo from thermochemical data for the (CN)Cl and (CN)I linkages (74·7 k.cal. and 52·0 k.cal. respectively) are in good agreement with Eucken's mean values.

Measurements are given of band edges of the C_2N_2 absorption spectrum in the region 2380 A–1850 A. The influence of temperature on the relative intensities of the bands has been studied.

The long wave-length limits of the regions of continuous absorption due to the cyanogen halides are: CNCl, 2240 A; CNBr, 2540 A; CNI, 3100 A and 2150 A.

We desire to thank Dr Baker, of the Royal Observatory, Edinburgh, for making photometer records of several spectra, Dr Ludlam for his helpful interest in our work, and Imperial Chemical Industries, Ltd., for a grant towards the cost of quartz apparatus.

(*Issued separately February* 15, 1932.)

IV.—The Effect of Malnutrition on Root Structure. By George Bond, B.Sc., Ph.D., Carnegie Teaching Fellow in the Botany Department, University of Glasgow. *Communicated by* Professor J. WALTON. (With Two Plates.)

(MS. received October 24, 1931. Read February 15, 1932.)

THE effect of the malnutrition induced by removal of the cotyledons from seedlings at an early stage in germination is variable, depending chiefly on the species employed and upon the conditions of growth. Goebel (7), speaking of *Phaseolus vulgaris*, says that provided decotylation is delayed until after the radicle is at least 1 cm. long, the seedlings can be induced, by the provision of favourable cultural conditions, to continue development, even as far as the production of flower buds, although such plants are weakly when compared with normal plants. Flaskämper (6) obtained similar results with Phaseolus sp. and with *Vicia Faba*. Reed (10), probably employing less favourable conditions, found that death followed decotylation in *Vicia Faba* and in *Phaseolus multiflorus* unless the operation was delayed until the third or fourth pair of leaves was expanded, while in different varieties of *Phaseolus vulgaris* decotylation after the first pair of plumular leaves was unfolded in no way affected the subsequent growth. It is worthy of mention that Reed's results possibly arose from the greater photosynthetic efficiency which may prevail in epigean seedlings as compared with hypogean types (Briggs (3)), as a result of which the latter would be more dependent on the cotyledonary reserves.

In the present instance the method has been employed with the possibility in view of obtaining facts bearing on the concept of the Size Factor, recently advanced by Bower (2). As a result of decotylation the dimensions of the subsequent growth of the various parts of the seedling undergo reduction,* and an opportunity is offered for the study of the effect of Size upon Structure in an individual organ. Flaskämper's observations (*loc. cit.*) suggest that the primary root of the seedling is the most favourable organ for the purpose, and in the present work consideration will be confined to that organ. A review of Flaskämper's results is desirable at this point since the present work was undertaken with the object of confirming and extending his observations.

* A similar effect can be produced after the seedling stage is passed by leaf-pruning. This method has recently been employed by Parker and Sampson (9) in experiments to which reference is made later.

Flaskämper's Results.

The following is a summary of the statements made by Flaskämper on the basis of his experiments:—

The root apex was much less active as the result of decotylation, the tissues being produced on a reduced scale. Thus in Phaseolus sp. there was a marked reduction in the number of vessels in the post-decotylation growth as compared with the number in corresponding parts of roots of controls. The changes induced by decotylation were not, however, those of degree only. Flaskämper states that definitely new features appeared. In Phaseolus a disappearance of the pith usually followed decotylation. Thus in one experiment the roots were 5–7 cm. long at decotylation, and after a subsequent growth of 1 cm. the pith had generally disappeared. "Man kann allgemein sagen, dass nach umgefähr 1 cm. dass Mark verschwunden ist" (p. 205). In a second experiment one particular seedling, six weeks old, possessed a main root 26 cm. in length, decotylation having been effected when the root was rather less than 6 cm. long. Up to a point 5 cm. from the base* of the root a typical Phaseolus structure was displayed, a "tetrarch stele with well-developed pith" being present (see later for criticism of this latter statement). Beyond this point, however, pith-reduction ensued, and at a point 7 cm. from the base was complete, no pith being present. This reduced condition persisted up to a point 20 cm. from the base, at which point the pith reappeared, and extended thence to the apex. This reappearance of the pith was correlated by Flaskämper with the improved nutrition that obtains as the number of expanded leaves borne by the seedling increases.

A similar reaction was detected in *Vicia Faba*, a disappearance of the pith being observed after a growth of about 1 cm. The reduced condition persisted for a shorter zone than in Phaseolus, the pith reappearing in one root, for example, after a growth of 5–6 cm. An additional feature in Vicia, also stated to be consequent on decotylation, was a reduction in the number of protoxylem poles. This second reduction phenomenon was first noticed after a growth of 3 cm. subsequent to decotylation, and was most frequent in roots originally hexarch. Reduction to pentarchy and sometimes subsequently to tetrarchy was observed in these cases. It should be noted that no mention is made of a restoration of the original protoxylic symmetry as growth of the root continued.

If the above observations are correct they appear to be of some

* In the present paper the term "base" is employed in contradistinction to the *apex* of the root—that is, indicative of the proximal portion of the root lying immediately below the hypocotyl.

significance in a consideration of the effect of Size on Structure. As Bower (*loc. cit.*) has pointed out, the efficiency of the root as an organ of absorption depends to a considerable extent upon the maintenance of a sufficiently large surface of contact between the dead tracheal elements and the living cells surrounding them. The facts advanced by Bower to show that when increase in size of the stele of the root occurs, modifications ensue which result in the maintenance of the original extent of this surface, relative to the bulk of the xylem, suggest that the surface is in fact a limiting one. The usual modifications consist in the increase in the number of xylem rays and medullation. The demonstration of the relation of medullation to stelar size is not so conclusive as that in the case of the number of xylem rays, and Bower suggests that in the former instance the inherited structure hinders individual reaction to the Size Factor, *i.e.* the root is insufficiently plastic.

In Flaskämper's experiments we appear to be getting, in the first place, not increased stelar size, accompanied by greater complexity of arrangement, but a reduction, and with this a more simple type of construction. Regarded from the standpoint of the Size Factor hypothesis, these facts would be expressed by saying that reduction in the amount of xylem is accompanied by a disappearance of those modifications originally introduced to maintain the necessary ratio of area of contact between the tracheal elements and the surrounding parenchyma *to* the increased bulk of the tracheal elements. Bower would ascribe the disappearance of these features as due to the proleptic action of the Size Factor on the apical meristem. As the nutrition again approaches the original level, the size of the stele, and along with this its arrangement, returns to normal. Here there appears to be a double incidence of the Size Factor, one following a reduction in size, the second an increase.

It was in view of these considerations that the present work was embarked upon. It must be pointed out, however, at this point, that in several respects Flaskämper's account of his methods and results is unsatisfactory. It is much to be regretted that he fails to mention the species of Phaseolus employed, making it very difficult for his results to be checked. Secondly, in no case does he specify the number of plants examined in an experiment, and he neglects to indicate the degree to which the description of a single seedling, with which he frequently illustrates a statement, applies to the other seedlings examined. The most serious criticism is, however, that Flaskämper appears to have made insufficient examination of his controls. As will be detailed below, it now appears probable that practically all the phenomena that Flaskämper

attributed to the effect of decotylation are merely normal features in the development of the species concerned.

The Present Experiments.

The species employed in these experiments were as follows: Broad Bean (*Vicia Faba* L.), Austin & M'Aslan's * Giant Windsor; Runner Bean (*Phaseolus multiflorus* Willd.), A. & M.'s Mammoth Scarlet; Dwarf Bean (*Phaseolus vulgaris* L.), A. & M.'s Canadian Wonder. It is assumed that the species of Phaseolus employed by Flaskämper was one of these two common ones.

Vicia Faba L.

These experiments were carried out in early spring. Decotylation was effected 7-8 days after sowing, and in the earlier experiments precautions were taken to prevent infection of the wounded plants, the cotyledons being severed with a sterilised scalpel and the wounds sealed with hot paraffin wax. The subsequent growth of these seedlings in no wise differed from that of others in which these precautions were omitted, and in view of this, in later experiments with this and other species, no particular precautions were taken to guard against infection. The seedlings were grown in a good quality soil under normal greenhouse conditions. The growth of the decotylated seedlings was followed for 3-4 weeks and compared with that of controls of the same age growing under identical conditions. During this period the decotylated seedlings showed very little growth above or below ground. Thus the epicotyl attained a height of only 2-5 cm., and the plumular bud rarely showed any signs of opening, while the controls grew to a height of 20-30 cm., each plant bearing several foliage leaves. The average length of the main root of the decotylated seedlings at the end of the period was 7·6 cm., giving a growth of 1·6 cm., the length at decotylation having been 6 cm. The average length of the roots of the controls at the end of the period was 23 cm. In this species, unlike those to be described later, the diameter of the root was not markedly affected by the condition of malnutrition induced by decotylation, although it might have been had the elongation of the root continued for a longer period. A number of seedlings failed to survive the period of the experiment, and a high proportion of the twenty-five that did survive were, at the end of the experiment, showing a slight blackening of the tissues of the root-tip or epicotyl, a reaction which usually precedes or accompanies death in the Broad Bean. In experiments continued for a longer period than the above, the discolora-

* Seedsmen to University of Glasgow.

tion became more extensive, and very shortly the death of all the seedlings occurred.

An examination of the root structure in decotylated and in normal seedlings was made. With regard to the pith the condition of affairs under normal circumstances is much the same in this species as in the genus Phaseolus, described in detail below. While a true pith is present in the basal part of the root, being in continuity with the hypocotyledonary pith, yet, after the root has attained a certain length, a central strand of procambial tissue is produced in place of the pith, and from this a core of metaxylem ultimately develops (Pl. I, figs. 1 to 7). In five out of six control roots examined a solid core of metaxylem was present at least from a midway point in the length of the root down to a point distant one-fifth of the total length of the root from the base. In a sixth root the zone showing a xylem core was very short.

In all the decotylated seedlings a pith was present from base to apex, presumably because the elongation of the root had not proceeded to the stage at which the change in apical behaviour occurs. No signs were detected of any elimination of the pith as a result of decotylation, but the early death of the seedling made it impossible to demonstrate conclusively that decotylation does not induce pith-disappearance. As is indicated below for Phaseolus, there is, however, nothing to suggest that the pith-disappearance described by Flaskämper, and assumed by him to be a result of decotylation, was anything other than the normal process referred to above, for it is quite possible that in the variety used by him this normal change occurred shortly after the stage at which decotylation was effected.

The question of the symmetry of the root was also investigated. No such results as Flaskämper described can be recorded, and in any case reduction in the number of protoxylem poles during the development of the root appears to be sufficiently common in the Broad Bean under normal circumstances to throw serious doubt on Flaskämper's statements. Flaskämper makes no mention of any examination of his controls in this connection, obviously a very necessary procedure. In the present work six roots of control plants have been sectioned from base to apex. Only one of these showed constant symmetry throughout, namely, hexarchy. Three showed a reduction from pentarchy to tetrarchy at a point near the middle of the root, *i.e.* about 12 cm. from the base in these examples. In one of these roots reduction occurred as the result of a protoxylem strand ending blindly, while in the other examples two root poles gradually converged, one diminishing in size meanwhile, and finally fusing more or less

completely with its larger neighbour, the sequence being illustrated in Pl. I, figs. 1 to 6. One of these three roots showed an additional reduction from hexarchy to pentarchy, just below the hypocotyl.

In each of the remaining two roots a reduction from hexarchy to pentarchy had occurred at a point 3–4 cm. from the base of the root.

A similar reduction was also detected in three out of eight decotylated seedlings that were examined, in one case from hexarchy to pentarchy at a point 4 cm. from the base, in a second a similar reduction 2 cm. from the base, and in a third from heptarchy to pentarchy shortly below the hypocotyl. In all these cases the reduction had occurred before decotylation and therefore could have no connection with it. It seems more than probable that in Flaskämper's experiments, in which growth in the decotylated seedlings proceeded to a much greater extent than in the present work, the above normal reduction sometimes occurred subsequent to decotylation, and was erroneously assumed by him to be a result of that process. This probability is strengthened by the absence from Flaskämper's account of any mention of a restoration of the original symmetry as the nutrition returned to normal.

Phaseolus multiflorus Willd.

These experiments were conducted in November and December, over periods of 17 and 24 days. A number of decotylated seedlings failed to survive even the shorter period. Those that did showed very little growth compared with the controls, the epicotyls being from 2–5 cm. in height and the plumules unopened and chlorotic, while the controls attained a height of 15 cm., the first pair of foliage leaves being expanded. The length of the radicle at the time of decotylation, effected 6 days after sowing, was 4·9 cm. (average of 43 seedlings). The length of the main root of those seedlings which were left intact to serve as controls was 30·9 cm. (average of 10) after 17 days, and of the decotylated seedlings 9·5 cm. (average of 14). After 24 days the latter had increased to 10·6 cm. (average of 6). The apical portion of the roots of decotylated seedlings showed a definite attenuation, and these were also distinguished from those of the controls by their poor production of lateral roots.

In the study of the internal structure, transverse sections were cut from corresponding levels in the roots of controls and of decotylated seedlings, seven levels being sectioned in each root. Seven decotylated seedlings and four controls were treated in this way, and the figures given below are average values based on the examination of these seedlings.

As mentioned above, the roots of decotylated seedlings showed an

apical attenuation, the range of diameter in these being from ·639 mm.* (at the sectioned level nearest the apex) to 1·833 mm. (at the level nearest to the base), while the corresponding range in the controls was 1·006 mm. to 2·375 mm., the greatest difference being thus near the apex.

This reduction in diameter was accompanied by a general reduction in the tissues. In the case of the stele, the range of diameter in the decotylated seedlings was from ·349 mm. to ·888 mm., the corresponding figures for the controls being ·433 mm. to 1·356 mm. Evidently the greatest difference in stelar size is towards the base, this being due to the decreased production of metaxylem and of secondary xylem in the decotylated seedlings.

Comparison of the number of vessels seen in the transverse sections at different levels showed a range of 49 to 88 in the decotylated seedlings, against 39 to 121 in the controls. The greater number of vessels in the apical region of the decotylated seedlings is due to the more extensive development of what Dodel (5) termed "Zwischenstränge." These latter, characterising some species of Phaseolus, are extra strands of xylem lying alternately with the normal root poles. Like the latter these "Zwischenstränge" pass into the cotyledons. Their number and development is irregular. In the basal part of the root, lateral roots may be developed in connection with them, but they do not normally extend far into the more apical part of the root. Apart from the extreme basal part of the root these strands, according to Dodel, consist entirely of metaxylem. In agreement with Dodel's statements these strands were detected in the more basal parts of the roots of the control plants, but never near the apex (Pl. I, figs. 8 to 11). Evidently the root apex produces these extra strands until the root has attained a certain length, when their deposition ceases. The roots of the decotylated seedlings, by reason of their very slow elongation, were, at the end of the experimental period, of such a length that "Zwischenstränge" were still being differentiated closely behind the apex (Pl. II, fig. 21), and hence the greater number of vessels in the apical region of these roots than in corresponding regions of the controls.

Particular attention was paid to the pith, since Flaskämper reported a disappearance of this tissue subsequent to decotylation. As stated by Dodel (*loc. cit.*) and by Compton (4), a pith is present in the basal and first formed part of the root in the genus Phaseolus, being continuous with the hypocotyledonary pith. The metaxylem in this region develops tangentially from the protoxylem groups and forms a layer round the margin of the pith. A change in the behaviour of the apex occurs, however, as the

* Measurements made with a standardised eyepiece micrometer.

root elongates. A pith is no longer produced. Instead, a strand of procambial cells occupies the medullary position, and from this the metaxylem develops a short way behind the apex. Examination of the controls showed that the structure agreed with the above description given by Dodel and Compton, in these seedlings the production of a pith having ceased at a point distant from the base about one-third of the total length of the root at the end of the experimental period (Pl. I, figs. 8 to 11).

In the decotylated seedlings the root showed a pith practically throughout its entire length, doubtless as a result of the much slower growth as compared with the controls. In four of these seedlings, however, the change in apical behaviour had obviously been occurring at the period when they were uprooted, for in these a few thin-walled metaxylem vessels were present in the "pith" in the sections from nearest the apex (Pl. II, fig. 20). There is nothing to indicate that this change is in any way different from the normal change occurring in the controls, and if this was the species studied by Flaskämper, then it is probable that he mistook this normal disappearance of the pith and its replacement by a core of metaxylem for a change induced by decotylation, an error easily committed in the absence of a thorough examination of the controls. In his seedlings the change must have occurred shortly after decotylation. The reappearance of the pith, reported by him to occur after the nutrition returned to normal, is capable of explanation in the same way, for the metaxylem does not develop immediately behind the apex, so that there is constantly in the apical part of the root a medullary strand of procambial tissue simulating a pith.

Phaseolus vulgaris L.

These experiments were carried out in October and November, over periods of 26–33 days. At the end of these periods practically all the decotylated seedlings showed more or less advanced signs of succumbing, and it appeared very unlikely that any would have survived a much longer period. The hypocotyl of the decotylated seedlings attained during the period of the experiments a length of 2–5 cm., but the epicotyl was practically undeveloped, and the plumular leaves were yellowish green, rarely showing any signs of opening. The controls were very much stouter, having hypocotyls 7 cm. in height and bearing a fully expanded pair of foliage leaves. The length of the radicle at decotylation, effected 6–10 days after sowing, was 5·1 cm. (average of 23 seedlings), and at the end of the period 9·6 cm. (average of 10). The main root of the controls during the same period attained a length of about 30 cm. The apical

part of the main root of the decotylated seedlings showed attenuation. Lateral roots were very few on these roots, while there was a copious production of them in the case of the controls.

The internal structure of these seedlings was not examined in such detail as in the previous species, but the general features were much the same as in the latter. As is suggested by a comparison of the photographs at the end of this paper, the diameter of the stele in the roots of decotylated seedlings was smaller throughout than in the controls (see Pl. II, figs. 17 and 18, and 13 to 16). The number of vessels was much the same in the apical part of the roots of decotylated seedlings as in those of the controls, but there were fewer in the basal region of the former, due to decreased production of metaxylem and of secondary xylem (Pl. II, figs. 13 and 17). "Zwischenstränge" were present in the basal part of the roots of controls (Pl. II, figs. 13 and 14), but were not detected in any of the decotylated seedlings.

As in the previous species, a true pith is present in the basal part of the root in normal circumstances, in continuity with that of the hypocotyl, but for the greater proportion of the length of the root a central core of metaxylem is present, with no semblance of a pith (Pl. I, fig. 12; Pl. II, figs. 13 and 14). The medullary core of tissue, which in sections taken from near the apex appears to be a pith, is in reality a strand of procambial tissue, from which metaxylem vessels develop a short way behind the apex (Pl. II, figs. 15 and 16). In decotylated seedlings the differentiation of this metaxylem tends to be delayed, with the result that the xylem core is confined to the basal part of the root.

In view of the absence of a pith from the root of this species except at its very base, it is improbable that this was the species used by Flaskämper, since he refers to the normal root as showing a well-marked pith.

Discussion.

As appears from the foregoing account, in the present work decotylation affected the seedlings more adversely than with Flaskämper, presumably owing to his use of more favourable cultural conditions or of more virile varieties. Although, for this reason, circumstances have not permitted of a complete repetition of his experiments, yet it is very probable that Flaskämper was in error in stating that decotylation results in a non-formation of the pith in the subsequent growth of the root in seedlings of Phaseolus sp. and of *Vicia Faba*, and further, that a reduction in the number of root poles is induced in the latter. It is

pointed out that these changes occur during the normal development of the above species, and therefore the latter are obviously unsuited for use in an investigation of the possibility of the same changes being induced by decotylation. In fact there is nothing in Flaskämper's account of his work to suggest that the phenomena he observed and attributed to the effect of decotylation were anything other than these normal changes. The present work indicates that in these species decotylation has no marked effect on root structure.

Contrary to expectation, therefore, so far as the roots of these particular plants are concerned, it has not been possible to demonstrate that the experimentally produced diminution in size of the stele is followed by the morphological changes such as loss of pith and decrease in the number of xylem rays which might have been expected to result from the operation of the Size Factor.* It is, however, obvious that such a simplification of root structure will not have the critical importance to the plant that a corresponding but opposite increased complexity accompanying increase in bulk of the stelar tissues may have, for economy of material is the only advantage likely to accrue from the former process. Experimentally produced *increase* in the normal size would probably yield results of more significance in an investigation of the operation of the Size Factor.

Although the facts presented in this paper do not warrant a definite conclusion on this point, it is very probable that the reduction in the number of protoxylem strands occurring in *Vicia Faba* and also in Phaseolus sp., if the "Zwischenstränge" are regarded as serving as extra poles, can be related to the fact that during the earlier development of the root, in the period when it is the sole absorptive organ of the seedling, more xylem is developed behind the apex than in the later stages of root development, when the main root is supplemented by laterals. In order that this greater bulk of xylem may maintain the required efficiency in the absorption (or reception) of water from the surrounding living tissue, a higher degree of subdivision into rays is necessary than in the later-developed portion of the root. Such an explanation of the change in symmetry is admittedly teleological, but it is all that can be attempted at present. It is probable that more extensive investigation would reveal the occurrence of such simplifications in symmetry during development in many other roots. Aldrich-

* It is probable, however, that reduction in root size *may* be accompanied by a reduction in the number of protoxylem rays, particularly where the normal number of the latter is high. The figures in Parker and Sampson's recent paper (9) suggest that such is the case in roots of defoliated cereals.

Blake (1, p. 18) notes a similar change in symmetry during the development of the seedling-root in the Corsican Pine. Liese (8) found reduction to be of normal occurrence in *Pinus sylvestris*. A converse type of change, *i.e.* an increase in the number of poles from diarchy to tri- and sometimes tetrarchy, was detected by Aldrich-Blake (*loc. cit.*, p. 26) in several lateral roots in the Corsican Pine, and he was able to relate this definitely to an increase in the bulk of the xylem occurring shortly afterwards.

It remains to consider the pith. In the species studied here a true pith is only produced during the earlier growth of the radicle. Possibly the innermost portion of the plerome, as a result of the activity of which the pith is produced, undergoes elimination during the later development of the root. No causal explanation of this modification in the products of the plerome can be attempted. One can only give a teleological reason by indicating any obvious advantage that is likely to accrue as a result of the presence of the pith in the basal part of the root.

As mentioned previously, Bower, chiefly from a consideration of the more primitive vascular plants, looks upon medullation as an adaptation, frequently accompanying increase in size of the stele, which helps to maintain a suitable area of contact between xylem and living tissue, particularly when the xylic ring is interrupted, the living cells of the pith being thereby brought into continuity with those abutting on the radial and outer surfaces of the xylem strands, so that the latter become surrounded by living tissue.

In the roots under consideration here it seems very improbable that the greater production of xylem during the earlier development of the root should necessitate a higher division into rays and also the formation of a large pith in order to maintain efficiency in absorption. The writer inclines to the view that no question of limiting surface need enter into an explanation of the presence of a pith in the basal part of these roots. Primarily the appearance of a pith there is a purely structural feature necessary to allow of a suitably gradual transition from the relatively stout hypocotyl, with its stele distended by the presence of a bulky pith, a type of construction necessary where rigidity is needed, to a typical root structure with its centrally condensed stele. A secondary advantage accruing from the presence of the pith, resulting as it does in a larger stele, is that it allows of a more intensive production of secondary xylem, which may be of considerable importance to the basal part of the tap root, since this serves as the main channel for transport of water from the entire root system to the shoot.

Summary.

1. Reference is made to experiments conducted by Flaskämper as a result of which he stated that the removal of the cotyledons at an early stage from seedlings of Phaseolus sp. and of *Vicia Faba* resulted in an elimination of the pith from the subsequently formed part of the root in both species, and further induced a reduction in the number of poles in the root of the second species.

2. The significance of such observations in a consideration of the theory of the Size Factor is pointed out.

3. The experiments have been repeated. The malnutrition resulting from the decotylation affected the varieties employed in the present work more adversely than with Flaskämper, and usually caused the death of the seedling after a shorter or longer period.

4. Sufficient evidence has been obtained, however, to throw very serious doubt on Flaskämper's conclusions. The two reduction phenomena he described are now shown to occur in the normal development of the roots of these species, and there is no indication that their occurrence is affected by decotylation.

5. A teleological explanation, based on the theory of the Size Factor recently advanced by Bower, is made of this reduction in the number of protoxylem poles which occurs in the normal development of the roots of these plants.

6. The significance of the appearance of a pith in the basal parts only of these roots is discussed.

In conclusion, I desire to express my indebtedness to Professor F. O. Bower, F.R.S., for his constant interest in this work, and to thank Dr S. Williams and other members of the Botany Department for much valuable help. Also my best thanks are due to the Executive Committee of the Carnegie Trust for a grant towards the expense of the illustrations both of the present work and those of a recent paper on "The Stem-Endodermis in the Genus Piper," published in the *Transactions of the Royal Society of Edinburgh.*

References.

(1) Aldrich-Blake, R. N., "The Plasticity of the Root System of Corsican Pine in Early Life," *Oxford Forestry Memoirs*, No. 12, 1930.

(2) Bower, F. O., *Size and Form in Plants.* London, 1930.

(3) Briggs, G. E., "Experimental Researches on Vegetable Assimilation and Respiration. XV. The Development of Photosynthetic Activity during Germination of Different Types of Seeds," *Proc. Roy. Soc.*, B, **94**, 1922.

(4) Compton, R. H., "On Seedling Structure in the Leguminosæ," *Journ. Linn. Soc.*, **41** (Bot.), 1912–13.

(5) Dodel, A., "Der Uebergang des Dicotyledonen-Stengels in die Phahl-Wurzel," *Jahrb. f. Wiss. Bot.*, **8**, 1872.

(6) Flaskämper, P., "Untersuchungen über die Abhängigkeit der Gefäss- und Sklerenchymbildung von äusseren Faktoren nebst einigen Bemerkungen über die angebliche Heterorhizie bei Dikotylen," *Flora*, **1** (Neue Folge), 1910.

(7) Goebel, K., *Einleitung in die experimentelle Morphologie der Pflanzen.* Leipzig und Berlin, 1908.

(8) Liese, "Beitr. z. Kennt. des Wurzelsystems der Kiefer," *Zeit. f. Forst. und Jagdw.*, 1926.

(9) Parker, K. W., and A. W. Sampson, "Influence of Leafage Removal on Anatomical Structure of Roots of *Stipa pulchra* and *Bromus hordeaceus*," *Plant Physiology*, **5**, 1930.

(10) Reed, "Some Points in the Morphology and Physiology of Fasciated Seedlings," *Ann. Bot.*, **26** (1), 1912.

EXPLANATION OF PLATES.

Note.—The nature of this investigation has demanded an extensive use of hand sections. As a result of this the microphotographs forming the plates are in some respects unsatisfactory, but they are considered to show all that is necessary.

All the photographs were taken from gentian violet-glycerine jelly preparations. Unless otherwise stated, the magnification in Plate I is × 56, and in Plate II × 90.

Plate I.

Figs. 1 to 7, *Vicia Faba* L.

Figs. 1 to 6 inclusive are from transverse sections at different levels of one control root of the Broad Bean, the root being 18 cm. long. The respective distances of the sectioned levels from the base of the root were as follows: Fig. 1, 3 cm.; fig. 2, 6 cm.; fig. 3, 10 cm.; fig. 4, 12 cm.; fig. 5, 13 cm.; fig. 6, 16 cm. The series shows how, as the root has elongated, tetrarchy has gradually superseded pentarchy as the result of the convergence and ultimate fusion of two neighbouring poles, these two poles being the lowermost as the photographs are orientated in the plate. The establishment of tetrarchy is completed in fig. 6. Fig. 1 (× 50) indicates the presence of a pith at the base of the root, "mixed" at this particular level. Fig. 6 shows the presence at levels close behind the apex of a central strand of procambial tissue from which, as the other sections show, a central core of metaxylem develops later.

Fig. 7. Transverse section from another control root at a point 8 cm. from the apex, showing an early stage in the differentiation of the metaxylem from the central procambial tissue.

Figs. 8 to 11, *Phaseolus multiflorus* Willd.

These photographs are of a series of transverse sections of a control root, 35 cm. in length, of the Runner Bean. The respective distances of the sectioned levels from the base of the root were as follows: Fig. 8, 4 cm.; fig. 9, 12·5 cm.; fig. 10, 20 cm.; fig. 11, 27·5 cm. Figs. 8 (× 47) and 9 indicate that a true pith is present in the basal part of the root. In the more central region of the root a core of metaxylem occupies the centre of the stele (fig. 10). Fig. 11 shows the development of this metaxylem from the medullary strand of procambial tissue. The series also shows the presence of "Zwischenstränge," diminishing in number from three (?) in fig. 8 to a single one in fig. 10.

Fig. 12, *Phaseolus vulgaris* L.

From a transverse section at the base of a control root, 16 cm. long, of the Dwarf Bean. Note the presence of a pith at this level. (× 47.)

Plate II.

Figs. 13 to 18, *Phaseolus vulgaris* L.

Figs. 13 to 16, a series of transverse sections of the same control root as that from which the section depicted in fig. 12 was taken. The respective distances from the base of the root of the points at which sections were taken were as follows: Fig. 13, 3 cm.; fig. 14, 6 cm.; fig. 15, 9 cm.; fig. 16, 12 cm. In fig. 13 the pith has already been replaced by a core of metaxylem which extends down to the level of fig. 15, where differentiation of the metaxylem is proceeding. In fig. 16, from the section nearest to the apex, a pseudo-pith of metaxylem mother-cells is present. "Zwischenstränge" are to be seen in figs. 13 and 14.

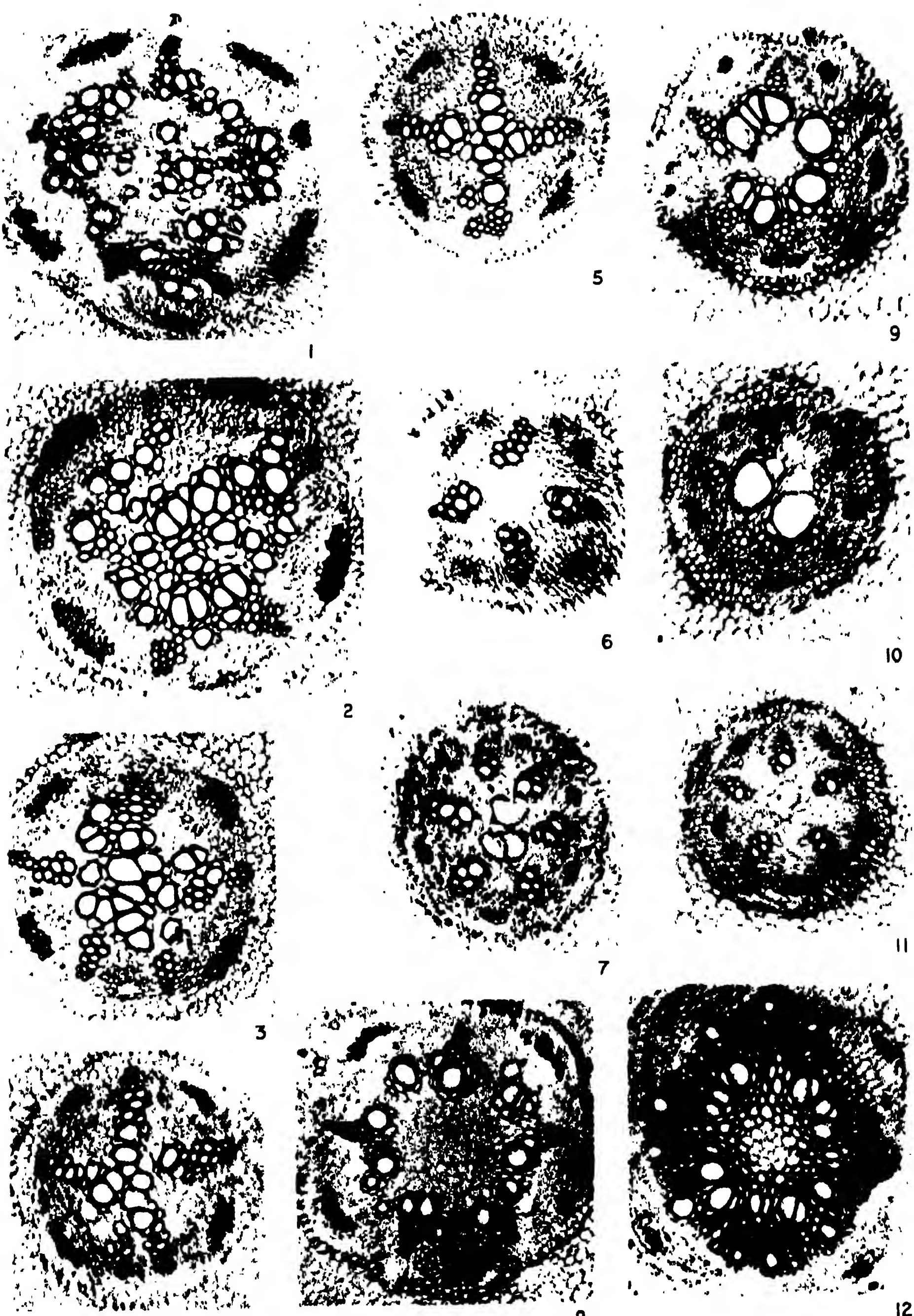
Fig. 17. Transverse section through the root of a decotylated seedling of the Dwarf Bean at a point 4 cm. from the base and 7 cm. from the apex. This particular root was 6·5 cm. long at decotylation. The photograph illustrates the reduced dimensions of the stele as compared with the steles of control roots at corresponding levels. Note that the "Zwischenstränge" have failed to develop.

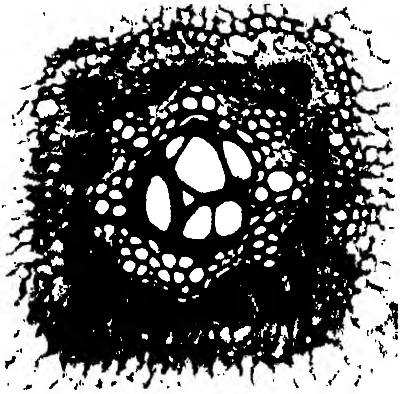
Fig. 18. Of the same root as the preceding figure, at a point 8 cm. from the base (3 cm. from the apex). Note the small relative size of the stele. A metaxylem element is developing from the "pith."

Figs. 19 to 21, *Phaseolus multiflorus* Willd.

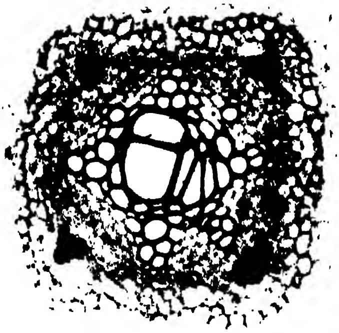
Fig. 19. From a transverse section through the root of a decotylated seedling of the Runner Bean. Decotylation was effected at a stage when the root was 4·5 cm. in length, and the latter had increased to 10 cm. when the seedling was removed from the soil. The section illustrated was from a level distant 7·2 cm. from the base of the root, and shows the presence of a pith. The latter extended from this level right down to the base of the root.

Fig. 20. From the same root as the previous figure, at a point 8·6 cm. from the base (1·4 cm. from the apex). The replacement of the true pith by a core of metaxylem mother-cells, from which a vessel is seen to be developing, has evidently occurred at this level.


1
5
9
2
6
10
3
7
11
4
8
12

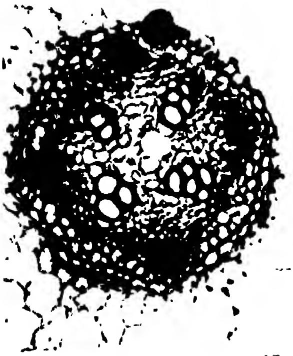


13



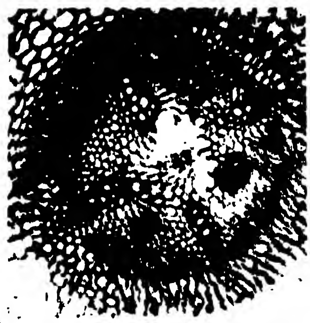
14

18

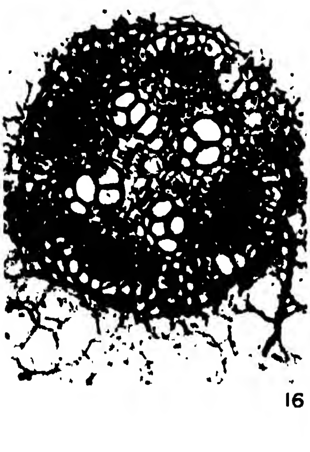


15

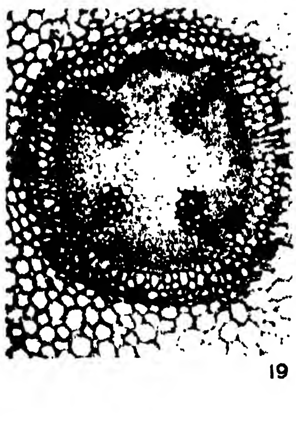
17



21



16



19

20

Fig. 21. Transverse section from the root of another decotylated Runner Bean seedling, at a point 2·3 cm. from the apex and 8·7 cm. from the base. This root was 6·5 cm. long at decotylation. In this root, more typical in this respect than that from which the sections represented in the two previous photographs were taken, two "Zwischenstränge" were present close behind the apex, as the figure indicates.

Issued separately February 19, 1932.)

V.—On the Orthogonal Polynomials in Frequencies of Type B.

By A. C. Aitken, M.A., D.Sc.

(MS. received October 30, 1931. Read January 11, 1932.)

1. Introductory.

The orthogonal properties of the Hermite polynomials, their relation to the normal frequency function, and the part they play in expressing the more general frequency function of Type A, are matters of common knowledge in mathematical statistics. Less well known are the very similar properties of a class of polynomials related in the same kind of way to Poisson's frequency function of rare events and to the function of Type B which generalizes Poisson's function. These polynomials have been interestingly studied during the last few years by Ch. Jordan and by H. Pollaczek-Geiringer. (References are given at the end of this paper.) In preparation for our main object we shall gather together the principal results concerning these polynomials, together with a few others not mentioned in the extant literature.

Our object is to investigate, in relation to the function of Type B, a suggestion made by Charlier in regard to Type A, namely, that certain well-known irregularities of convergence in the series of Type A might be obviated by expanding not the frequency but its logarithm, in a series of Hermite polynomials. Charlier's conjecture, proved in part by him, has been justified in general by Dr A. Oppenheim and the present writer. The corresponding peculiarities of convergence in the terms of Type B are not so serious in practice, but it is a question of interest whether here also an expansion of the logarithm of the frequency in a series of "polynomials of Type B" may not give a more regular convergence. We shall prove that this is actually the case; but we shall have occasion to observe that no great practical advantage ensues, the established methods, in particular that of fitting by factorial moments, being still the most convenient and just as reliable.

The frequency function of Type B is expressible in the form of the series

$$f(x) = \psi(x) + \beta_2\nabla^2\psi(x) + \beta_3\nabla^3\psi(x) + \dots, \qquad (1.1)$$

where

$$\psi(x) = e^{-m}m^x/\Gamma(x+1), \quad \text{and} \quad \nabla\psi(x) = \psi(x) - \psi(x-1), \qquad (1.2)$$

a receding finite difference.

The function $\psi(x)$ is Poisson's frequency function, m being the mean value of x. The coefficient β_2 is known to be of order N^{-1}, β_3 and β_4 being of order N^{-2}, β_5 and β_6 of order N^{-3}, and so on, where N is the number of causes of deviation, or of elementary increments. (In many discussions x is regarded as capable of assuming discrete values only, $\Gamma(x-1)$ being then written $x\,!$, but the series of Type B is just as applicable to continuous variates as that of Type A, and can be derived in a similar manner.)

It will be proved below that if $\log f(x)$ be expanded as a series of orthogonal polynomials $K_r(x\,;\,m)$, or $K_r(x)$, as follows,

$$\log f(x) = \log \psi(x) + k_2 K_2(x) + k_3 K_3(x) + \ldots, \quad . \quad . \quad . \quad (1.3)$$

where

$$K_r(x)\psi(x) = (-)^r \nabla^r \psi(x), \quad . \quad . \quad . \quad . \quad . \quad (1.4)$$

then the coefficient k_r is of the order N^{1-r}, that is, the terms of the expansion decrease uniformly. The double induction by which the result is established follows the lines of the corresponding induction for Hermite polynomials, due to Dr Oppenheim, by which the analogous problem for Type A was resolved. Though the final result is similar, noteworthy features of contrast occur in the intermediate stages.

2. Properties of Polynomials of Type B.

From the definition (1.4) above, we have readily

$$K_0(x) = 1,$$
$$K_1(x) = (x-m)/m,$$
$$K_2(x) = \{x(x-1) - 2mx + m^2\}/m^2,$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$K_r(x) = \left\{x^{[r]} - rmx^{[r-1]} + \binom{r}{2} m^2 x^{[r-2]} - \ldots + (-)^r m^r\right\} \Big/ m^r,$$

where

$$x^{[r]} = x(x-1)\ldots(x-r+1). \quad . \quad . \quad . \quad . \quad . \quad (2.1)$$

Symbolically,

$$K_r(x) = e^{-m\Delta} x^{[r]}/m^r. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.2)$$

We have at once

$$\nabla K_r(x) = \frac{r}{m} K_{r-1}(x-1), \quad . \quad . \quad . \quad . \quad . \quad (2.3)$$

or

$$\Delta K_r(x) = \frac{r}{m} K_{r-1}(x), \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.4)$$

where Δ is the operator of *advancing* differences, defined by

$$\Delta\psi(x) = \psi(x+1) - \psi(x).$$

Also

$$\nabla\{K_r(x)\,.\,\psi(x)\} = \nabla\psi(x)\,.\,K_r(x-1) + \psi(x)\nabla K_r(x),$$

that is,

$$K_{r+1}(x)\,.\,\psi(x) = \psi(x)\left\{K_1(x)K_r(x-1) - \frac{r}{m}K_{r-1}(x-1)\right\}, \quad . \quad . \quad (2.5)$$

giving a recurrence relation,

$$K_{r+1}(x) - K_1(x)K_r(x-1) + \frac{r}{m}K_{r-1}(x-1) = 0, \quad . \quad . \quad . \quad (2.6)$$

analogous to that for the Hermite polynomials,

$$H_{r+1}(x) - H_1(x)H_r(x) + rH_{r-1}(x) = 0. \quad . \quad . \quad . \quad . \quad (2.7)$$

Any arbitrary polynomial can be expanded according to polynomials of Type B, the expansion for factorials $x(x-1)(x-2)\ .\ .\ .\ (x-r+1)$ taking a specially simple form, proved at once by inverting the symbolic relation (2.2) above and using (2.4), namely

$$x^{[r]} = m^r e^{m\Delta} K_r(x), \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.8)$$

or

$$x(x-1)\ .\ .\ .\ (x-r+1) = m^r\left\{K_r(x) + rmK_{r-1}(x) + \frac{r(r-1)}{2!}m^2K_{r-2}(x) + \ .\ .\ .\ + m^r\right\}.$$

Once again the reciprocal relations (2.2) and (2.8) may be compared with the expansions of the Hermite polynomials $H_r(x)$ in powers of x and with those of x^r in terms of Hermite polynomials, which exhibit a similar reciprocity; symbolically, these are

$$H_r(x) = e^{-\frac{1}{2}D^2}x^r, \quad \text{and} \quad x^r = e^{\frac{1}{2}D^2}H_r(x). \quad . \quad . \quad . \quad (2.9)$$

By Sturmian methods it may be proved from the recurrence relation (2.6) that $K_r(x)$ has r real roots, all positive, and that the roots of $K_r(x)$ and $K_{r-1}(x)$ alternate. Now by (2.4) we see that, wherever $K_{r-1}(x)$ is zero, we must have

$$K_r(x+1) - K_r(x) = 0.$$

But this, taken in conjunction with the fact that the roots of $K_r(x)$ and $K_{r-1}(x)$ alternate, means that it must be possible in the segment of the curve $y = K_r(x)$ between any two consecutive roots of $K_r(x)$ to place a chord parallel to the x-axis and of length unity. Hence the range of such a segment must exceed unity; that is, consecutive roots of $K_r(x)$ must differ by more than unity, for every value of r. (The argument is an application to the calculus of finite differences of a principle analogous to Rolle's Theorem in the differential calculus.)

The above facts are in evidence in the following short table of the roots of $K_r(x)$ for $m=1$; $r=1, 2, 3, 4, 5$: and for $m=0·5, 2, 5$; $r=1, 2, 3$.

r.	$m=1$.
1	1·000.
2	0·382, 2·618.
3	0·139, 1·746, 4·115.
4	0·044, 1·332, 3·080, 5·544.
5	0·0114, 1·131, 2·541, 4·388, 6·928.

r.	$m=0\cdot5$.	$m=2$.	$m=5$.
1	0·500.	2·000.	5·000.
2	0·104, 2·396.	1·000, 4·000.	3·209, 7·791.
3	0·0304, 1·297, 3·173.	0·5107, 2·711, 5·778.	2·166, 5·686, 10·148.

3. Orthogonal Properties of the Polynomials.

The orthogonal properties of the polynomials $K_r(x)$ again resemble those of the Hermite polynomials, summations replacing integrations. We have first that

$$\sum_{x=0}^{\infty}\psi(x)=1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.1)$$

Also, by (2.1), for $r>0$,

$$\begin{aligned}\sum_0^{\infty}K_r(x)\psi(x)&=\sum_0^{\infty}\left\{\psi(x)-r\psi(x-1)+\binom{r}{2}\psi(x-2)-\ldots+(-)^r\psi(x-r)\right\}\\&=(1-1)^r\\&=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.2)\end{aligned}$$

So, using repeated summation by parts, in the form

$$\sum_a^b U(x)\nabla V(x)=U(b)V(b)-U(a)V(a-1)-\sum_a^{b-1}V(x)\nabla U(x+1), \quad . \quad (3.3)$$

suited to the receding difference-operator ∇, we have, since the integrated part vanishes always at the limits,

$$\begin{aligned}\sum_0^{\infty}K_r(x)\,.\,\nabla^s\psi(x)&=-\sum_0^{\infty}\nabla^{s-1}\psi(x)\,.\,\nabla K_r(x+1)\\&=-\frac{r}{m}\sum_0^{\infty}K_{r-1}(x)\,.\,\nabla^{s-1}\psi(x)\\&=\frac{r(r-1)}{m^2}\sum_0^{\infty}K_{r-2}(x)\,.\,\nabla^{s-2}\psi(x)\\&=\ldots\ldots\ldots\ldots\ldots\\&=(-)^r\frac{r!}{m^r}\sum_0^{\infty}K_0(x)\,.\,\nabla^{s-r}\psi(x).\end{aligned}$$

Hence, by (3.1) and (3.2),

$$\sum_0^{\infty}K_r(x)\,.\,K_s(x)\,.\,\psi(x)=\begin{matrix}0, & r\neq s,\\ (-)^r r!/m^r, & r=s.\end{matrix} \quad . \quad . \quad . \quad (3.4)$$

These orthogonal properties can be used to expand an arbitrary function $\phi(x)$ as a series of the polynomials, necessary conditions being that

$$\sum_0^{\infty}\phi(x)\,.\,\psi(x) \quad \text{and} \quad \sum_0^{\infty}K_r(x)\,.\,\phi(x)\,.\,\psi(x) \quad . \quad . \quad (3.5)$$

must converge. For if such an expansion

$$\phi(x) = k_0 + k_1 K_1(x) + k_2 K_2(x) + \ldots$$

be possible, then we must have

$$\sum_0^\infty \phi(x) \,.\, K_r(x) \,.\, \psi(x) = (-)^r k_r r!/m^r, \qquad . \quad . \quad . \quad (3.6)$$

whence the coefficient k_r is determined. Sufficient conditions for the existence of such an expansion are obtained in the paper of Pollaczek-Geiringer cited at the end.

4. Application to Frequency of Type B.

The frequency function of Type B may be written in symbolic form,

$$f(x) = \exp\,(b_2\nabla^2 + b_3\nabla^3 + \ldots)\psi(x), \qquad . \quad . \quad . \quad (4.1)$$

where $\exp\left(\sum_1^\infty b_r t^r\right)$ is the generating function of the *factorial* moments of the distribution, and b_r is of the order N^{1-r}.

In order to prove the result of (1.3) of § 1 it will be necessary to demonstrate that the coefficient of $K_r(x)$ in the expansion in polynomials $K_r(x)$ of

$$\log\,\{1 + \beta_2 K_2(x) + \beta_3 K_3(x) + \ldots\}, \qquad . \quad . \quad . \quad . \quad (4.2)$$

where

$$\exp\,(b_2 t^2 + b_3 t^3 + \ldots) = 1 + \beta_2 t^2 + \beta_3 t^3 + \ldots, \qquad . \quad . \quad (4.3)$$

is of order N^{1-r}.

Now the term in $b_\lambda b_\mu b_\nu$, for example, in the expansion of (4.2), is readily found to be

$$b_\lambda b_\mu b_\nu \{K_{\lambda+\mu+\nu} - 1\,!\,(K_{\lambda+\mu}K_\gamma + K_{\lambda+\nu}K_\mu + K_{\mu+\nu}K_\lambda) + 2\,!\,K_\lambda K_\mu K_\nu\}, \quad (4.4)$$

or the same divided by a combinatory number involving factorials, if there happen to be equalities among λ, μ, ν.

The coefficient $b_\lambda b_\mu b_\nu$ which appears in this term is itself of the order $N^{3-\lambda-\mu-\nu}$, *i.e.*, of the order of $b_{\lambda+\mu+\nu-2}$.

We shall prove that the co-factor of $b_\lambda b_\mu b_\nu$, namely, the isobaric function of K's in the bracket of (4.4) above, reduces to a *linear* function of K's involving a term of highest order $K_{\lambda+\mu+\nu-2}$. More generally, for any number of suffixes $\lambda_1, \lambda_2, \lambda_3, \ldots \lambda_n$, we shall prove that the function V defined by

$$\begin{aligned} V_n(\lambda_1, \lambda_2, \ldots, \lambda_n\,;\,x) \equiv K_{\lambda_1+\lambda_2+\ldots+\lambda_n} &- 1\,!\,(K_{\lambda_1}K_{\lambda_2+\ldots+\lambda_n} + \ldots) \\ &+ 2\,!\,(K_{\lambda_1}K_{\lambda_2}K_{\lambda_3+\ldots+\lambda_n} + \ldots) - \ldots, \end{aligned} \qquad (4.5)$$

where the suffixes in the bracket preceded by $(-)^s s!$ comprise all partitions of the partible integer $\lambda_1 + \lambda_2 + \ldots + \lambda_n$ into $s+1$ parts, reduces to

a linear function of K's, of highest order $\lambda_1+\lambda_2+\ldots+\lambda_n-n+1$. Then on re-arrangement of terms each K_r will have a coefficient k_r of the desired order N^{1-r}.

5. Recurrence Relation for V_n.

It does not affect the problem, as now formulated, if we introduce initial terms b_1t and β_1t in the respective expansions (4.2) and (4.3).

The polynomial V_n, being of degree $\lambda_1+\lambda_2+\ldots+\lambda_n$ at most, must be expressible as

$$V_n=v_0K_{\lambda_1+\lambda_2+\ldots+\lambda_n}+v_1K_{\lambda_1+\lambda_2+\ldots+\lambda_n-1}+\ldots+v_{\lambda_1+\lambda_2+\ldots+\lambda_n}. \quad (5.1)$$

It will be proved that

(I_n) $\quad v_i=0, \quad$ for $i=0, 1, 2, \ldots, n-2$;

$$v_{n-1}=(-)^{n-1}\frac{\lambda_1\lambda_2\ldots\lambda_n}{m^{n-1}}\cdot\frac{(\lambda_1+\lambda_2+\ldots+\lambda_n-n)!}{(\lambda_1+\lambda_2+\ldots+\lambda_n-2n+2)!}.$$

In the first place, $V_n=0$ if any λ is zero. This is a combinatory result, and has been proved already in the paper on the problem for Type A, p. 37. In the second place we regard V_n as zero when any n takes a negative value.

The following recurrence formula for the V's, an extreme generalization of the formula (2.6), is fundamental:

(A)

$$V_n(\lambda_1+1, \lambda_2, \ldots, \lambda_n)-V_1V'_n(\lambda_1, \lambda_2, \ldots, \lambda_n)+\frac{\lambda_1}{m}V'_n(\lambda_1-1, \lambda_2, \ldots, \lambda_n)$$

$$=-\frac{1}{m}\{\lambda_2V'_{n-1}(\lambda_1+\lambda_2-1, \lambda_3, \ldots, \lambda_n)+\ldots+\lambda_nV'_{n-1}(\lambda_2, \lambda_3, \ldots, \lambda_1+\lambda_n-1)\},$$

where V' denotes that in the corresponding V all K's with suffixes involving λ_1 have the argument $x-1$ instead of x.

To prove (A) we group together, as in the former paper, such terms on the left of (A) as have the λ_1+1, λ_1, or λ_1-1 respectively *in the same part of similar partitions.* The left side of (A) then falls into groups of terms like

$$K_{\lambda_1+1+\mu_1}K_{\mu_2}K_{\mu_3}\ldots K_{\mu_s}-K_1K'_{\lambda_1+\mu_1}K_{\mu_2}\ldots K_{\mu_s}+\frac{\lambda_1}{m}K'_{\lambda_1-1+\mu_1}K_{\mu_2}\ldots K_{\mu_s} \quad (5.2)$$

where $\mu_1, \mu_2, \ldots, \mu_s$ is a partition of $\lambda_2+\lambda_3+\ldots+\lambda_n$, any of the places μ being possibly unoccupied. We thus obtain $K_{\mu_2}K_{\mu_3}\ldots K_{\mu_s}$ multiplied by

$$K_{\lambda_1+1+\mu_1}-K_1K'_{\lambda_1+\mu_1}+\frac{\lambda_1}{m}K'_{\lambda_1-1+\mu_1},$$

that is, in view of (2.6), by

$$-\frac{\mu_1}{m}K'_{\lambda_1-1+\mu_1}. \quad (5.3)$$

Thus the terms in which the place μ_1 is unoccupied vanish identically. In the remaining terms μ_1 contains parts of $\lambda_2+\lambda_3+\ldots+\lambda_n$. We pick out the coefficient of λ_2 by remarking that, by (5.2), λ_2 and λ_1-1 must always occur in the same part of a partition. Hence the terms of the coefficient of λ_2 comprise all partitions of the $n-1$ elements

$$\lambda_1-1+\lambda_2,\ \lambda_3,\ \ldots,\ \lambda_n,$$

and each term has its proper sign and factorial multiplier. But the aggregate of these constitutes $-\frac{1}{m}V'_{n-1}(\lambda_1-1+\lambda_2,\ \lambda_3,\ \ldots\ \lambda_n)$; and by symmetry the relation (A) follows.

6. Induction from the Recurrence Relation.

We next prove inductively that if the identity I_{n-1} holds for all $V'_{n-1}(\mu_2, \mu_3, \ldots, \mu_n)$, then I_n holds for $V_n(1, \lambda_2, \lambda_3, \ldots, \lambda_n)$.

The reason is that, on the given assumption, if $\lambda_1=0$ in (A), on the left there is only $V_n(1, \lambda_2, \lambda_3, \ldots, \lambda_n)$, and all the terms on the right are of degree $\lambda_2+\lambda_3+\ldots+\lambda_n-n+2$. This is the correct degree. Also the coefficient of $K_{\lambda_2+\lambda_3+\ldots+\lambda_n-n+2}$ in V_n is

$$(-)^{n-1}m^{1-n}\lambda_2\lambda_3\ldots\lambda_n\frac{(\lambda_2+\lambda_3+\ldots+\lambda_n-n)!}{(\lambda_2+\lambda_3+\ldots+\lambda_n-2n+3)!}(\lambda_2+\lambda_3+\ldots+\lambda_n-n+1)$$
$$=(-)^{n-1}m^{1-n}1\,.\,\lambda_2\lambda_3\ldots\lambda_n\frac{(1+\lambda_2+\lambda_3+\ldots+\lambda_n-n)!}{(1+\lambda_2+\lambda_3+\ldots+\lambda_n-2n+2)!}, \quad (6.1)$$

which is as it should be.

Finally, we shall prove that if I_{n-1} is valid for all V'_{n-1}, and if I_n is valid for all V'_n up to $V'_n(\lambda_1, \lambda_2, \ldots, \lambda_n)$, then I_n is valid also for $V_n(\lambda_1+1, \lambda_2, \ldots, \lambda_n)$.

For we see by reference to (A) that the degree is correct, while the coefficient of $K_{\lambda_1+\lambda_2+\ldots+\lambda_n-n+2}$ is found to be

$$(-)^{n-1}m^{1-n}\Big[\lambda_1\lambda_2\ldots\lambda_n\frac{(\lambda_1+\lambda_2+\ldots+\lambda_n-n)!}{(\lambda_1+\lambda_2+\ldots+\lambda_n-2n+2)!}$$
$$+\lambda_2\lambda_3\ldots\lambda_n\frac{(\lambda_1+\ldots+\lambda_n-n)!}{(\lambda_1+\ldots+\lambda_n-2n+3)!}\{(n-1)\lambda_1+\lambda_2+\ldots+\lambda_n-n+1\}\Big]$$

which reduces simply to

$$(-)^{n-1}m^{1-n}(1+\lambda_1)\lambda_2\ldots\lambda_n\frac{(1+\lambda_1+\lambda_2+\ldots+\lambda_n-n)!}{(1+\lambda_1+\lambda_2+\ldots+\lambda_n-2n+2)!}, \quad (6.2)$$

the desired value.

This completes the induction, for $I_1(\lambda)$ is the manifest identity $V_1(\lambda)=K_\lambda$, while $I_2(1, \lambda)$ is simply the recurrence relation (2.6) in the form

$$K_{\lambda+1}-K_1K'_\lambda=-\frac{\lambda}{m}K'_{\lambda-1}.$$

It is remarkable that in spite of several points of difference in the proof, the coefficient v_{n-1} differs from the corresponding coefficient in the case of the Hermite polynomials in Type A only in sign and in the factor m^{1-n}.

In re-arranging the terms in the logarithmic expansion (4.2) we have assumed the absolute convergence of the series for $\log f(x)$ in these polynomials of Type B. As earlier remarked, the validity of this depends on the sufficient conditions for convergence, and these have been given by H. Pollaczek-Geiringer.

7. The Fitting of Data by these Polynomials.

We shall denote the relative frequencies as given by observation or experiment by f_x, the corresponding theoretical frequencies by $f(x)$. To graduate given frequencies f_x by these orthogonal polynomials, we first evaluate the mean m, then the function $\psi(x; m)$ and its receding differences $\nabla^r\psi$. The function to be fitted is then $\log f(x) - \log \psi(x)$, let us say $F(x)$, from data $\log f_x - \log \psi(x)$, say F_x. The computed differences of $\psi(x)$ serve a double purpose: they are used in successive *columns* to evaluate the coefficient k_r by means of

$$k_r = (m^r/r!)\sum_0^{\infty} F_x \nabla^r \psi(x), \quad . \quad . \quad . \quad . \quad . \quad (7.1)$$

and then in successive *rows* to evaluate

$$F(x)\psi(x) = k_0\psi(x) + k_1\nabla\psi(x) + k_2\nabla^2\psi(x) + \ldots, \quad . \quad . \quad (7.2)$$

whence $F(x)$ is determined, and so $f(x)$.

Theoretically k_0 and k_1 should be zero: actually, because of fortuitous errors in the data f_x, they always prove to have small but appreciable values. Also, since observational frequencies after the first few are zero, it is advisable in all cases to *cumulate* the residual differences at the last non-zero frequency retained, in order to secure the theoretical agreement of moments.

We may illustrate the scheme of calculation by graduating a short series of data, a well-known example of von Bortkiewicz. In several other examples we found, however, that short series gave trouble, doubtless for the reason that the small relative frequencies in the extreme classes, though the least important and most subject to error, receive undue weight when logarithms are taken.

x	0	1	2	3	4	5+	Sum
f_x	144	91	32	11	2	0	280.

The mean $m = 0{\cdot}700$.

x	$\psi(x)$.	$\nabla\psi$.	$\nabla^2\psi$.	$\nabla^3\psi$.	F_x.
0	·4966	4966	4966	4966	·0349
1	·3476	−1490	−6456	−11422	−·0672
2	·1217	−2259	−0769	5687	−·0628
3	·0284	−0933	1326	2095	·3245
4	·0057	−0284	0933	−1326	·2231
	$\Sigma F\psi$.	$\Sigma F\nabla\psi$.	$\Sigma F\nabla^2\psi$.	$\Sigma F\nabla^3\psi$.	
	−·0032	·0049	·1294	·0968	
	k_0.	k_1.	k_2.	k_3.	
	−·0032	·0034	·0317	·0055	

x.	0	1	2	3	4	5
$F(x)$	0374	−0·816	−·0038	·1742	·3708	—
$f(x)$	144	90	34	9	2	1
f_x	144	91	32	11	2	0

The agreement is good, indeed too good, but the process involves many multiplications and divisions, and although these are easily performed by machine, the method of fitting by factorial moments is much more rapid, and the difficulty of the small frequencies is really serious. We consider therefore that while both in Type A and Type B the expansion of the logarithm of frequency by the appropriate orthogonal polynomials ensures regular decrease of terms and avoids the occurrence of negative frequencies, the logarithmic function is not the ideal function to employ in this connection.

REFERENCES.

Aitken, A. C., and A. Oppenheim, *Proc. Roy. Soc. Edin.*, 51 (1930), 35–41.

Bortkiewicz, L. von, *Das Gesetz der Kleinen Zahlen* (Leipzig, 1898), 24.

Charlier, C. V. L., *Meddel. Lunds Astr. Observ.*, (2), 51 (1928), "A New Form of the Frequency Function."

Jordan, Ch., *Bull. Soc. Math.*, 14 (1926), 101–137.

Pollaczek-Geiringer, Hilda, *Zeits. f. angew. Math. u. Mech.*, 8 (1928), 292–309.

(*Issued separately February* 17, 1932.)

VI.—On the Definition of Spatial Distance in General Relativity.

By H. S. Ruse, B.A.(Oxon.).

(M.S. received October 15, 1931. Read November 2, 1931.)

In a recent paper,* Professor E. T. Whittaker discussed the problem of defining, in a general riemannian space-time, the concept of spatial distance between material particles. It is the object of this paper to give an alternative definition, and to compare the new formula with that of Whittaker.

§ 1. Whittaker's Definition of Spatial Distance.

It is pointed out in the above-mentioned paper that if an observer O in a general riemannian space-time makes an assertion regarding the distance from himself of a particle S, he is really stating a relation between the world-point of S at the instant when light left it, and his own world-point at the instant when the light arrives. In other words, he is giving a relation between two world-points which lie on the same null geodesic. It is indeed obvious that, if O is to measure the "distance" between himself and a distant point S, he must be able to *see* S; that is, O and S must lie on the world-line of a light-pulse, which is a null geodesic.

Now astronomers sometimes determine the distance of a star from the earth by finding the square root of the ratio of its absolute brightness to its apparent brightness. The geometrical statement of this fact constitutes Whittaker's definition of spatial distance, which may be expressed as follows:—

The spatial distance between a star S and an observer O (on the same null geodesic) is proportional to the square root of the two-dimensional cross-section made by a thin pencil of null geodesics, with vertex S and passing near O, on the instantaneous three-dimensional space of the observer.†

The definition is further particularised by requiring that when O and S are near one another the "spatial distance" shall reduce to the element of length at O of the instantaneous three-dimensional space.

* *Proc. Roy. Soc.*, A, **133** (1931), 93.

† The instantaneous three-dimensional space of the observer consists of those world-points in his immediate neighbourhood which he regards as simultaneous. Geometrically, it is a small portion near O of the hypersurface formed by the geodesics through O which are perpendicular at this point to the observer's world-line.

It is shown that an application of this general definition to the space-time of constant curvature $-1/R^2$ specified by

$$(1.1) \quad \frac{ds^2}{R^2}=\frac{du^2-dx^2-dy^2-dz^2}{1+x^2+y^2+z^2-u^2}+\frac{(udu-xdx-ydy-zdz)^2}{(1+x^2+y^2+z^2-u^2)^2}$$

leads to the following conclusion:

The spatial distance between a star whose world-coordinates are (u, x, y, z) and an observer whose world-line is the geodesic $x=\bar{x}$, $y=\bar{y}$, $z=\bar{z}$, where $\bar{x}$, $\bar{y}$, $\bar{z}$ are constants, is

$$(1.2) \quad \Delta=\frac{R\sin\rho}{\cos(\sigma+\rho)},$$

where

$$\rho=\text{arc}\cos\left\{\frac{1+x\bar{x}+y\bar{y}+z\bar{z}}{(1+x^2+y^2+z^2)^{\frac{1}{2}}(1+\bar{x}^2+\bar{y}^2+\bar{z}^2)^{\frac{1}{2}}}\right\}, \qquad (0\leqslant\rho\leqslant\pi)$$

and

$$\sigma=\text{arc}\sin\left\{\frac{u}{(1+x^2+y^2+z^2)^{\frac{1}{2}}}\right\}, \qquad \left(-\frac{\pi}{2}\leqslant\sigma<\frac{\pi}{2}\right)$$

the radicals being taken positively. Substitution of these values of ρ and σ in (1.2) gives

$$(1.3) \quad \Delta=\frac{R(1+\Sigma x^2)^{\frac{1}{2}}\{\Sigma(x-\bar{x})^2+\Sigma(y\bar{z}-\bar{y}z)^2\}^{\frac{1}{2}}}{(1+\Sigma x\bar{x})(1-u^2+\Sigma x^2)^{\frac{1}{2}}-u\{\Sigma(x-\bar{x})^2+\Sigma(y\bar{z}-\bar{y}z)^2\}^{\frac{1}{2}}},$$

the summations in each case being for x, y, z; so, for example, Σx^2 denotes $x^2+y^2+z^2$.

§ 2. Spatial Distance in Galilean Space-time.

In restricted relativity, if an observer O is at the point $(\bar{x}, \bar{y}, \bar{z})$ with respect to a galilean frame of reference relative to which he is at rest, then his distance from a star at the point (x, y, z) is

$$(2.1) \quad \Delta=\{(x-\bar{x})^2+(y-\bar{y})^2+(z-\bar{z})^2\}^{\frac{1}{2}}.$$

Suppose that his measurement of the distance is made at time $\bar{t}$, and that the light from the star which reaches him at this instant left it at time t. Then, as is well known,

$$(2.2) \quad \bar{t}=t+\{(x-\bar{x})^2+(y-\bar{y})^2+(z-\bar{z})^2\}^{\frac{1}{2}},$$

the velocity of light being unity.

In geometrical language the above statements may be expressed as follows. If

$$(2.3) \quad ds^2=dt^2-dx^2-dy^2-dz^2$$

defines the metric of space-time, and if the world-line of the observer O is the geodesic $x=\bar{x}$, $y=\bar{y}$, $z=\bar{z}$, where $\bar{x}$, $\bar{y}$, $\bar{z}$ are constants, then the spatial distance between a star S at the world-point (t, x, y, z) and the observer is

given by (2.1). If at the instant of making the observation the world-coordinates of the observer are $(\bar{t}, \bar{x}, \bar{y}, \bar{z})$, then the relation between them and the world-coordinates of the star is given by equating to zero the geodesic distance (interval) s between $(\bar{t}, \bar{x}, \bar{y}, \bar{z})$ and (t, x, y, z), thus expressing the fact that the two points lie on the same null geodesic. But

(2.4) . . . $$s^2 = (t-\bar{t})^2 - (x-\bar{x})^2 - (y-\bar{y})^2 - (z-\bar{z})^2,$$

so $s=0$ gives

(2.5) . . . $$t-\bar{t} = \pm\{(x-\bar{x})^2 + (y-\bar{y})^2 + (z-\bar{z})^2\}^{\frac{1}{2}}.$$

The negative sign must be chosen since we are dealing with rays of light *from* the star *to* the observer (so that $\bar{t}>t$), and not vice versa.

Any definition of spatial distance in a general riemannian space-time must reduce to (2.1) when applied to the above particular case, which will be referred to as the "fundamental galilean case."

It may be remarked that the corresponding formulæ for an observer who is moving relative to the given system of reference may be obtained from the above by applying an appropriate Lorentz transformation.

§ 3. A Definition of Spatial Distance.

Let

(3.1) . . . . . $$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu$$

define the metric of space-time in terms of a coordinate system (x^0, x^1, x^2, x^3) selected by an observer O.

It will be assumed that the quadratic form on the right-hand side of (3.1) is indefinite and of signature -2; that is, that it can be reduced by a real point-transformation $x^\mu = x^\mu(\xi^\nu)$ to the form

$$(\lambda_0 d\xi^0)^2 - (\lambda_1 d\xi^1)^2 - (\lambda_2 d\xi^2)^2 - (\lambda_3 d\xi^3)^2,$$

where the λ's are real functions of the ξ's. It will be seen that this assumption involves no real loss of generality. It will be convenient to use the symbol e_a defined by

(3.2) . . . . $$\begin{cases} e_a = 1 & (a=0) \\ e_a = -1 & (a=1, 2, 3). \end{cases}$$

Let the world-coordinates of the star S be $(x^\mu) \equiv (x^0, x^1, x^2, x^3)$, and those of the observer be $(\bar{x}^\mu) \equiv (\bar{x}^0, \bar{x}^1, \bar{x}^2, \bar{x}^3)$. Then, if Ω denote one-half of the square of the geodesic distance between (x^μ) and $(\bar{x}^\mu)$, the fact that these two points lie on a null geodesic may be expressed by the equation

(3.3) . . . . $$\Omega = 0.$$

Let τ be the proper-time of the observer at the point $(\bar{x}^\mu)$. That is, let τ be the length of the arc of his world-line (a geodesic) measured from

some given fixed point on it to the point $(\bar{x}^\mu)$. Then each of the $\bar{x}$'s is a function of τ, say

(3.4) . . . . . $$x^\mu = x^\mu(\tau), \qquad (\mu = 0, 1, 2, 3)$$

while by (3.1),

(3.5) . . . . . . $$d\tau^2 = \bar{g}_{\mu\nu}\, d\bar{x}^\mu\, d\bar{x}^\nu,$$

where $\bar{g}_{\mu\nu}$ is the value of $g_{\mu\nu}$ at $(\bar{x}^\mu)$. Moreover, since in a space of four dimensions ∞^3 geodesics pass through any point, equations (3.4) will involve three given constants which may be eliminated, so that τ is expressible as a function of $\bar{x}^0, \bar{x}^1, \bar{x}^2, \bar{x}^3$.

Let

(3.6) . . . . . $$\bar{k}^\mu = \frac{d\bar{x}^\mu}{d\tau} \qquad (\mu = 0, 1, 2, 3)$$

be the contravariant components of the unit vector in the direction of the tangent at $(\bar{x}^\mu)$ to the world-line of O. That $\bar{k}^\mu$ is a unit vector follows from (3.5), which gives

(3.7) . . . . . . $$\bar{g}_{\mu\nu}\, \bar{k}^\mu\, \bar{k}^\nu = 1.$$

It must be observed that the $\bar{k}^\mu$ are each functions of τ, and therefore of $\bar{x}^0, \bar{x}^1, \bar{x}^2, \bar{x}^3$.

It will now be shown that the observer can set up a system of reference with his world-point $(\bar{x}^\mu)$ as origin such that

(i) the "coordinate-axes" are geodesics which are mutually perpendicular at $(\bar{x}^\mu)$, one of these axes being his own world-line;

(ii) the corresponding coordinate-system * $(\eta^a) \equiv (\eta^0, \eta^1, \eta^2, \eta^3)$ is such that the four-dimensional distance (interval) from the origin to the point (η^a) is $[(\eta^0)^2 - (\eta^1)^2 - (\eta^2)^2 - (\eta^3)^2]^{\frac{1}{2}}$;

(iii) in terms of the new coordinates the equations of the geodesics (and in particular of the null geodesics) through the observer's world-point are linear.

This system of coordinates is the nearest approximation to a galilean system that can be obtained in his curved space; in fact, in the case when space-time is flat, $(\eta^0, \eta^1, \eta^2, \eta^3)$ are nothing other than the galilean coordinate-system (t, x, y, z), with respect to which the observer's world-line is $x=0$, $y=0$, $z=0$. Now in the galilean case the four-dimensional distance of (t, x, y, z) from the origin $(0, 0, 0, 0)$ is $(t^2 - x^2 - y^2 - z^2)^{\frac{1}{2}}$, and the spatial distance is $(x^2+y^2+z^2)^{\frac{1}{2}}$. It will therefore be argued that a reasonable definition of spatial distance in the more general case is

* The η's are in fact a particular set of Riemann normal coordinates. Systems of this type have recently been employed by T. Y. Thomas, *Proc. Nat. Acad. Sci.*, **16** (1930), 761.

$\{(\eta^1)^2+(\eta^2)^2+(\eta^3)^2\}^{\frac{1}{2}}$, and it is shown that, in terms of the original coordinates (x^μ), this formula can be expressed in a very simple form.

Moreover, since the null geodesic joining the world-point of the observer to that of the star has linear equations in the new coordinate-system, it is a straight line relative to this system. The formula for spatial distance determined by this method will therefore be one which allows the observer to assume that a light-signal received from the star travels in a straight line.

But this is precisely the principal assumption made when astronomers determine the distance from the earth of the nearer celestial objects by parallax-measurements,* a fact which suggests, though does not prove, that the new definition may provide a general formula for spatial distance as determined by the measurement of parallaxes.

Suppose then that the observer selects at his world-point $(\bar{x}^\mu)$ a set of four mutually orthogonal real directions, the unit contravariant vectors in these directions being ${}_0\bar{h}^\mu$, ${}_1\bar{h}^\mu$, ${}_2\bar{h}^\mu$, ${}_3\bar{h}^\mu$, the lower index specifying the vector, the upper being an ordinary tensor-suffix. The superposed bars indicate that each of the sixteen h's is a function of the $\bar{x}^\mu$. These vectors will be denoted by the single symbol ${}_a\bar{h}^\mu$, where the lower (Latin) index specifies the vector and the upper (Greek) index the component of the vector. Repeated Greek suffixes will indicate, as above, summations from 0 to 3, but repeated Latin indices will *not* indicate summations unless preceded by the symbol Σ.

Suppose that the observer selects these directions so that

$$(3.8) \qquad {}_0\bar{h}^\mu = k^\mu,$$

while ${}_1h^\mu$, ${}_2h^\mu$, ${}_3h^\mu$ have any values consistent with the orthogonality condition. Let

$$(3.9) \qquad {}_a\bar{h}_\mu = \bar{g}_{\mu\nu}\, {}_ah^\nu.$$

That ${}_a\bar{h}^\mu$ are unit mutually orthogonal vectors may be expressed in the form

$$(3.10) \qquad {}_a\bar{h}_\mu\, {}_b\bar{h}^\mu = e_a\, \delta_{ab} \qquad (a, b = 0, 1, 2, 3)$$

where e_a is defined by (3.2) and $\delta_{ab}=0$ or 1 according as $a \neq b$ or $a=b$. Moreover, it is easily shown † that

$$(3.11) \qquad \sum_{a=0}^{3} e_a\, {}_a\bar{h}_\mu\, {}_a\bar{h}_\nu = \bar{g}_{\mu\nu}.$$

Now let $(y^\mu)\equiv(y^0, y^1, y^2, y^3)$ be the Riemann-Veblen normal coordin-

* Eddington, *Mathematical Theory of Relativity* (1924), 163.

† See, for example, Eisenhart, *Riemannian Geometry* (1926), ch. iii, (29.3).

ates * having $(\bar{x}^\mu)$ as origin. These are defined as functions of the original coordinates (x^μ) by the equation †

(3.12) . . . . $y^\mu = -\bar{g}^{\mu\alpha}\dfrac{\partial\Omega}{\partial\bar{x}^\alpha}$, $(\mu = 0, 1, 2, 3)$

where Ω, as above, represents one-half of the square of the geodesic distance between $(\bar{x}^\mu)$ and (x^μ). For the moment Ω is not zero, since the x^μ are being employed as current coordinates instead of as the world-coordinates of the star.

In the fundamental galilean case,

$$\Omega = \tfrac{1}{2}\{(t-\bar{t})^2 - (x-\bar{x})^2 - (y-\bar{y})^2 - (z-\bar{z})^2\},$$
$$\bar{g}^{00} = 1,\ \bar{g}^{11} = \bar{g}^{22} = \bar{g}^{33} = -1,\ \ \bar{g}^{\mu\nu} = 0 \text{ if } \mu \neq \nu, \quad \text{so}$$
$$y^0 = -\partial\Omega/\partial\bar{t} = t - \bar{t}$$

and similarly $y^1 = x - \bar{x}$, $y^2 = y - \bar{y}$, $y^3 = z - \bar{z}$. Hence in this case the transference to normal coordinates reduces to a mere change of origin without rotation.

Expressed in terms of the new variables, the equations of any geodesic through $(\bar{x}^\mu)$ are of the form

(3.13) . . . . $y^\mu = a^\mu s$, $(\mu = 0, 1, 2, 3)$

where s is the length of its arc from the origin $(y^\mu) = 0$ to the point (y^μ), and the constants a^μ determine the direction of the geodesic. In fact

(3.14) . . . . . . $a^\mu = \dfrac{dy^\mu}{ds} = \left(\dfrac{dx^\mu}{ds}\right)_0$

where $\left(\dfrac{dx^\mu}{ds}\right)_0$ is the value at $(\bar{x}^\mu)$ of $\dfrac{dx^\mu}{ds}$ for the geodesic in question. In particular, the observer's world-line has the equations

(3.15) . . . . $y^\mu = k^\mu s$. $(\mu = 0, 1, 2, 3)$

It should be noticed that, in terms of the new coordinates, the equations (3.13) of the geodesics through $(\bar{x}^\mu)$ are linear.

By (3.14),

$$\bar{g}_{\mu\nu}\, a^\mu\, a^\nu = \bar{g}_{\mu\nu}\left(\frac{dx^\mu}{ds}\right)_0\left(\frac{dx^\nu}{ds}\right)_0$$
$$= 1 \qquad\qquad \text{by (3.1).}$$

So, multiplying by s^2 and using (3.13), we deduce that the square of the four-dimensional distance from the observer of a star whose normal world-coordinates are (y^μ) is given by

(3.16) . . . . $s^2 = \bar{g}_{\mu\nu}\, y^\mu\, y^\nu$.

---

* Veblen, *Invariants of Quadratic Differential Forms* (Camb. Math. Tract No. 24, 1927), ch. vi.

† Ruse, *Proc. London Math. Soc.*, **32** (1931), 90.

Hence

$$\delta^2 = -\sum_{a=0}^{3} e_a (\eta^a)^2 + (\eta^0)^2$$
$$= -s^2 + (\eta^0)^2 \quad \text{by (3.19).}$$

We can now use the fact that the star and the observer lie on the same null geodesic, so that $s=0$. Hence $\delta^2=(\eta^0)^2$ and therefore

(3.21) . . . . $\delta = -\eta^0,$

the negative sign being chosen since in the fundamental galilean case $\eta^0 = t - \bar{t}$ and $t < \bar{t}$. So

$$\begin{aligned} \delta &= -{}_0\bar{h}_\mu\, y^\mu && \text{by (3.17)} \\ &= -\bar{g}_{\mu\nu}\, \bar{k}^\nu\, y^\mu && \text{by (3.9) and (3.8)} \\ &= \bar{g}_{\mu\nu}\, \bar{g}^{\mu\alpha}\, \bar{k}^\nu \frac{\partial \Omega}{\partial \bar{x}^\alpha} && \text{by (3.12)} \\ &= \bar{k}^\nu \frac{\partial \Omega}{\partial \bar{x}^\nu} \end{aligned}$$

since $\bar{g}_{\mu\nu}\, \bar{g}^{\mu\alpha}$ is equal to the Kronecker symbol δ^α_ν.

Hence, by (3.6),

(3.22) . . . . $\delta = \dfrac{\partial \Omega}{\partial \bar{x}^\nu} \dfrac{d\bar{x}^\nu}{d\tau}.$

Since Ω is a function of the world-coordinates (x^μ) of the star and of the world-coordinates $(\bar{x}^\mu)$ of the observer, it is, by (3.4), a function of the x^μ and of the observer's proper-time τ. So, by (3.22),

$$\delta = \frac{\partial \Omega}{\partial \tau}.$$

We can therefore finally frame the following definition of spatial distance in a general riemannian space-time:

The spatial distance between a star at the world-point $(\mathbf{x}^\mu)$ *and an observer at the world-point* $(\bar{\mathbf{x}}^\mu)$*, these points being on the same null geodesic, is*

(3.23) . . . . $\delta = \dfrac{\partial \Omega}{\partial \tau},$

where (i) Ω *is one-half the square of the interval between* $(\mathbf{x}^\mu)$ *and* $(\bar{\mathbf{x}}^\mu)$;

(ii) τ *is the proper-time of the observer at* $(\bar{\mathbf{x}}^\mu)$;

(iii) *one of the variables may be eliminated, after performing the differentiation required by* (3.23), *by means of the equation* $\Omega = 0$.

It must be remembered that in solving the equation $\Omega=0$ for one of the variables, the sign of the radical which in general appears must be determined from the consideration that the light travels *from* and not *to* the star.

In the fundamental galilean case,

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2,$$

the star is at (t, x, y, z) and the observer at $(\bar{t}, \bar{x}, \bar{y}, \bar{z})$ where $\bar{x}, \bar{y}, \bar{z}$ are constants; so

$$\begin{aligned} d\tau^2 &= d\bar{t}^2 - d\bar{x}^2 - d\bar{y}^2 - d\bar{z}^2 \\ &= d\bar{t}^2 \end{aligned}$$

and we may therefore take $\tau = \bar{t}$. Then

$$\begin{aligned} 2\Omega &= (t-\bar{t})^2 - (x-\bar{x})^2 - (y-\bar{y})^2 - (z-\bar{z})^2 \\ &= (t-\tau)^2 - (x-\bar{x})^2 - (y-\bar{y})^2 - (z-\bar{z})^2. \end{aligned}$$

Hence

$$\begin{aligned} \delta &= \partial\Omega/\partial\tau \\ &= -(t-\tau). \end{aligned}$$

The equation $\Omega = 0$ gives

$$t - \tau = \pm \{(x-\bar{x})^2 + (y-\bar{y})^2 + (z-\bar{z})^2\}^{\frac{1}{2}},$$

and we choose the negative sign since $\tau = \bar{t} > t$.

Hence

$$\delta = \{(x-\bar{x})^2 + (y-\bar{y})^2 + (z-\bar{z})^2\}^{\frac{1}{2}}.$$

§ 4. Comparison with Whittaker's Formula.

We now apply the formula of the last section to the de Sitter world, whose metric in Beltrami's coordinates is given by (1.1), namely

$$(4.1) \quad . \quad . \quad \frac{ds^2}{R^2} = \frac{du^2 - dx^2 - dy^2 - dz^2}{1 + x^2 + y^2 + z^2 - u^2} + \frac{(udu - xdx - ydy - zdz)^2}{(1 + x^2 + y^2 + z^2 - u^2)^2}.$$

Suppose the star to be at (u, x, y, z) and the observer at $(\bar{u}, \bar{x}, \bar{y}, \bar{z})$, where $\bar{x}, \bar{y}, \bar{z}$ are constants* and $\bar{u}$ is a function of the observer's proper-time τ. Then†

$$(4.2) \quad . \quad . \quad . \quad . \quad . \quad . \quad 2\Omega = R^2(\text{arg cosh } Q)^2$$

where

$$Q = \frac{1 + x\bar{x} + y\bar{y} + z\bar{z} - u\bar{u}}{(1 + x^2 + y^2 + z^2 - u^2)^{\frac{1}{2}}(1 + \bar{x}^2 + \bar{y}^2 + \bar{z}^2 - \bar{u}^2)^{\frac{1}{2}}}.$$

So

$$\begin{aligned} \delta &= \partial\Omega/\partial\tau \\ &= \frac{R^2 \text{ arg cosh } Q}{(Q^2-1)^{\frac{1}{2}}} \frac{\partial Q}{\partial \bar{u}} \frac{d\bar{u}}{d\tau}. \end{aligned}$$

The equation $\Omega = 0$ gives $Q = 1$. Since $\lim\limits_{Q \to 1} \dfrac{\text{arg cosh } Q}{(Q^2-1)^{\frac{1}{2}}} = 1$, we get

$$(4.3) \quad . \quad . \quad . \quad . \quad . \quad . \quad \delta = R^2 \frac{\partial Q}{\partial \bar{u}} \frac{d\bar{u}}{d\tau}.$$

* $x = \bar{x}$, $y = \bar{y}$, $z = \bar{z}$ are then the equations of a geodesic, since the equations of geodesics in this space are all of the form $x = au + b$, $y = a'u + b'$, $z = a''u + b''$, where the a's and b's are constants.

† Whittaker, *loc. cit.*, equation (4).

Now by (4.1),

$$\frac{d\tau^2}{R^2}=\frac{d\bar{u}^2}{1+\bar{x}^2+\bar{y}^2+\bar{z}^2-\bar{u}^2}+\frac{\bar{u}^2d\bar{u}^2}{(1+\bar{x}^2+\bar{y}^2+\bar{z}^2-\bar{u}^2)^2},$$

whence

$$\frac{d\bar{u}}{d\tau}=\frac{1}{R}\frac{1+\bar{x}^2+\bar{y}^2+\bar{z}^2-\bar{u}^2}{(1+\bar{x}^2+\bar{y}^2+\bar{z}^2)^{\frac{1}{2}}}.$$

Also

$$\frac{\partial Q}{\partial \bar{u}}=\frac{\bar{u}(1+x\bar{x}+y\bar{y}+z\bar{z})-u(1+\bar{x}^2+\bar{y}^2+\bar{z}^2)}{(1+x^2+y^2+z^2-u^2)^{\frac{1}{2}}(1+\bar{x}^2+\bar{y}^2+\bar{z}^2-\bar{u}^2)^{\frac{3}{2}}}.$$

Substituting in (4.3) we get

$$(4.4)\qquad \frac{\delta}{R}=\frac{\bar{u}(1+\Sigma x\bar{x})-u(1+\Sigma\bar{x}^2)}{(1+\Sigma\bar{x}^2)^{\frac{1}{2}}(1-u^2+\Sigma x^2)^{\frac{1}{2}}(1-\bar{u}^2+\Sigma\bar{x}^2)^{\frac{1}{2}}},$$

the summations being for x, y, z. From this $\bar{u}$ (say) may be eliminated by means of the equation $Q=1$, which gives

$$(4.5)\qquad \bar{u}=\frac{u(1+\Sigma x\bar{x})+(1-u^2+\Sigma x^2)^{\frac{1}{2}}\{\Sigma(x-\bar{x})^2+\Sigma(y\bar{z}-\bar{y}z)^2\}^{\frac{1}{2}}}{1+\Sigma x^2}.$$

Now compare this formula for δ with the formula for Whittaker's spatial distance Δ. Referring to (1.3), it is easily seen by the use of (4.5) that

$$(4.6)\qquad \frac{\Delta}{R}=-\frac{u(1+\Sigma x\bar{x})-\bar{u}(1+\Sigma x^2)}{(1+\Sigma x^2)^{\frac{1}{2}}(1-\bar{u}^2+\Sigma\bar{x}^2)^{\frac{1}{2}}(1-u^2+\Sigma x^2)^{\frac{1}{2}}}.$$

A comparison of (4.4) and (4.6) shows that in this special case the formula for Δ may be obtained from that of δ by interchanging the observer's and the star's world-coordinates, and altering the sign.

In terms of the quantities ρ and σ appearing in (1·2), equation (4.5) may be written

$$(4.7)\qquad \bar{u}=(1+\Sigma\bar{x}^2)^{\frac{1}{2}}\sin(\sigma+\rho),$$

and it is easily shown that

$$(4.8)\qquad \frac{\delta}{R}=\frac{\sin\rho}{\cos\sigma}.$$

This may be compared with (1.2), namely

$$\frac{\Delta}{R}=\frac{\sin\rho}{\cos(\sigma+\rho)}.$$

§ 5. Spatial Distance in the de Sitter World when the Metric is given in the Standard Form.

The more usual form of the metric of the de Sitter world is

$$(5.1)\qquad ds^2=\left(1-\frac{r^2}{R^2}\right)dt^2-\frac{dr^2}{1-r^2/R^2}-r^2d\theta^2-r^2\sin^2\theta d\phi^2.$$

This is derivable from the form (4.1) by putting *

$$(5.2) \quad \begin{cases} x = r(R^2 - r^2)^{-\frac{1}{2}} \sin\theta\cos\phi \operatorname{sech}(t/R) \\ y = r(R^2 - r^2)^{-\frac{1}{2}} \sin\theta\sin\phi \operatorname{sech}(t/R) \\ z = r(R^2 - r^2)^{-\frac{1}{2}} \cos\theta \operatorname{sech}(t/R) \\ u = \tanh(t/R). \end{cases}$$

If $(\bar{t}, \bar{r}, \bar{\theta}, \bar{\phi})$ are the world-coordinates of the star, then of course they are connected with the original coordinates $(\bar{u}, \bar{x}, \bar{y}, \bar{z})$ by relations corresponding to (5.2); in fact

$$(5.3) \quad \begin{cases} \bar{x} = \bar{r}(R^2 - \bar{r}^2)^{-\frac{1}{2}} \sin\bar{\theta}\cos\bar{\phi} \operatorname{sech}(\bar{t}/R) \\ \dots\dots\dots\dots\dots \end{cases}$$

Using this, it is easily shown by substitution in (4.2) that

$$(5.4) \quad 2\Omega = R^2(\arg\cosh Q)^2$$

where

$$Q = \left(1 - \frac{r^2}{R^2}\right)^{\frac{1}{2}}\left(1 - \frac{\bar{r}^2}{R^2}\right)^{\frac{1}{2}} \cosh\frac{t - \bar{t}}{R} + \frac{r\bar{r}}{R^2}\{\cos\theta\cos\bar{\theta} + \sin\theta\sin\bar{\theta}\cos(\phi - \bar{\phi})\},$$

the star being at the world-point (t, r, θ, ϕ).

Suppose that the observer is "at rest" at the spatial origin; that is, suppose that $\bar{r} = 0$, an assumption consistent with the requirement that the observer's world-line shall be a geodesic. By (5·1) his proper-time τ is then given by

$$d\tau^2 = d\bar{t}^2$$

so we may take

$$(5.5) \quad \tau = \bar{t}.$$

Hence

$$(5.6) \quad Q = \left(1 - \frac{r^2}{R^2}\right)^{\frac{1}{2}} \cosh\frac{t - \tau}{R}.$$

Now

$$\begin{aligned} \delta &= \partial\Omega/\partial\tau \\ &\to R^2\partial Q/\partial\tau \end{aligned}$$

as above; so by (5·6),

$$(5.7) \quad \delta = R\left(1 - \frac{r^2}{R^2}\right)^{\frac{1}{2}} \sinh\frac{\tau - t}{R}.$$

Making use of the fact that $Q = 1$, which gives

$$\cosh\frac{\tau - t}{R} = \left(1 - \frac{r^2}{R^2}\right)^{-\frac{1}{2}}$$

and therefore

$$\sinh\frac{\tau - t}{R} = \frac{r}{R}\left(1 - \frac{r^2}{R^2}\right)^{-},$$

we at once get

$$(5.8) \quad \delta = r.$$

---

* Whittaker, *loc. cit.*, 96, and Eddington, *Mathematical Theory of Relativity* (1924), 161.

Hence:

The coordinate r *measures spatial distance from the (spatial) origin according to the definition of this paper.*

It is interesting to compare this with Eddington's remark that "in so far as the distances of celestial objects are determined by parallaxes or parallactic motions, the coordinate r will agree with their accepted distances." *

To find the formula for Δ corresponding to (5.8), it is necessary to substitute from (5.2) and (5.3) in (1.3). Since we are assuming $\bar{r}=0$, we get by (5.3)

$$(5.9) \qquad \begin{cases} \bar{x}=\bar{y}=\bar{z}=0 \\ \bar{u}=\tanh (t/\mathrm{R}). \end{cases}$$

So, by (1.3),

$$\Delta=\frac{\mathrm{R}(1+\Sigma x^2)^{\frac{1}{2}}(\Sigma x^2)^{\frac{1}{2}}}{(1-u^2+\Sigma x^2)^{\frac{1}{2}}-u(\Sigma x^2)^{\frac{1}{2}}}.$$

Substituting from (5.2), we get

$$\Delta=\frac{r\mathrm{R}}{(\mathrm{R}^2-r^2)^{\frac{1}{2}}}\left\{\frac{\mathrm{R}+r\tanh (t/\mathrm{R})}{\mathrm{R}-r\tanh (t/\mathrm{R})}\right\}^{\frac{1}{2}}.$$

This therefore is Whittaker's formula for the spatial distance of a star from the origin in the de Sitter world.

* *Op. cit.*, 163.

(*Issued separately February* 26, 1932.)

VII.—Graphical Classification of Carbonaceous Minerals: the Place of the Constituents of Common Coal. By Professor Henry Briggs, D.Sc., Ph.D.

(MS. received November 12, 1931. Read January 11, 1932.)

THIS communication extends the method of classifying carbonaceous minerals already introduced * in order to include the three components of common coal. The existence of the three constituents has long been recognised, but their close study, and the nomenclature now generally accepted, date from 1919, when Dr M. C. Stopes described their salient characteristics in a paper to the Royal Society.† In that paper she gave the name *durain* to the hard, lustreless constituent, that of *vitrain* to the lustrous part, and borrowed the French miner's name *fusain* for the charcoal-like material commonly found between the bands of a coal seam. A bright variety, called *clarain* by Dr Stopes, is now usually regarded as a blend rather than as a separate substance, and may be disregarded in a classification such as this.

The method of classification depends on the ultimate analysis of the minerals, each analysis being reduced to terms of carbon, hydrogen, and oxygen only. It was shown in my previous paper (*loc. cit.*) that if carbon percentage be plotted against oxygen percentage, a series of straight lines are obtained, one for each mineral species. These lines were named "development lines," since by their course they indicate the development or evolution of the several species from mineral low in rank (*i.e.* low in carbon and high in oxygen) to mineral high in rank (*i.e.* high in carbon and low in oxygen); they express graphically the histolysis of the original vegetable substances and the histogeny of the minerals derived from them.

Minerals like torbanites and cannels are generally of fairly homogeneous structure; but a seam of bituminous coal is far from uniform, being a compound bed built up of lenticular bands of vitrain, durain, and fusain. Moreover, these constituents occur in different proportions in different seams, or even in different areas of the same seam. An attempt

* "Classification and Development of Carbonaceous Minerals," *Proc. Roy. Soc. Edin.*, li, p. 54, 1931.

† *Proc. Roy. Soc.*, B, 98, p. 470, 1919.

to classify so complex and variable a stratified mass by means of the analysis of a bulk sample, though it may be moderately successful from the commercial standpoint, leaves much to be desired from the scientific point of view. In the present state of knowledge the only satisfactory procedure with common coal appears to be that of subjecting the three components to individual examination. This fact was recently recognised by Dr E. S. Grumell,* who applied the graphical method to bright coal and dull coal separately.

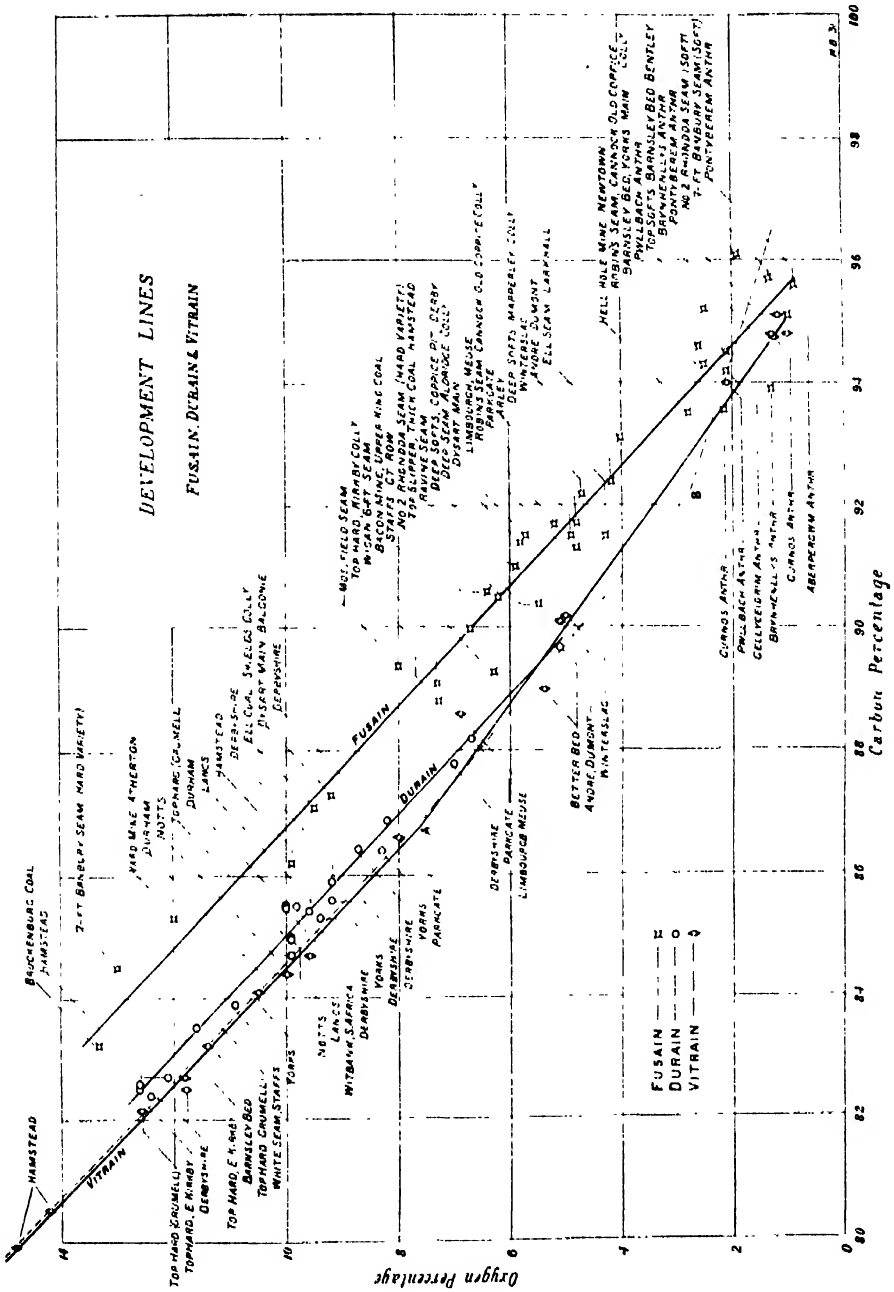
During the last few years, ultimate analyses of vitrain, durain, and fusain have been made available in sufficient numbers to enable the development lines of these substances to be drawn independently and with some claim to accuracy, and they are given in the subjoined graph (fig. 1). The analyses themselves are taken from papers by Baranov and Francis, Beet, Davis, Graham, Grounds, Grumell, Sinnatt, and Tideswell and Wheeler. All but eight are of British coals.

The dotted lines on the chart is Hickling's mean line for common coals. It is identical with the line *LABC* of fig. 2 of my previous paper (*loc. cit.*, p. 57). Its direction is deflected slightly at *A* and in a more pronounced manner at *B*. The first of these bends now appears to be attributable to the vitrain constituent of the coal; for some reason not yet understood the vitrain development line suffers a change of course in the neighbourhood of the point *A*. Information in regard to the trend of both the vitrain and durain lines below *A* is scanty, and the elucidation of the problem presented by these changes of direction is not yet feasible. It would, however, seem probable that the second kink, *B*, was due to an approximately simultaneous swing of the durain and vitrain lines in that vicinity.

So far as it has been traced, the fusain line exhibits no tendency to any similar deviation.

If the positions in fig. 1 of vitrain, durain, and fusain from the same seam be compared, it becomes clear that the fusain of any given bituminous coal is more advanced in rank than the other components of the seam—an observation supporting the prevailing view that fusain was, in its original form, more completely carbonised than its associates. It is also to be noticed that, as the coal becomes more anthracitic, the vitrain, by evolving more rapidly than the fusain, diminishes its handicap. In the end, when the coal has reached the state of anthracite, the vitrain has caught up with the fusain and sometimes even passed it. In contrast with the disparity between the vitrain and fusain of bituminous coal, the

* "A Contribution to the Study of Coals," *Trans. Inst. Min. Engs.*, lxxxi, p. 214, 1931.



DEVELOPMENT LINES
FUSAIN, DURAIN & VITRAIN
FUSAIN
DURAIN
VITRAIN
Carbon Percentage
Oxygen Percentage

FIG. 1.

difference in rank between vitrain and durain from the same seam is small. In some instances the durain is slightly the more advanced of the two. The especial importance of vitrain (or of blends in which vitrain is the principal ingredient) is indicated by the close agreement in their upper portions of the mean coal line (dotted) and the vitrain line.

Making use of the data in this and my previous paper, I have transferred to a single graph (fig. 2) the development lines of the seven species

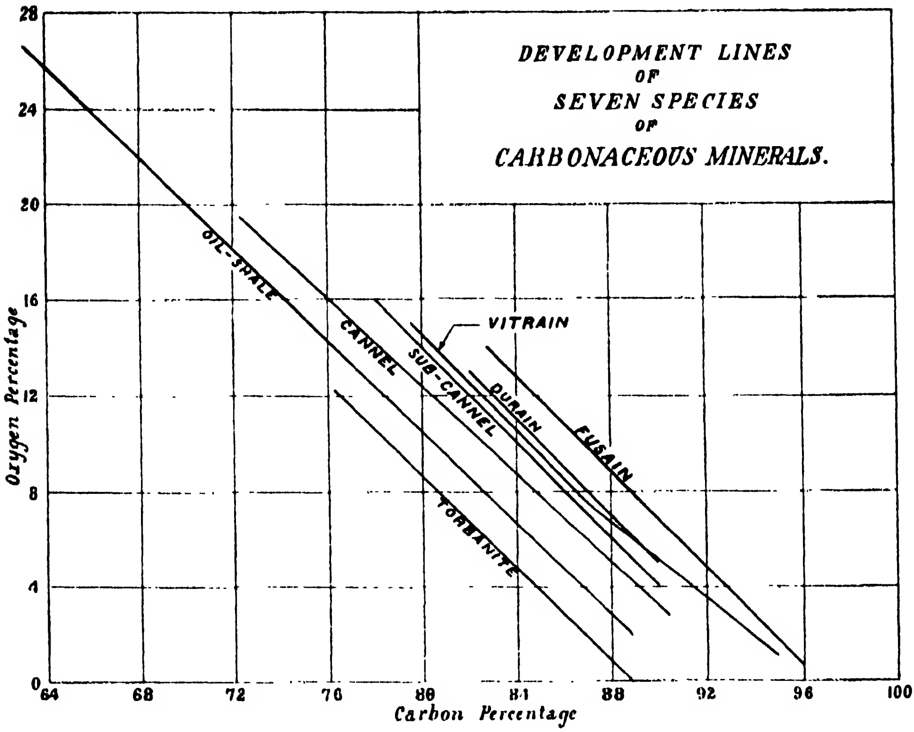


FIG. 2.

of carbonaceous mineral of organic origin, so far as it has been possible to trace them. Without doubt they will all eventually be found to extend over a much wider range.

In the previous paper it was pointed out that each of these lines is directed towards a base-line product whose composition is capable of being expressed by a simple formula of the type C_mH_{6n}. Continuation of the lines downwards to the carbon axis of the graphs indicates that, unless checked or deflected, the process of evolution will bring the minerals to the final products stated below.

Torbanites	$C_{4n}H_{6n}$
Oil-shales	$C_{5n}H_{6n}$
Cannels	$C_{7n}H_{6n}$
Sub-cannels	$C_{8n}H_{6n}$
Vitrain, upper portion of curve (carbon <86·7 per cent.)	$C_{8n}H_{6n}$
Vitrain, lower portion of curve (carbon >86·7 per cent.)	?
Durain	$C_{9n}H_{6n}$
Fusain	$C_{14n}H_{6n}$

(Issued separately February 26, 1932.)

VIII.—On the Structure of Vertebraria. By Professor John Walton, M.A., D.Sc., and Miss Jessie A. R. Wilson, B.Sc. (With Two Plates.)

(MS. received December 21, 1931. Read February 15, 1932.)

VERTEBRARIA is one of the commonest and most characteristic fossils in those Upper Carboniferous and Permian strata in which Glossopteris is the dominant fossil plant. It has been found in the form of coaly incrustations, impressions, and casts, and accounts of these various forms have been given by Arber* and Seward.† In view of our very inadequate knowledge of Vertebraria and Glossopteris any information about them is of importance. The genus *Vertebraria Royle* was founded on impressions of plant axes, some of which, from the evidence of the attachment of roots, must be judged to be roots or rhizomes. Vertebraria axes are usually found lying in the plane of the bedding of the matrix (Pl. I, fig. 2) and less frequently at right angles to the plane of bedding (Pl. I, fig. 3). Specimens found preserved in the latter position were placed in the species *Vertebraria radiata* by Feistmantel,‡ but it is clear that they cannot be distinguished by any definite specific characters from *V. indica.*

Zeiller § claims to have demonstrated the connection between Vertebraria and Glossopteris fronds, but the demonstration is not entirely convincing. He also interpreted these peculiar axes, with their transverse ridges extending from the centre to the sides, in terms of a stem such as that of *Onoclea* (*Struthiopteris*) *germanica* (Polypodiaceæ), in which in cross-section the stem has a roughly stellate outline. Oldham ‖ on the other hand described Vertebraria as built up of a central axis connected to an outer region by means of radial septa which were connected in places by transverse septa. We consider that Oldham's description and interpretation is probably nearer the truth. Seward suggests that the spaces between the longitudinal flanges and the transverse septa may have been occupied in part, at least, by secondary wood and the transverse septa as the medullary rays and leaf traces combined. No secondary wood had been proved to exist by any of these earlier investigators.

Other evidence for the supposed connection between Glossopteris and

* Arber (1905), p. 93.

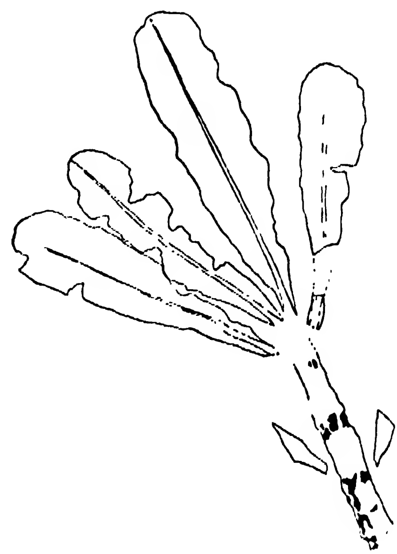
† Seward (1910), p. 501.

‡ Feistmantel (1880), vol. iii, pt. 2, p. 71.

§ Zeiller (1896), pp. 356–362 ; and (1897).

‖ Oldham (1897), p. 48.

Vertebraria has been furnished by the specimen collected at Vereeniging some years ago by Professor Molengraaff of Delft and figured and described by Professor Seward.* Dr Jongmans, who has also examined this specimen, very kindly furnished us with some excellent photographs of it. From an examination of these photographs it is clear that while there is little doubt about the connection between the leaves and the axis there is, as Dr Jongmans points out, no satisfactory evidence that the axis is Vertebraria. The axis appears to have transverse nodes, and the arrangement of the leaves and the obscure markings, giving the appearance of nodes, suggest that the leaves may have been arranged in whorls (text-fig. 1). It would appear, therefore, that while there may have been two types of axis bearing leaves of the *Glossopteris Browniana* type, one type with the leaves borne in whorls as in the Delft specimen, and the other type described by Zeiller and others, and figured here (Pl. I, figs. 1 and 2), we do not consider that the connection between Glossopteris and Vertebraria has as yet been adequately proved.



TEXT-FIG. 1.

Glossopteris Browniana Bgt.

Outline drawing of the specimen in Professor Molengraaff's collection at Delft from photographs provided by Dr W. J. Jongmans.

Irregularities of the surface of the stem are shown in black. The relations between the leaves and these irregularities suggest that the leaves may have been in whorls. ½ nat. size.

In view of the fact that secondary wood is clearly demonstrable in some specimens of typical Vertebrarias, which were kindly sent to one of us by Mr H. B. Maufe Director of the Geological Survey of Southern Rhodesia, our conception of the organisation of the Vertebraria axis is considerably altered.

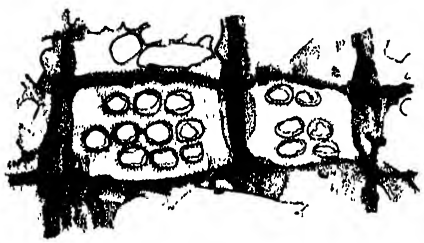
The specimens in which secondary wood is preserved were found in a core from a boring in the neighbourhood of the Wankie Colliery in Southern Rhodesia in rocks of Carboniferous or Lower Permian age. The two specimens of Vertebraria shown in Pl. I, figs. 1 and 2, are on the same side of a fragment of core. The rock is a black shale in which the only recognisable fossils are Vertebraria axes. Parts of these axes were covered with a syrupy solution of cellulose nitrate in amyl acetate. This was

* Oldham (1897), p. 504.

allowed to dry. The cellulose film then peeled off brought with it some of the coaly matter. In places this coaly material was brown and translucent and showed structure under the microscope.

Three types of tissue were recognisable in these peels: (*a*) Secondary wood with small medullary rays, (*b*) elongated thin-walled cells, and (*c*) large, thin-walled, angular parenchymatous tissue.

(*a*) The secondary wood is composed of tracheids of gymnospermous type in regular series and averaging 26·2 μ in breadth (Pl. II, fig. 4). The pits on the tracheid walls are in two to three series, slightly flattened, and tend to be arranged in horizontal rows (Pl. II, fig. 4); less frequently the pits are circular and disposed alternately (Pl. II, fig. 5). Occasionally tracheids occur showing a single row of pits, which are widely separated and more circular. The pits in what are probably the most completely preserved parts are irregularly bordered, an appearance possibly due to imperfect preservation, having an overhanging flap on either side and an elliptical, horizontally placed pore (text-fig. 2). Narrow, annular, or scalariform tracheids occur in places (Pl. II, fig. 7); these are probably parts of the primary xylem. No xylem parenchyma can be detected amongst the tracheids, the medullary rays are usually only one cell deep (Pl. II, fig. 6), occasionally two. On the wall separating ray cell and tracheid there are bordered pits similar to those on the radial walls of the tracheids. Parts of the wood have larger and more irregular pits (Pl. II, fig. 5), but this may be due to poor preservation.



TEXT-FIG. 2.—*Vertebraria indica* Royle. Drawing of the pitting on the surfaces common to the medullary-ray cells and tracheids. Mag. × 360.

(*b*) There are regions in which there are elongated thin-walled cells parallel to the adjacent tracheids on what would appear to be the outer side of the secondary wood. The longitudinal limit of these cells cannot usually be determined. They are approx. 20 μ in breadth and with a minimum length in some cases of 0·26 mm. Associated with these long thin-walled cells are other similar cells possessing very dense contents.

(*c*) Usually towards the outer limits of the axes, as shown in these peels, occurs a tissue composed of very large cells (Pl. II, fig. 8) arranged in radial rows. These large cells are thin walled and rectangular in shape, 80 μ in breadth and 0·22 mm. in length. This tissue has been compressed, and there is the appearance of the superposition in many cases of series

of these cells which suggest that it was produced by very regular cell divisions and may be a secondary tissue.

Associated with the Vertebraria axes on the samples of the core are impressions of long narrow structures, which bear numerous branches as shown in Pl. I, fig. 1. These branches in turn have laterals, which in one specimen appear to have been endogenous. At a few places their attachment to the axes can be established, and it is perhaps significant that in most cases these points of junction appear to coincide, as Zeiller * observed, with the transverse grooves. Similar structures have been described by Zeiller, who from evidence of their connection to Vertebraria, interpreted them as the roots of that plant. Tracheids of the same type as those found in the Vertebraria axes are found in these more slender axes.

From the distribution of the secondary wood in the peels from the surface of the axis it is clear that secondary wood extended from very close to the centre of the axis to very close to the edges. This indicates, firstly, that the primary wood and pith, if one was present, occupied a relatively small space; and, secondly, that the tissues external to the xylem were possibly small in bulk and have almost entirely disappeared.

In another specimen of Vertebraria from New South Wales (Pl. I, fig. 3) the axes are embedded at right angles to the bedding. There are portions of five axes shown in this specimen. The largest (*e*) is 2 cm. in diameter, while the smallest (*d*) is about 2 mm. In transverse section they appear as thin-rayed stars, one stem (*b*) with ten rays and the smaller (*a*) with five.

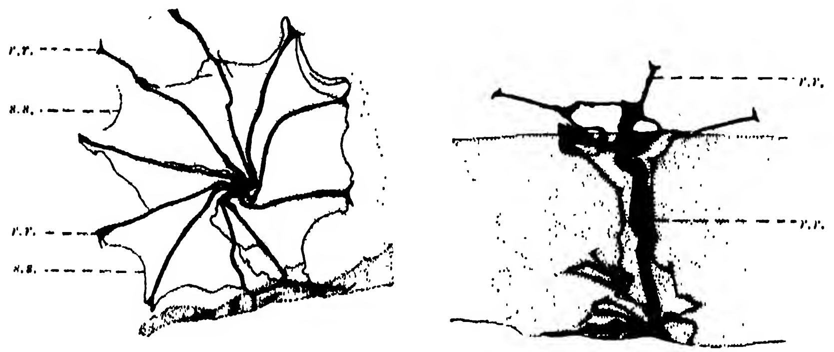
One of these axis (*b*) is shown enlarged in a drawing in text-fig. 3, A. There is a narrow central core of carbonaceous material, and the rays, which are apparently of the same coaly substance, radiate from it. The rays are the transverse sections of irregular, roughly vertical plates of carbonaceous material, as was seen when the side of this block was ground down to expose a tangential longitudinal section of one of these axes (Pl. I, fig. 3, *c*). On the left-hand side of the axis (Pl. I, fig. 3, *e*) these plates are occasionally curved over and anastomose, as is seen at (*e*) and also at three places at (*c*). The outer edges of the plates therefore form a reticulum when the specimen is examined from the side (text-figs. 3, A and 3, B).

In the specimen shown in Pl. I, fig. 3, *b* and in text-fig. 3, A there are thin layers of carbonaceous matter present attached to the extremities of the rays and extending across the spaces between the rays. These

* Zeiller (1896), p. 355.

layers afford evidence that the Vertebraria axis was not in its original intact condition stellate in section, but that it was probably cylindrical or nearly so. They in all probability represent the shrivelled remains of cortical tissues which have been drawn inward as a result of the collapse of the medullary ray tissue.

Cellulose peels taken from the surfaces, seen in Pl. I, fig. 3, show that these plates of carbonaceous matter, where they are bent over as on the left-hand side of (*e*), fig. 3, consist of wood containing tracheids with multiseriate bordered pits arranged in regular series and with clear indication of medullary rays. This tissue is seen to extend from the outer edge



A. B.

TEXT FIG. 3.—*Vertebraria indica* Royle.

A. Drawing of end of axis shown at Plate I, fig. 3 (*b*).
r.r. Secondary wood.
s.s. Remains of tissue external to the secondary wood.

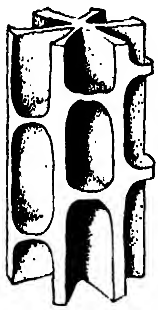
B. Median section of specimen shown in Plate I, fig. 3 (*e*), revealed by grinding down the side of the rock fragment.
The lighter stippled area represents the top of the rock fragment; the darker area the side or vertical face of the rock fragment. Mag. × 3.

of the plates to quite near the centre of the axis. The appearance and dimensions of the tracheids and pits compare so closely with those of the corresponding tissue in the Wankie specimens as to leave no doubt that we are dealing with parts of similar plants.

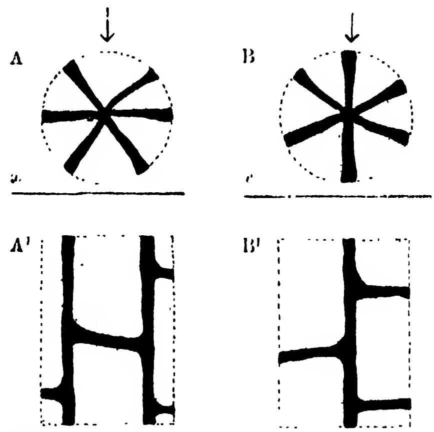
In a specimen of *Vertebraria indica* Royle, kindly lent to one of us by Professor Birbal Sahni of Lucknow University, there are areas on the surface of parts of the axis covered with a very fine network. The dimensions of the meshes of which accord very well with the dimensions of the large thin-walled cells found in the Wankie Vertebrarias (Pl. II, fig. 8). On other parts of the axis there are longitudinal and transverse striations, representing no doubt the tracheids and medullary rays of the secondary wood.

These Vertebraria axes would thus appear to be stems or roots in which

the pith or primary wood occupied a very small space, but with a large development of secondary wood, consisting of tracheids and small medullary rays. The vascular tissue had possibly a form, such as is shown in text-fig. 4. This vascular skeleton is reconstructed upon the evidence afforded by the fossils, but most probably the living plant possessed a very much larger amount of secondary wood than is shown here, which has become shrunken and compressed tangentially before and during fossilisation into narrow vertical plates. Thus the large medullary rays would not originally have been as broad as indicated in the drawing: these rays in all probability consisted of the large thin-walled cells, previously described (p. 202 (*c*)), which have shrivelled up, the spaces becoming filled with matrix. The traces of tissue (text-fig. 3, A, *s.s.*) linking up the protecting flanges of secondary wood show that the stem was not originally stellate in section but nearly cylindrical. Applying this interpretation to these Vertebraria casts—the rectangular areas represent the large medullary rays and the longitudinal and transverse grooves or ridges the secondary wood. The flattened axes usually show one to three longitudinal grooves or ridges dividing the surface of the axes into two to four longitudinal series of rectangular areas. The number of the grooves depends upon the number of rays in the stem and the direction of compression (see explanation to text-fig. 5 and the illustrations of Vertebraria given by Feistmantel.* The extent of these rectangular areas corresponds to that of the large medullary rays, and they have on their surfaces traces of the secondary wood which bounded them.



TEXT-FIG. 4.—Reconstruction to show the form of the xylem in the Vertebraria axis.



TEXT-FIG. 5.—Diagram to show how the position of the axis in relation to the direction of compression would determine the form of the resulting fossil.

A. One position of axis in relation to plane of bedding *x* and direction of compression (indicated by arrow).

B. Another possible position of axis in relation to plane of bedding and direction of compression.

A^1. Resultant form of fossil.

B^1. Resultant form of fossil.

(A and B are transverse sections of axis, and show no signs of the transverse connections seen in A^1, B^1.)

* Feistmantel, O. (1886), vol. iv, pt. 1, pl. xi, figs. 1–4, and pt. ii, pl. iva, figs. 4, 5, 7–11.

Feistmantel * considered that Vertebraria was related to the Equisetales, while Zeiller's † interpretation favoured a relationship with the Ferns. From the structure exhibited in the preparations from these Rhodesian and New South Wales Vertebrarias it is, however, practically certain that the plants which bore them were not Pteridophytes, but almost certainly belonged to one of the higher groups such as the Gymnosperms or Pteridosperms.

Although it is quite possible that some, at any rate, of the numerous species of Glossopteris were attached to Vertebraria, which formed their stems, we do not consider that the evidence so far brought forward is sufficiently conclusive. It may be noted in this connection that the older view that Glossopteris was a Pteridophyte has been seriously questioned and that recently Dr Hamshaw Thomas ‡ has pointed out the similarity between Glossopteris fronds and the leaflets of Sagenopteris, which, as he has shown, is probably the leaf of one of the Caytoniales, a group of plants whose nearest relations are probably Angiosperms. If, therefore, conclusive proof was forthcoming that Vertebraria bore Glossopteris fronds the evidence of the structure of Vertebraria would also support the attribution of Glossopteris to a gymnospermous or higher group. The relationship of Vertebraria to other known fossil plants must therefore be regarded as still obscure.

BIBLIOGRAPHY.

ARBER, E. A. N., *Catalogue of the Fossil Plants of the Glossopteris Flora in the British Museum*, London, 1905.

ETHERIDGE, R., "On the Mode of Attachment of the Leaves or Fronds to the Caudex in Glossopteris," *Proc. Linn. Soc. New S. Wales*, vol. ix, 1894.

FEISTMANTEL, O., "The Fossil Flora of the Godwana System," *Mem. Geol. Surv. India*, Pal. Ind., ser. 12, vols. iii and iv, 1879–1886.

OLDHAM, R. D., "On a Plant of Glossopteris with Part of the Rhizome attached, and on the Structure of Vertebraria," *Rec. Geol. Surv. India*, vol. xxx, pt. 1, 1897.

SEWARD, A. C., *Fossil Plants*, vol. ii, Cambridge, 1910.

THOMAS, H. H., "The Caytoniales, a New Group of Angiospermous Plants from the Jurassic Rocks of Yorkshire," *Phil. Trans. Roy. Soc. London*, Ser. B, vol. ccxiii, 1925.

ZEILLER, M. R., "Étude sur quelques plantes fossiles, en particulier Vertebraria et Glossopteris, des environs de Johannesburg (Transvaal)," *Bull. de Soc. Geol. France*, 3 ser., t. xxiv, 1896. The reference is to the genus Vertebraria. (Transl. fr. *Compt. Rend.*, t. cxxii.) *Rec. Geol. Surv. India*, vol. xxx, pt. 1, 1897.

---

* Feistmantel, O. (1879), vol. iii, pt. i, p. 8.

† Zeiller (1896), p. 360.

‡ Thomas (1925), p. 353.

Fig. 1.

Fig. 2.

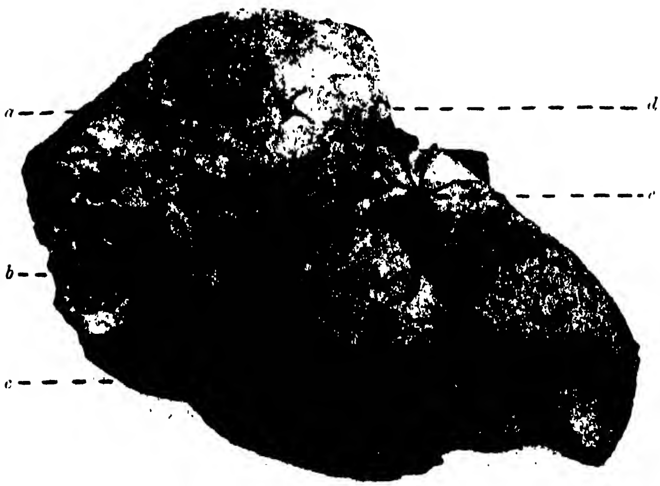


Fig. 3.

Fig. 1.

Fig. 5.

Fig. 6.

Fig. 7.

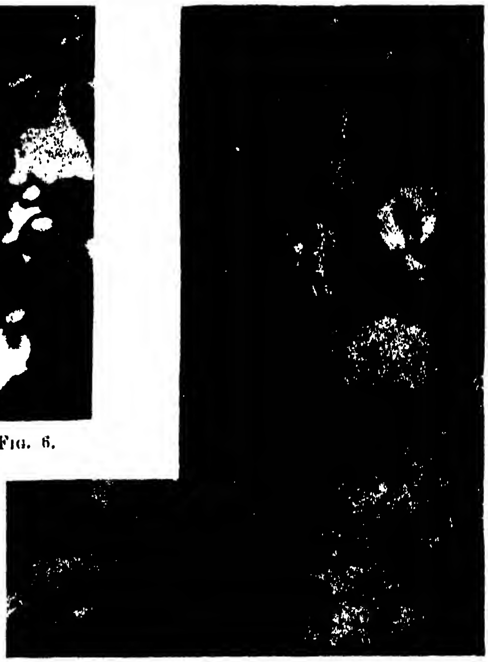


Fig. 8.

DESCRIPTION OF PLATES.

Plate I.

Vertebraria indica Royle.

Fig. 1. Part of a core from a boring at the Wankie Colliery, Southern Rhodesia, showing an axis of Vertebraria, and in the top of the right-hand corner a branching root. Nat. size. Glasgow Univ. Bot. Dept. Museum, No. Pb. 1.

Fig. 2. Another example of Vertebraria from the same piece of core as that in fig. 1. Nat. size.

Fig. 3. Fragment of shale from New South Wales with several Vertebraria axes (*a*, *b*, *c*, *d*, and *e*) embedded at right angles to the plane of the bedding of the rock. Nat. size. Manchester Museum, No. 10538, A.

Plate II.

Fig. 4. Cellulose-pull (see p. 201) from one of the flat surfaces on the axis shown in fig. 1, Plate I, showing tracheids of the secondary wood with multiseriate pitting and indications of medullary rays. Mag. × 185. Glasgow Univ. Bot. Dept., Figured Slide Collection, No. 1.

Fig. 5. Part of another cellulose-pull showing parts of four tracheids. Mag. × 370. G.U.B.D., Figured Slide Collection, No. 2.

Fig. 6. Part of a cellulose-pull showing a uniseriate medullary ray with the pitting between the ray cells and the underlying tracheids. Mag. × 370. G.U.B.D., Figured Slide Collection, No. 1.

Fig. 7. Part of a cellulose-pull showing parts of small annular or spiral tracheids and narrow tracheids with uniseriate pits. Mag. × 185. G.U.B.D., Figured Slide Collection, No. 2.

Fig. 8. Part of a cellulose-pull showing the large, thin-walled cells described on p. 202 of the text. Mag. × 185. G.U.B.D., Figured Slide Collection, No. 3.

(*Issued separately May 2, 1932.*)

IX.—Graphical Analysis of Internal Combustion Engine Indicator Diagrams. By Alex. R. Horne, O.B.E., B.Sc.(Lond.), Professor of Mechanical Engineering, Heriot-Watt College, Edinburgh.

(MS. received February 22, 1932. Read May 2, 1932.)

The analysis of the performance of an internal combustion engine, based upon the indicator diagram, for the purpose of investigating such matters as the cyclical changes of temperature and internal energy content of the charge, and the heat flow between the working agent and the cylinder walls, is of considerable importance.

If a complete analysis is to be undertaken in order to determine the continuous fluctuations of the quantities referred to over the complete cycle, the calculations are very laborious; they are especially so if allowance is to be made for the variation of specific heat with temperature. Moreover, the interpretation of the results is difficult unless curves are subsequently plotted.

In what follows, wholly graphical methods of deducing these curves of temperature, internal energy, work and heat transfer, are described and illustrated. The processes involved are based upon graphical methods of deducing from a curve of Y against X, curves which represent (1) the product XY, and (2) the integral YdX. The method of the "product curve" would appear to have a wide application to mechanical and physical problems. The system here used to construct the integral curve is an improvement upon one which is sometimes made use of in mechanics. For the sake of brevity, the principles underlying the constructions are described with reference to the functions involved in an internal combustion engine.

Let P be the absolute pressure of the charge whose mass is M and whose specific heat at constant volume is C_V, when the charge occupies a volume V at absolute temperature T. The characteristic constant for the gas mixture is R. The units here used are the pound, the foot, and the degree centigrade. C_V is a function of T. Further, E is the internal energy of the mixture in C.H.U. The work done in C.H.U. is represented by W.

The scales of the curves which constitute the data of the problem, namely of P, V, and $\bar{C}_V$, where $\bar{C}_V$ is the mean value of C_V over a given range of temperature, are denoted by p, v, and c; while those involving

the deduced quantities T, E, and W are indicated by t, e, and w respectively. Thus the scales are $1'' = p$ lb./ft.^{2}; $1'' = v$ ft.^{3}, etc.

The Product Curve: Temperature.—Let the primary curve in fig. 1 represent the known relation between P and V, given by the indicator diagram. A vertical line is drawn at a convenient distance, r_T, from the P axis. This line is conveniently called the reference line, and r_T the reference distance.

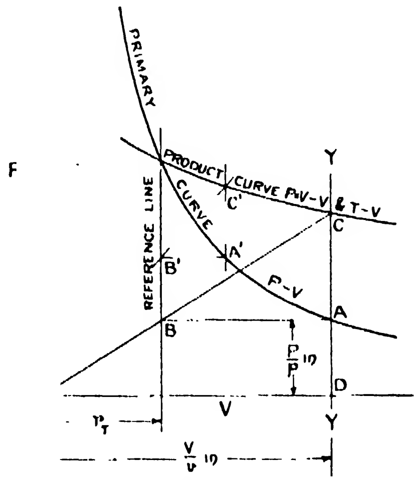


FIG. I

Consider any ordinate YY. Draw AB to cut the reference line in B, and a ray from the origin 0, through B, to cut YY in C. The co-ordinates of A obviously are $\frac{P}{p}$ and $\frac{V}{v}$ inches. From the properties of similar triangles it may be shown that

$$CD = \frac{1}{p \cdot v \cdot r_T} \cdot P.V. \quad . \quad . \quad . \quad . \quad (1)$$

Since the scales p and v, and the distance r_T, are defined, it follows that CD is proportional to the product P.V. The scale on which this product is represented by CD may be adjusted by altering r_T. The construction relating to another ordinate, and the resulting product curve, are shown in the figure.

The complete cycle in the engine may be divided into two parts: (1) that portion which extends from the closing of the suction valve to the opening of the exhaust valve (see fig. 4), during which the mass of the charge remains constant, and conveniently referred to as the "closed" part of the cycle; and (2) the remainder, which relates to the suction and exhaust processes—the "open" part of the cycle. During (2) the mass of the charge is continually varying. For the present, attention will be confined to the closed part of the cycle.

Considering the law of gases,

$$PV = MRT \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

the value of T is unknown at any point in the cycle. Its experimental determination is difficult, but its value just after the admission valve has closed may be estimated from experience and the result of research. Choosing such a point, as a on the indicator diagram, fig. 4, P and V may be read. The value of R is known for the mixture, hence the mass present, M, may be found from equation (2).

Substituting in (1) from (2)

$$CD = \frac{M \cdot R}{p \cdot v \cdot r_T} \cdot T \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The only variable in the closed cycle is T, hence CD is proportional to the temperature of the charge. The product curve therefore is one of temperature on a volume base. The scale to which CD represents temperature obviously is

$$t = \frac{p \cdot v \cdot r_T}{M \cdot R} \text{ degrees C. per inch.} \quad . \quad . \quad . \quad . \quad . \quad (A)$$

The Product Curve: Internal Energy.—The specific heat at constant volume, C_V, of unit mass of a gas may be expressed by

$$C_V = A + BT + CT^2,$$

hence the internal energy at T degrees above that at absolute zero is, per unit mass,

$$E = \int_0^T C_V dT = AT + \frac{B}{2}T^2 + \frac{C}{3}T^3,$$

and the mean specific heat over the range $T = 0$ to $T = T$ is

$$\bar{C}_V = \frac{E}{T} = A + \frac{B}{2} \cdot T + \frac{C}{3} \cdot T^2 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

in which A, B, and C are constants for any one gas mixture.

Again, reckoning energy above absolute zero, for a mass M

$$E = M \cdot \bar{C}_V \cdot T \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

For simplicity of explanation the term in T^2 in (4) is here omitted.

In fig. 2 the $T - V$ curve deduced in fig. 1 is repeated. EF is a curve of $\bar{C}_V$ plotted horizontally against T vertically, in accordance with equation (4).

A reference line is set up, distant r_E from the axis. Consider any ordinate YY. A horizontal line HK is drawn through C, and a ray from 0, through H, to meet a vertical from K in L. A horizontal line through L cuts YY in M. By similar triangles, introducing the scale values,

$$MD = \frac{1}{c \cdot t \cdot r_E} \cdot \bar{C}_V \cdot T = \frac{1}{M \cdot c \cdot t \cdot r_E} \cdot M \cdot \bar{C}_V \cdot T.$$

Substituting for t from (A) and using (5)

$$MD = \frac{R}{p \cdot v \cdot c \cdot r_E \cdot r_T} \cdot E.$$

The only variable being E, MD represents the internal energy of the mass of gas present, the scale being

$$e = \frac{p \, . \, v \, . \, c \, . \, r_E \, . \, r_T}{R} \text{ C.H.U. per inch} \qquad . \quad . \quad . \quad (B)$$

The curve of internal energy is shown in fig. 2.

Temperature and Internal Energy in the "open" part of the Cycle.

Since it has been assumed in the development of these constructions that the mass M is constant, the methods described would appear to be

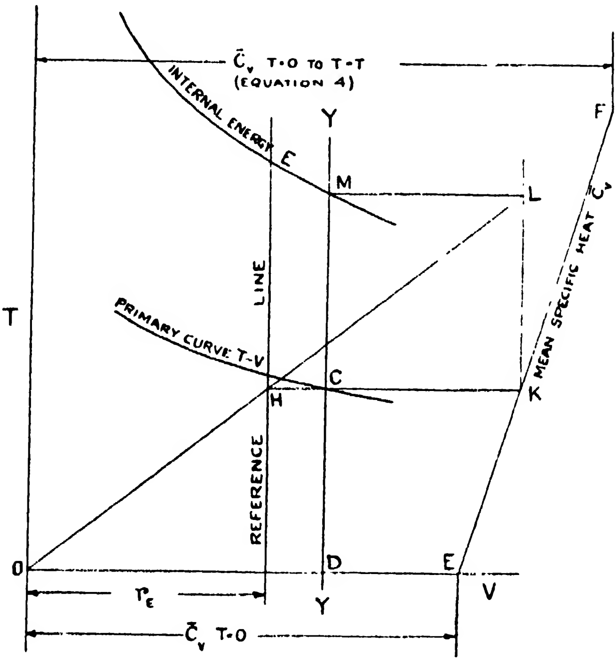


FIG. 2

inapplicable to the open part of the cycle. This is true in respect of temperature, but not so of internal energy.

If at a point where the pressure is P and the volume V the mass present were M (the mass in the closed part of the cycle), the temperature would be T, such that PV = MRT, and the internal energy would be MT × mean specific heat. If, in fact, the mass present is m and the temperature θ, then PV $= m\text{R}\theta$, and the internal energy, $m\theta \times$ mean specific heat. But $m\theta =$ MT since each equals PV/R. It follows that, if the processes described are followed for the open cycle, a fictitious value will be obtained for temperature; but the value deduced for internal energy will be correct if the value of $\bar{C}_V$ over the ranges T = 0 to T = T, and T = 0 to T = θ are equal.

While this is not strictly true, the error introduced by making this assumption is small because the rate of variation of $\bar{C}_V$ with temperature is small. Therefore the normal method of constructing the curve of internal energy may be followed in the open part of the cycle, and almost exact results obtained.

The Work Integral.—The P–V curve in fig. 1 is repeated in fig. 3. A reference line is drawn at a distance r_W from the vertical axis. Consider the ordinate YY. Draw AN, and the ray from 0, through N, to cut the boundaries of a small volume δV, around YY, in P and Q. By similar triangles

$$QR = \frac{\delta V}{V} \cdot SD = \frac{\delta V}{V} \cdot \frac{P}{p} \cdot \frac{V}{v} \cdot \frac{1}{r_W} = \frac{1}{p \cdot v \cdot r_W} \cdot P \cdot \delta V \quad . \quad . \quad . \quad (6)$$

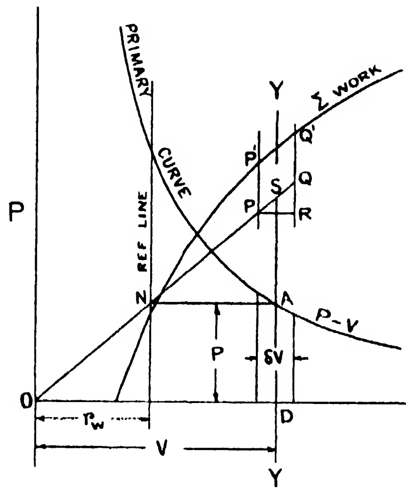


FIG. 3

But $P \cdot \delta V$ is the work done in foot-pounds, while the volume changes by δV. Or, expressed in C.H.U., the work done is $\delta W = P \cdot \delta V / J$, where J is Joule's equivalent. Hence QR represents the work performed while the piston displaces the volume V to a scale.

$$w = \frac{p \cdot v \cdot r_W}{J} \text{ C.H.U. per inch} \quad . \quad (C)$$

In order to construct the curve of work, or work integral, it is necessary to begin with ordinates at that end of the pressure curve which is nearest to 0, and to build it up by drawing lines parallel to those corresponding to PQ; or, more simply, to the rays of which 0N is representative. The resulting integral curve is shown in fig. 3. The element P′Q′ is, for example, drawn parallel to PQ or 0N.

It is to be noted that very few of the construction lines shown in the foregoing figures need actually be drawn. With the exception of the reference lines and ordinates such as YY, and the verticals from the $\bar{C}_V$ line in fig. 2, it is necessary only to mark the several points. This greatly facilitates the work, not only directly, but indirectly also; for the confusion which would arise from the presence of many construction lines is avoided. With a little experience the work can be done very rapidly.

Choice of Scales.—Of the various scales, p, v, and c are selected so as to give a diagram of suitable size. As a guide, it may be said that

$p=7200$ lb./ft.^{2} (equivalent to 50 lb./in.^{2}) is suitable for engines of ordinary type. This figure may be doubled for Diesel engines. The scale value, v, ought to be chosen to give an indicator diagram about 6 inches long.

In the investigation to be described, $p=7200$ lb./ft.^{2}. As the piston displacement is 0·35 ft.^{3}, a scale $v=0{\cdot}05$ ft.^{3} was selected, giving an indicator diagram 7 inches long. A convenient value for the scale of $\bar{C}_V$ is $c=0{\cdot}02$ C.H.U. per inch.

These primary scales being selected, an examination of equations (A), (B), and (C) discloses that the scales t, e, and w are dependent upon one or more of the magnitudes of the reference distances, r_T, r_E, and r_W. The procedure will be explained with reference to the case illustrated in figs. 4 and 5. Repeating the equations (A), (B), and (C)

$$t=\frac{p\,.\,v\,.\,r_T}{M\,.\,R}\ \ldots\ (A)\,;\quad e=\frac{p\,.\,v\,.\,c\,.\,r_E\,.\,r_T}{R}\ \ldots\ (B)\,;\quad w=\frac{p\,.\,v\,.\,r_W}{J}\ \ldots\ (C),$$

in which $p=7200$, $v=0{\cdot}05$, $c=0{\cdot}02$, $J=1400$, and R is taken as 96. For a diagram about 6 inches long, the reference distances are conveniently made between 2 and 5 inches.

At the point a on the compression curve, fig. 4, by measurement $P=15\times144=2160$ lb./ft.^{2}, and V, including the cylinder clearance space $=0{\cdot}425$ ft.^{3}. The temperature at a is estimated as 100° C., or $T=373$. Then, since $P_aV_a=MRT_a$, $M=0{\cdot}0256$ pound.

Substituting in (A), $t=146{\cdot}5r_T$. By making $r_T=2{\cdot}73$ inches, $t=400$ degrees per inch.

Proceeding to (B) there results $e=r_E/4{\cdot}88$. If r_E is made 4·88 inches, $e=1$ C.H.U. of internal energy per inch.

For purposes of comparison, it is necessary that the scales of internal energy and work in C.H.U. should be equal. Hence in (C) $w=1$, which makes $r_W=3{\cdot}89$ inches.

These several scales are tabulated alongside fig. 5. It is to be remembered that they relate to the original drawing, which is here reduced.

Application to typical Indicator Diagram.—The indicator diagram is shown in fig. 4. The clearance volume, 0·1 ft.^{3}, is set off on the left. The four strokes are developed in fig. 5 in their correct order in respect to time, beginning with the suction stroke. The compression and exhaust strokes of fig. 4 are reversed in fig. 5. The pressure curve is indicated.

It is convenient to begin with the construction of the curves which relate to the power stroke. The point 0, of zero volume, corresponds to 0 in fig. 4 being distant from the beginning of the power stroke by the clearance volume. The several reference lines are set up at distances r_T, r_E, and r_W from 0, in agreement with the values in the table.

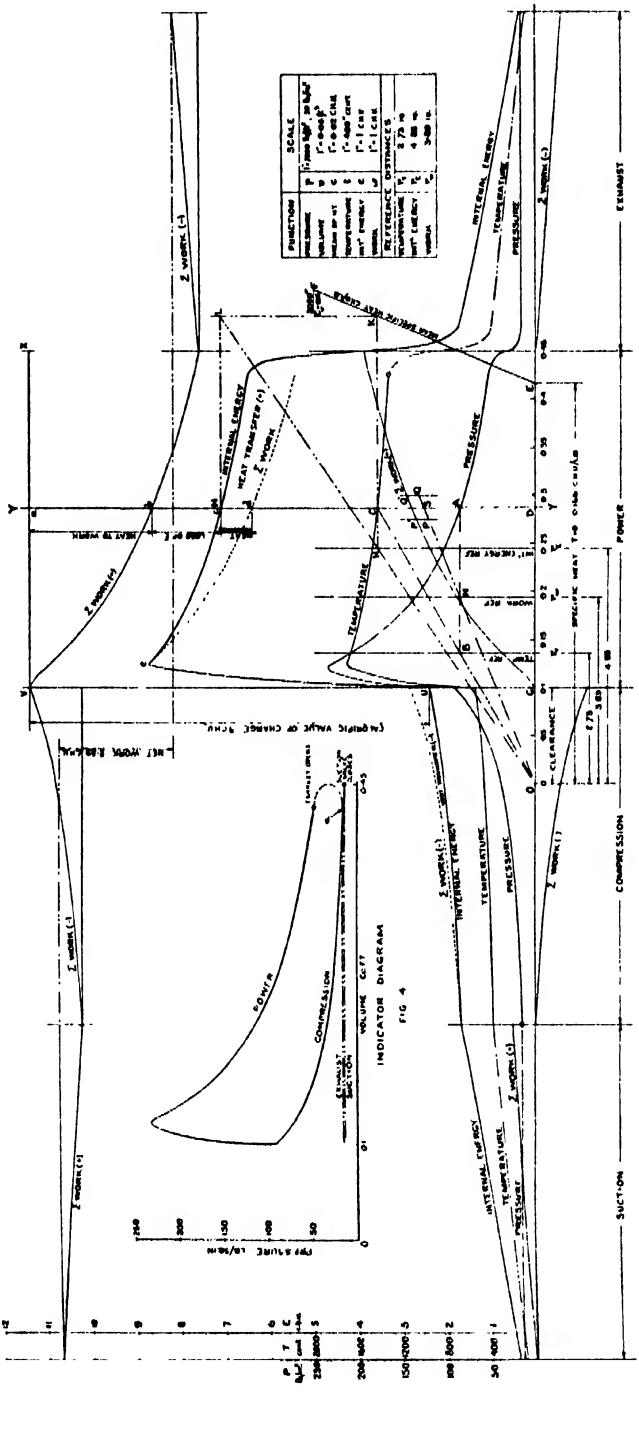

SUCTION
COMPRESSION
POWER
EXHAUST
INDICATOR DIAGRAM
FIG 4

FIG 5

The specific heat, per pound, has been assumed to be $0{\cdot}166 + 0{\cdot}00004\ T$; hence, from equation (4); $\bar{C}_V = 0{\cdot}166 + 0{\cdot}00002\ T$. When $T = 0$, $\bar{C}_V = 0{\cdot}166$, and when $T = 2000$, $\bar{C}_V = 0{\cdot}206$. The scale being $1'' = 0{\cdot}02$, these values are represented by 8·3 and 10·3 inches respectively. These lengths are set off to the right of 0, the former on the base line, where $T = 0$, and the latter on the level $T = 2000$ on the temperature scale on the extreme left. The inclined line EF through these points is drawn. Had the term in T^2 in (4) been included, this line would have been curved.

Selecting the ordinate YY, B is marked on the r_T line opposite to A on the pressure curve. C is on the ray from 0 through B. This point defines the temperature.

To obtain the point on the internal energy curve, C is projected to H on the r_E line and to K on the $\bar{C}_V$ line. A ray from 0, through H, cuts the vertical through K in L. Finally, M is marked on YY, opposite to L. MD is the value of the internal energy appropriate to the ordinate YY.

To determine the slope of the work integral in the region of YY, A on the pressure line is projected to N on the r_W axis. The slope is given by ON. The process of drawing the integral curve must be begun from the left-hand end of the stroke, and the curve gradually built up from the various slopes, corresponding to ON, with the aid of a clinograph. Thus, for example, P′Q′ on the graph is parallel to PQ. The diagram is lettered similarly to figs. 1, 2, and 3 in order that the methods may be clearly followed.

This work integral need not be drawn from the point G. It is, in fact, more convenient to draw it elsewhere. It may be inverted by inverting the clinograph.

Similar constructions are made for the remaining strokes. It is to be noted that the points corresponding to 0, will, for the compression and exhaust strokes, be set off to the *right*, since the lines of pressure in fig. 4 which relate to these strokes have been reversed in fig. 5.

The curves of temperature and internal energy are shown for the complete cycle. Those portions of the former which relate to the open cycle, and give fictitious values of temperature, are in dotted lines.

At the end of the compression stroke the internal energy content of the charge is indicated by GU. Ignition, which is assumed to occur at this point, results in the liberation of the potential heat of the charge. The calorific value of the charge is here assumed to be 9 C.H.U., and is set up from U as shown by UV, so that GV represents the total energy which would have been present had the full calorific value of the charge found its way into the products of combustion. Actually, a portion fails to do so.

For the purposes of analysis, it is convenient to begin the work integral from V instead of from G, and to draw it inverted as shown. Referring now to the ordinate YY, a portion of the total energy at V, namely *ab*, has been converted to work, while *bc* represents the missing energy arising mainly from the presence of the water-jacket.

Again, if the appropriate portion of the work integral is repeated through *e*, then *cd* is a measure of the heat which has come into the charge while the piston has moved from the position corresponding to *e* to that corresponding to YY.

The work integral curves for the remaining three strokes may be drawn in the positions shown, near the top of the figure; that for compression terminating at V. It will be observed that positive work, *i.e.* work done by the charge, is set *downwards*, while negative work is set *upwards*. Also, that the heat transfer from the charge to the cylinder walls, piston, etc., up to any point in the compression stroke, is shown by the ordinate between the dotted work integral and internal energy curves.

The net work done during the cycle is given by the difference of the levels of the extreme ends of the work integral line, and amounts to 2·59 C.H.U. Since the calorific value has been assumed as 9 C.H.U., the thermal efficiency of the engine is 2·59/9, or 28·8 per cent.

The Law of the Expansion and Compression Curves.—It is commonly assumed that the expansion and compression curves are of the form $PV^n = \text{constant}$. This implies that the rate of interchange of heat between the gas mixture and its container bears a constant ratio to the rate at which work is being performed. As the conditions do not in general permit of this being so, the law does not hold. Hopkinson and others have found n to vary considerably over the various sections of the curve.

The chief purpose of determining n is to facilitate the calculation of the rate of heat interchange with respect to the rate of change of volume, from

$$\frac{dH}{dV} = \frac{\gamma - n}{\gamma - 1} \cdot \frac{P}{J} \quad \text{C.H.U. per cubic foot.}$$

If the value of $\dfrac{dH}{dV}$ is required with respect to any point on the curve, the instantaneous value of n appropriate to a small section in the region of the point may be found by measuring the values of P and V at two points in the curve, close to the point to be considered, one on each side, and solving for n in $P_1V_1^n = P_2V_2^n$. This method cannot be relied upon to give an accurate result as it implies the measurement of very small

differences of P and V. The graphical method described below permits of the instantaneous value of n being found.

In fig. 6, the axes of pressure and volume, and a portion of the curve, are shown. Lines AB and CD are drawn parallel to the axis of volume,

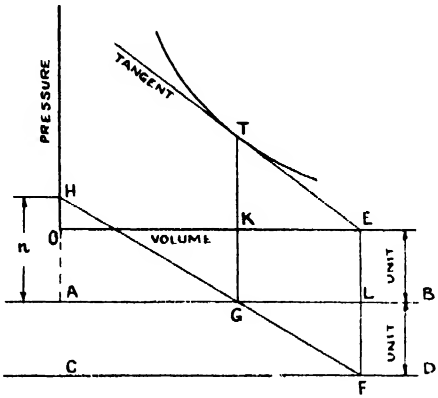


FIG. 6

equally spaced at a convenient distance, which is taken as a unit. A point E is selected, and from it a tangent to the curve is drawn, the point of contact being T. The points G and F are marked below T and E. H is on the pressure axis in line with F and G.

The value of n in the region of T is given by AH. The proof is as follows:—

$$\text{If } PV^n = C \text{ then } \frac{dP}{dV} = -n\frac{P}{V}; \text{ but } \frac{dP}{dV} = -\frac{KT}{KE} \text{ and } \frac{P}{V} = \frac{KT}{OK},$$

hence

$$n = \frac{OK}{KE} = \frac{AH}{EL} = AH$$

since EL has been selected as unit length.

(*Issued separately May* 2, 1932).

X.—The Pigmentary System and the Dopa Reaction. By Evlyn Boyd, M.Sc., 1851 Exhibition Research Student, University of South Africa. *Communicated by* Professor F. A. E. Crew, M.D. (With Four Plates.)

(MS. received December 4, 1931. Read February 15, 1932. MS. revised February 18, 1932.)

Pigment occurs in every group in the animal kingdom, and its origin and precise nature have for years been the subject of much research. In mammals the characteristic pigment, melanin, occurs normally in the skin and its derivatives, the eye, and certain parts of the nervous system. It also occurs in some pathological conditions of the ovary, uterus, adrenals, rib cartilages, peritoneum, and in rapidly growing melanotic tumours.

It is usual to consider the physiological mechanism in the body which produces melanin under the collective title of "the pigmentary system" without reference to the actual site in the body where that mechanism is functioning. The constituent elements of this mechanism are always the same no matter where melanin is being produced, but they may vary quantitatively within limits. (It should be clearly recognised that the use of the phrase "pigmentary system" does not include those varieties of pigment derived from blood, bile, or urine.) The pigmentary system itself essentially consists of a chromogen and a catalytic enzyme or enzymes, and it is the interaction of these which produces melanin. The nature of the basic chromogen and the catalytic enzymes still gives rise to controversy, but recent work (Bloch, 1917) has made it practically certain that tyrosine is the most commonly occurring chromogenic substance, and that upon its oxidation by enzyme action melanin is produced. It has been shown that the action of tyrosinase on tyrosine produces a red pigment which in alkaline solutions changes spontaneously to a colourless substance, and this in the presence of molecular oxygen or a specific oxidising agent in the cell is converted into melanin. Raper (1926) showed that tyrosine on oxidation gives rise to dihydroxyphenylalanine, and this in turn, when further oxidised, produces melanin; dihydroxyphenylalanine may thus be accepted as an intermediate product in melanin formation (see Table I). Unlike tyrosine, dihydroxyphenylalanine can be converted into melanin in the epidermis, and for this reason—"dopa,"

as it is now called, has been employed to test the pigment-forming ability of epidermal cells. In mammals these pigment-forming cells are situated in the lowermost part of the epidermis, the Malpighian layer, and with suitable staining will give a precise and specific reaction. As yet it has only been possible in a few instances to obtain extracts containing those specific oxidases concerned in melanin formation, and little is known concerning them. It seems, however, that the oxidase which transforms tyrosine to dihydroxyphenylalanine is distinct and different from that which converts dopa into melanin and must occur elsewhere than in the epidermal cells, for tyrosine must undergo the preliminary change before it reaches the epidermal tissues, the cells of which are powerless to transform tyrosine directly into melanin.

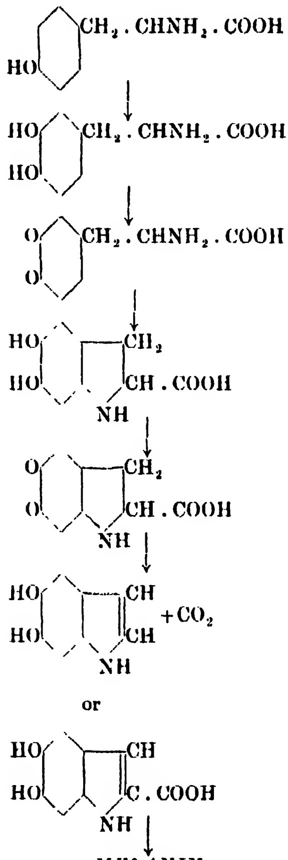
The steps in the production of melanin from its source (tyrosine) can be best illustrated by reference to Table I, where the successive stages in melanogenesis are represented by chemical formulæ from Pryde (1928).

Pigment is deposited either in the form of discrete granules or in a diffuse state, and commonly, although not invariably, the two types occur together. The view has been expressed many times that three distinct pigments exist—red, yellow, and brown; or yellow, brown, and black (Little, 1913)—but in certain *in vitro* experiments with dopa it was possible to produce every shade of colour from red and yellow to brown and black, and this would suggest that further research may fail to demonstrate the existence of more than one pigment. This melanin pigment, however, may vary considerably in its colour expression, depending on chemical and physical factors. When distinct colours such as red, brown, or black, which characterise the coat of certain breeds of cattle, are considered, the difference between each is striking and unmistakable, but if these same colours be examined microscopically the fact emerges that each is fundamentally identical with the other. The individual pigment granules are similar, and it is their disposition in the fibre cells, their quantity, association with diffuse pigment, and the accomplished degree of oxidation which materially affect the colour that is obvious to the naked eye. It is certain also that the physical structure of the fibre vastly influences the final visible colour.

Criticism has been raised concerning the reality of the existence of only one pigment, since breed colours are apparently so rigid and distinct. The explanation for the maintenance of these distinct colour differences is a simple one; since every colour expression is genetic, the differences in genetic constitution between breeds determines also the extent to which the colour genes shall find expression. By appropriate stimuli an animal

which is normally red could be made to exhibit black coloration, and so on. This experimental interference naturally can have only an extremely limited application.

TABLE I.



I. *Tyrosine.*

II. *Dihydroxyphenylalanine* or *dopa.*

III.

IV.

V. *Red Substance.*

VI.

VII. *MELANIN.*

To illustrate the Production of Melanin from Tyrosine.

The precise site of conversion of Tyrosine (I) into Dopa (II) has not yet been discovered, but when the latter has been reduced it is supplied to the melanin-producing terraces Substances III, IV, V, and VI are probably formed within the cells. When substance VI is reached no further enzyme action is necessary. The final product, Melanin, is the result of oxidation of the indole derivative VI. From this it can readily be seen that in melanogenesis the intracellular environment is of great importance.

A deficiency of the oxidising enzyme or of the appropriate conditions for its action offers a better explanation of the restriction of pigment production than does the postulation of inhibitors or anti-inhibitors as specific entities.

Gortner (1911) found that the black pigment in the wool of sheep differed from that of the black rabbit in the quantity of residue after treatment with NaOH, but this seems to have been a quantitative and not

a qualitative difference. According to Wiedmer's (1923) researches light colour in horses is due to the absence of granular pigment and the presence of diffuse pigment. In black-haired horses granular pigment is always greatly in excess of the diffuse type. The same is true in man. Granular pigment is very stable and is known to be acid insoluble, differing greatly in this respect from diffuse pigment, which is soluble in acid.

The disposition and distribution of the pigment granules in fibres are determined primarily by the cellular structure of the particular fibre and by the rate at which the pigment is produced. In lightly pigmented fibres the pigment may be restricted to the medulla, but equally often it is found scattered through the cortex, and in heavily pigmented fibres both cortex and medulla contain pigment.

Hausman (1928) postulates a different form and mode of orientation of granules for the hair of different species, and apparently does not accept the fact that variations in fibre structure are sufficiently responsible for such difference.

The results of a previous study (Boyd, 1931) are in accordance with the observations of Haugg (1926), that colour tones in hair are quantitative and not qualitative, and are due to the arrangement and distribution of the pigment in the hair follicle. The precise colour of pigment is dependent on various factors; observations show that depth of colour is due to variations in quantity and disposition of the pigment which are determined by prevalent chemical factors present in the body during melanogenesis.

Both the fundamental chromogen and the oxidases may be present and yet melanin formation may not result, for in certain cases their interaction is rendered ineffective or impossible through the presence of inhibitors. The experiments with dopa suggest that an inhibitor is not necessarily a specific substance, and that the effects attributed to it are due to quantitative and qualitative differences in the constituents of the pigmentary system. Przibram (1921) is inclined to this view.

Melanin in tissue may exhibit every shade of colour from palest buff to most intense black. Genetic black is the most stable form of pigmentation, red the least; and the latter shows the greatest susceptibility to dilution factors. Every gradation between colourlessness and intense pigmentation may be found, and in wool fibres whiteness may be due to—

1. Absence of melanin.
2. Insufficiency of melanin.
3. Presence of "white" melanin.
4. Physical properties concerned with the structure of the fibre by means of which light rays are reflected.

The theory that white melanin was present in white horse hair was not confirmed by Gortner's experiments. White granular pigment has, however, been demonstrated in the medullary cells of certain white hairs from sheep, and these have since been observed in the white hairs of certain rabbits, being specially obvious after staining with Nissl's methyl blue. This white melanin may be the result of over-oxidation. A comparable phenomenon is seen in the bleaching or weathering which occurs in the tips of certain dark wools and in the coats of certain cattle. In these cases the lighter colour is due partly to pigment disintegration initiated by exposure to excessive light and partly to continued oxidation through the agency of the free oxygen of the air.

The natural bright brown or fawn coloration which is sometimes seen in the exposed portions of the fleeces of black sheep can be reproduced exactly by the application of hydrogen peroxide to the intensely pigmented portions of black fibres.

It is necessary to distinguish between this fading effect of the tips of exposed fibres and the occurrence of what are frequently called "agouti-tipped" fibres growing on the hair-bearing parts of black sheep—*e.g.* face, legs, etc.; in these agouti fibres the golden-red pigmentation of the tip is constant from the outset. It is succeeded by intense black pigmentation lower down on the shaft of each fibre, which resists weathering and oxidation and never loses its jet-black appearance.

The seasonal whitening which takes place in such animals as *Lepus Americanus* has been ascribed to an acute and complete bleaching of the pigment granules themselves (Hadwen, 1929) and not to the shedding of the coat as a whole. Atmospheric oxygen is powerless to produce such an exaggerated change, and since there is no known connection between the living body-cells and the wholly dead keratinized cells of the fibres, it is difficult to reconcile these facts with the theory. It is more probable that the whitening is due to the gradual replacement of the old coloured coat by a new white one without mass shedding of the coloured coat as a whole. Metchnikoff's belief in the reality of pigmentophagic cells which in greying individuals wandered up the medulla of the fibre and ingested the cortical pigment granules, thus leaving the fibre colourless, is now only historically interesting. His illustrations of pigmentophagic cells bear a startling likeness to certain types of ordinary commonly occurring pigment cells known as dendritic cells. It is only whilst melanin is in the living cell that it may be influenced by physiological conditions of the body, and melanin deposited in fibres is no longer controlled by body conditions, so that it is not possible to accept the suggestion that

overnight greying or sudden whitening of hair due to nerve shock does occur.

The precise origin of pigment granules is still a much discussed question. Their intracellular origin is undoubted, but beyond this no satisfactory hypothesis has yet been advanced. Recent tissue culture work (Koller, 1929) suggests that pigment granules arise *de novo* in the cytoplasm, other workers believe that mitochondria are transformed into granules, and a third school of workers supports the theory of the nuclear origin of pigment. No one has yet succeeded in satisfactorily demonstrating the rôle of the nucleus in melanin information, and whilst the results of Ludford's (1924) research on the melanotic sarcoma of the horse definitely supports this theory, Miescher's (1923) work on the eye of the chick, rabbit, and guinea-pig as definitely refutes it. Jeliaskowa-Paspalewa (1930) affirms that in the eyes of sheep, fowls, frogs, and toads pigment is nuclear in its origin, and even states that during mitosis whole chromosomes may become melanised. Makarov (1929) puts forward an interesting theory concerning the rôle of the chondriosomes in pigment formation. He states that the chondriosomes function as pro-pigment nuclei, becoming steeped in the basophilic mother substance of pigment. They are supposed to act on substances entering the cell, and he states that neither the Golgi apparatus nor the cell nucleus nor the nuclear bodies take any part in pigment formation. Observations on the epidermal pigment of sheep suggest that there is some close connection between melanin formation and the nucleus. Very often the nucleus of a cell is practically obscured by pigment granules, which cover it in such a way as to form a double cap. These nuclear caps, single or double, have been recorded in man (Percival and Stewart, 1930), cattle (Esskuchen, 1927), pigs (Teodoreanu, 1930), horses (Ludford, 1924), and we have recently observed them in many breeds of sheep. This disposition round the nucleus is certainly not fortuitous, and that this pigment has a protective function is certain. It is also possible that the aggregations indicate the site of origin. That is to say, the melanin is nuclear in origin in that the oxidases which convert the dihydroxyphenylalanine of the cytoplasm into melanin are elaborated by the nucleus itself. It is important to note that pigment production is always preceded by cellular activity, which ultimately depends on nuclear activity. Sewall Wright, in 1917, suggested that the oxidising agents which produced pigment were secreted by the nucleus, and to-day it seems that this is possibly true. Equally probable is the suggestion that nuclear activity may control the intracellular environment by a quantitative and qualitative discharge into the cytoplasm of such substances as

would act as inhibitors, or intensifiers, of pigment production. Sewall Wright thought that it was the oxidase which determined the precise colour, but *in vitro* experimentation has shown that this is not the case, and that colour is determined by those factors governing the interaction of the chromogen and the oxidases. The question then arises, Why is it that the nucleus of one cell may behave quite differently from that of its neighbour? Only a difference in genetic constitution can account for these variations. From the analytical studies carried out on the skin of variously coloured sheep it seems that some cells are more potentially pigment-producing than others; and just as there is in animals every grade between albinism and full self-colour, so is there every grade between the cell which cannot produce pigment and the cell which produces it to a maximum degree.

The Dopa Reaction.

The "Dopa" reaction was first described by Bloch (1917). He showed that when sections of skin were left in contact with 3-4-dihydroxyphenyl-alanine (dopa), melanin was produced in the epidermal cells. This melanin was distinct from and uninfluenced by already existing melanin present in the skin or its associated fibres. The use of the reaction may be looked upon as a method of revealing the maximum amount of pigment which the skin is capable of producing, or of causing the epidermal pigment cells to realise their full pigment potentiality. The reaction always took place in those parts of the skin which were normally pigmented (*i.e.* Malpighian layer), and was greater in heavily pigmented skin than in lightly pigmented skin. He concluded from this that any cell which produced melanin necessarily contained an oxidase which was capable of oxidising dopa, and which was specific in its action. This he called *dopa oxidase*. Percival and Stewart (1930) have reviewed all the recent investigations on melanogenesis and have given a concise account of the present position of the dopa oxidase theory. Our acceptance of dopa as a suitable substrate in melanin formation has been the basis of the following research. The work of Roberts (1924–1930) and Wassin (1928) on the genetics of colour inheritance in the sheep suggested to us that the use of a chemical test in genetic analysis would probably yield interesting results, and with this end in view an investigation of the dopa reaction in the skin of the sheep was undertaken.

Several research workers have used various skin extracts to demonstrate the presence or absence of oxidising enzymes and the existence of anti-enzymes or inhibitors. Young animals were always used, since it was apparently impossible to extract any active enzymes from the skin of

older animals. A search of the literature has not revealed any work of this nature in sheep, and it was thought that in the initial stages at least a study of the dopa reaction in skin sections would provide more information than would the skin extract method.

Findings with Dopa in Sections of Lamb and Adult Sheep Skin.

The material selected was classified strictly according to the age and colour of the sheep.

The dopa reaction technique is itself very simple, but the water used in preparing the dopa solution must be carefully distilled and the pH accurately adjusted (Percival and Stewart (1930) describe this method of preparation in detail). Fresh skin samples are fixed in 5 per cent. neutral formalin for four or five hours and then frozen sections are cut. These sections are then placed in dopa solution (1 mg. dopa to 1 c.c. prepared water; pH 7·34) and incubated for three hours at 37° C. or for twenty-four hours at room temperature. Experience showed that eighteen hours at room temperature gave the optimum results, and in our work with sections of the skin of the sheep this procedure was adopted.

The following two series are illustrative of the type of material used in the preparation of these sections. The genetic nature of the colour characterisation was previously known, since each of the sheep had been used or bred in connection with colour inheritance experiments.

Series I. Samples from lambs not more than two or three days old.

1. Recessive brown.
2. Recessive black.
3. Badger-face—white portion.
4. Badger-face—black portion.
5. Reversed badger-face—black portion.
6. Reversed badger-face—white portion.
7. White spot from black lamb—dominant white.
8. Dominant black.

Series II. Samples from adult sheep.

1. Recessive brown.
2. Heterozygous white (white × brown, F_1).
3. Recessive black.
4. Black with isolated white spot (pattern).
5. Reversed badger-face (wool showing banding).
6. Dominant white.
7. Brown and white area from face of Gritstone × Cheviot.
8. Dominant white (aged).

Dopa staining does not in any way affect the formed pigment in fibres, but it reveals those cells which are potentially pigment-producing and which in the untreated state are indistinguishable from any other cells of the Malpighian or basal layer. These dopa-positive cells are mainly confined to the epidermal layers of the skin, the fibre sheath and hair bulbs, but may appear in the dermis. In one instance (Series II, 5, in sections of a skin sample taken from a white area on the belly) dopa-positive melanoblasts were recorded in the dermis. The dendritic form of the dopa-positive cells occurs most frequently in pigmented skin which has been specially stimulated (*e.g.* by X-rays, sunlight, etc.), but is often seen naturally in fibre sheaths.

The branched fusiform pigmented cells of the dermis which do not give a dopa-positive reaction are chromatophores containing ingested pigment.

The following notes describe the microscopic findings in the various skin sections treated by dopa under identical conditions:—

Series I. Lambs.

1. *Recessive brown.*—Bright brown pigmented fibres; a chain of small regular dopa-positive cells in epidermis; minute brown granules throughout epidermis; dendritic cells in fibre sheaths (see Plate IV (*b*)).

2. *Recessive black.*—Heavily pigmented black fibres; dopa-positive and dendritic cells in Malpighian layer (Plate I (*a*)); brown granular pigment in epidermis (Plate I (*b*)); dendritic cells in fibre sheaths (Plate II (*c*)).

3. *Badger-face—white portion.*—Fibres colourless; practically continuous chain of contiguous dopa-positive basal cells in the epidermis; deposition of minute granules of brown pigment throughout epidermis; dendritic cells bearing large well-marked dendrites, especially in the fibre sheaths (Plate II (*d*)).

4. *Badger-face—black portion.*—Fibres densely black; numerous dopa-positive cells in epidermis irregularly arranged; many dendritic dopa-positive basal cells; minute granules in epidermis fewer than in the white areas of the same sheep (Plate III (*a*) and (*b*)).

5. *Reversed badger-face—black portion.*—Evenly pigmented fibres; numerous clean-cut basal dopa-positive cells; well-defined dendritic cells in certain follicle sheaths.

6. *Reversed badger-face—white portion.*—Most fibres colourless, but a few isolated pigmented ones; numerous well-formed basal dopa-positive cells; pigment production very restricted round certain follicles; dendritic cells very well defined, especially in fibre sheaths.

7. *Dominant white.*—(Spot from black lamb.) Complete absence of any reaction (Plate IV (*a*)).

8. *Dominant black.*—Fibres intensely pigmented; many small regularly shaped dopa-positive cells throughout Malpighian layer and follicle sheaths; isolated dendritic cells in follicle sheaths.

Series II. Adult Sheep.

1. *Recessive brown.*—Brown pigment granules in epidermal cells; no basal dopa-positive cells in epidermis: a few dopa-positive cells seen in cross-sections of follicle sheaths.

2. *Heterozygous white.*—Sections diffusely stained with yellowish brown; few dopa-positive basal cells visible.

3. *Recessive black.*—Some heavily pigmented black fibres and others colourless; small dopa-positive basal cells in epidermis and follicle sheaths; bulb cells of colourless fibres exhibit pigment formation.

4. *Black, with isolated white spot.*—Shafts of black fibres very dark; white fibres showing small amount of pigmentation: brown pigment in bulbs; a very few minute dopa-positive cells in epidermis.

5. *Reversed badger-face.*—This sheep showed colour-banding in its wool, the physical characters of which have been described elsewhere (Boyd, 1930). Bright brown fibres and occasional grey ones; few small dopa-positive cells in epidermis and follicle sheaths; aggregation of melanoblasts in dermis.

6. *Dominant white.*—No reaction beyond a general diffuse browning.

7. *Brown and white area from face of Gritstone × Cheviot.*—Diffuse brown staining restricted to epidermis: fibres white with pigment streaks: bulbs very black (*i.e.* intensive melanin formation); few minute dopa-positive cells in epidermis.

8. *Dominant white.*—No dopa-positive cells and absence of even a slight general dopa reaction.

A comparison between the results obtained with lambs and with adults illustrates the decreasing capacity of the tissues to respond to the dopa reaction. This is correlated with increasing age. In no case was the number of dopa-positive cells in the adult equal to that in the lamb, and in the former dendritic cells were never recorded, whilst they occurred frequently in the lamb. That the decreased dopa reaction was not due to the action of inhibitors was shown by the presence of heavily pigmented fibres in skin sections which gave only a weak reaction. This indicated that in life oxidase production was restricted to the cells of the bulb of the fibre. Apparently restriction in the production of dopa oxidase is a

natural accompaniment of age, and for this reason sections from lambs' skins are the more suitable for use in pigment studies. The presence of minute pigment granules throughout the epidermal layers in the lamb-skin sections is also indicative of the greater melanogenic power of the young skin. This pigment disappears in the form of dust during the process of desquamation characteristic of all epidermal tissues.

The appearance of dopa-positive cells in white portions of sections from badger-face skins would seem to indicate an inability on the part of the cells of the white area to make use of dopa in circulation, which, however, respond readily in a passive environment. The absence of any reaction in dominant white skin would indicate the presence of an inhibitor. Confirmation of this hypothesis will be sought when it is practicable to carry out skin extract experiments.

Beyond the differences in disposition and morphology of the dopa-positive basal cells and dendritic cells it is as yet difficult to define the distinction between colours. Recessiveness or dominance probably depends on the relative strength of the oxidases involved, and recent work on plants (Moncrieff, 1931) lends colour to this story.

In order to offer every opportunity for a dopa reaction in the adult skin series, sections were kept in the dopa solution for a much longer period of time than is normally required—thirty-six hours. This did not in any way influence the results. Small quantities of H_2O_2 (1 : 100) were added to certain sections, but again the reaction was not affected. Other sections, in an endeavour to destroy possible inhibiting enzymes, were boiled before staining, but there was no apparent change in the resultant reaction.

In Series I (lamb skins) only No. 8, the dominant black sections, were exposed to similar treatment. Neither after boiling the sections nor on the addition of H_2O_2 to the sections was there any change in the reaction. This experiment requires further confirmation and extension, for the result was not in accordance with that expected and quoted by other workers.

Experimental *in vitro* Standardisation of the Reaction.*

The results obtained by using the dopa reaction on skin sections suggested the desirability of attempting to define some of the factors controlling the reaction, and accordingly a series of *in vitro* experiments was planned. The selection of suitable substitutes was necessarily very restricted, and for this reason admittedly not beyond criticism. It is

* Carried out in collaboration with Dr C. I. B. Voge.

known that light, temperature, and pH variation may each influence the dopa reaction, but the extent of such influence has not been defined. The following is an outline account of some of the preliminary experiments in this series.

The following reagents were employed except when otherwise stated:—

Dopa (dioxyphenylalanine), 1 mg. per c.c. in pure distilled H_2O (substrate); *Hydrogen Peroxide*, 1 in 200 solution; *Phosphate buffer solutions*, adjusted to between pH 7·3 – 7·4; *Distilled water*, all the reagents were made up in distilled water specially prepared after the method described by Percival and Stewart (1930), using Jena glass apparatus. *Toluol* was added to the surface of the contents of most of the tubes to prevent the action of atmospheric oxygen.

The method of experimentation by which the pH as well as the concentration of dopa was varied is described by Kermack and Voge (1924–1925). The practice followed in setting up the experiments was to place the required quantity of dopa solution in a clean test-tube, to pour the peroxide very gently over the surface, and then to add toluol. Finally, the tubes were plugged with cotton-wool. Readings were taken at twenty-five, one hundred, and one hundred and eighty hours after setting up each series.

Each experiment was suitably controlled, and Ridgway's *Nomenclature of Colors* (1886) provided the colour standards for comparison, and his terminology is used throughout.

Experiment I.—By adding to some tubes a few c.cs. of concentrated hydrochloric acid and to others the same number of c.cs. of concentrated solution of sodium hydroxide, it was found that the acid inhibited while the alkali intensified the colour reaction.

Experiment II.—Concentrations of H_2O_2 greater than 1 : 200 inhibited a colour reaction, while dilutions less than this amount were unable to provoke a full response. The optimum concentration of H_2O_2, therefore, appeared to be 1 : 200.

Experiment III.—By using the same quantities of the reagents in test-tubes of varying diameters it was found, under otherwise constant conditions, that in those tubes with the greatest diameter, where a consequently larger area was exposed to the air, the colour reaction was obtained earlier than in narrow tubes.

Experiment IV.—The effect of varying the temperature, with other conditions constant, showed that at 0° C. no reaction took place, while at 55° C. the colour change occurred more rapidly than at room temperature. It was found that at the former of these temperatures the reaction could

still take place when the tubes were subsequently warmed to room temperature. This is of importance, since skin sections for dopa-staining may be preserved at 0° C. for considerable periods of time and subsequently yield a satisfactory colour reaction.

Experiment V.—By varying the concentration of dopa and altering the pH of the solution, keeping concentration and volume of hydrogen peroxide constant, results were obtained which are best indicated by reference to Table II.

TABLE II.

Experiment V.—Effect of variation of pH and Dopa concentrations on the production of pigment. The concentration and volume of Hydrogen Peroxide and the volume of Dopa are kept constant.

pH.	Time of Reading.	Concentration of Dopa.				
		–	+ –	+	+ +	+ + +
	Hrs.					
3	25	Colourless	Cream colour	Cream buff	Buff	Vinaceous cinnamon
	100	,,	Vinaceous buff	Fawn colour	Tawny ochraceous	Mars brown
	180	,,	Ochraceous buff	Vinaceous cinnamon	Mummy brown	Clove brown
5	25	,,	Pinkish buff	Ochraceous buff	Tawny ochraceous	Mars brown
	100	,,	Straw	Pinkish buff	Mummy brown	Slate grey
	180	,,	,,	Ochraceous buff	Tawny ochraceous	Mummy brown
7·4	25	,,	Tawny ochraceous	Mars brown	Slate grey	Black
	100	,,	Ochraceous buff	Slate grey	Clove brown	Vandyke brown
	180	,,	,, ,,	Tawny ochraceous	Mars brown	,, ,,
8·0	25	,,	Tawny ochraceous	Mars brown	Clove brown	Black
	100	,,	Ochraceous buff	Raw sienna	Mars brown	,,
	180	,,	,, ,,	Vinaceous buff	Russet	Seal brown
9·0	25	,,	Tawny ochraceous	Mars brown	Clove brown	Black
	100	,,	Straw	Tawny ochraceous	Seal brown	,,
	180	,,	Pale straw (·)	Ochraceous buff	Tawny ochraceous	,,

Experiment VI.—In another series the concentration of dopa and the volume of hydrogen peroxide were varied, the pH and the volume of dopa being constant. The results obtained are shown in Table III.

A summary of the results produced by these *in vitro* experiments shows that: (1) the pigmentation increases with concentration of dopa irrespective of the volume or source of oxygen; (2) depth of pigmentation bears a close relation to the volume of peroxide present; (3) the optimum pH for pigment deposition appears to be 7·4; (4) temperature influences colour formation in that at 0° C. it is inhibited, while at 55° C. the reaction was more rapid than at room temperature; (5) dopa in

presence of excess NaOH gives an intensified colour reaction, while excess HCl inhibits the reaction.

From these preliminary *in vitro* experiments emerge suggestions which lend some emphasis to the importance of the chemical environment at the site of melanin deposition. It was found that a wide range of colours could be produced by different chemical and physical variations, and although no suggestions are here offered as to the individual influence

TABLE III.

Experiment VI.—Effect of variation of Dopa concentration and volume of Hydrogen Peroxide on Pigment production. The concentration of Hydrogen Peroxide and the pH are kept constant and the volume of Dopa does not vary.

Volume of H_2O_2.	Time of Reading.	–	+ –	+	+ +	+ + + +
0 c.c.	Hrs. 25	Colourless	Colourless	Colourless	Colourless	Colourless
	100	,,	,,	,,	,,	,,
	180	,,	,,	,,	,,	,,
1·0 c.c.	25	,,	Pinkish buff	Tawny ochraceous	Mummy brown	Vandyke brown
	100	,,	Straw	Mars brown	Olive	Clove brown
	180	,,	,,	Raw sienna	Clove brown	Black
2·0 c.c.	25	,,	Vinaceous buff	Prouts brown	Mummy brown	Clove brown
	100	,,	Mummy brown	Olive	Clove brown	Black
	180	,,	Vinaceous buff	Fawn colour	Mars brown	Vandyke brown
4·0 c.c.	25	,,	Ochraceous buff	Mummy brown	Slate grey	Clove brown
	100	,,	Straw	Tawny ochraceous	Mummy brown	Olive
	180	,,	Paler straw	,, ,,	Tawny olive	Mars brown
10·0 c.c.	25	,,	Vinaceous buff	Mars brown	Mummy brown	Clove brown
	100	,,	Palest straw	Tawny ochraceous	Tawny olive	Vandyke brown
	180	,,	Colourless	Straw	Vinaceous buff	,, ,,

exerted by any one variant, it seems desirable to suggest that by further work along these lines it may be possible to define those chemical and physical optima which determine this or that shade of colour. It may eventually be possible to correlate the colour expression and the various degrees of pigmentation found in the animal kingdom with differential levels of metabolism, with the physical and chemical environment of dopa-positive cells in the skin, and with, what is of probably greater importance, the mode of deposition and disposition of the melanin in the cutaneous tissues.

One further observation seems worthy of mention. At a temperature of 0° C. pigment deposition was inhibited. It has not been determined at what level of temperature the threshold for colour formation *in vitro*

exists, but if this should be within the range of the seasonal variation of skin temperatures of hiberalbic animals, it may help to throw some light upon the occurrence of seasonal colour changes in them.

Summary.

1. A description of the mammalian pigmentary system is given together with an outline of the mechanism of melanin production and those factors which influence or control it.

2. The production of melanin by chemicophysical interaction between circulating 3-4-dihydroxyphenylalanine (dopa) in the blood-stream and an enzyme or enzymes of intracellular origin is accepted, but it is emphasised that in the interaction there are numerous opportunities for physiological interference which inhibit melanin formation.

3. The current theories of the origin of melanin are discussed and evidence supporting the nuclear origin of oxidising enzymes is given.

4. It is suggested that in the sheep some epidermal cells are more potentially pigment-producing than others, and that a gradation exists between the cell which cannot produce pigment and that which produces it to a maximum degree.

5. It is postulated that only one pigment exists, and that all animal colours are the result of a modification of this one pigment.

6. The results of staining sections of lamb and adult sheep skin with dopa are given.

7. A series of *in vitro* experiments planned to standardise the dopa reaction is described.

8. It is suggested that the dopa reaction may be used as a simple chemical test in genetic analysis whereby the pigmentogenic potentialities of a particular sheep may be analysed.

Acknowledgments.

I wish to express my thanks to Professor F. A. E. Crew, Director of the Institute of Animal Genetics, University of Edinburgh, for the scientific hospitality I have received in his Department.

Mr Wm. C. Miller has been extremely kind in giving me advice and encouragement throughout the course of this work, and I am greatly indebted to him and to Mr J. A. Fraser Roberts for the material they so kindly provided for this study.

BIBLIOGRAPHY.

BLOCH, B., 1917. "Chemische Untersuchungen über das spezifische pigmentbildende Fermente der Haut, die Dopaoxydase," *Ztschr. f. physiol. Chemie*, **98**.

BLOCH, B., 1917. "Das Problem der Pigmentbildung," *Arch. f. Dermat. u. Syphil.*, **145**, pp. 130-208.

BOYD, E., 1930. "Colour Banding in Fleeces," *Journ. Text. Instit.*, **21**, 6.

BOYD, E., 1931. *Pigmentation in Fibres of the Suffolk Breed of Sheep.* (*In Press.*)

ESSKUCHEN, E., 1927. "Über die Entstehung der Farbung der Haussaugetiere," *Zuchtungskunde*, **2**, Heft 27.

GORTNER, R. A., 1911. "Studies on Melanin," *Journ. Biol. Chem.* **10**, p. 113.

HADWEN, S., 1929. "Colour Changes in *Lepus Americanus* and other Animals," *Canad. Journ. of Res.*, **1**, 2, pp. 189–200.

HAUGO, R., 1926. "Über das Pigment des einfarbig graubraunen Gebirgsviehes," *Zuchtungskunde*, Band **1**.

HAUSMAN, L. A., 1928. "The Pigment Granules of Human Head Hair: A Comparative Racial Study," *Amer. Journ. Phys. Anthrop.*, **12**, 2, pp. 273–293.

JELIASKOWA-PASPALEWA, A., 1930. "Cytologische Untersuchungen uber die Entstehung des melanotischen Pigments," *Z. Zool.*, **137**, pp. 365–402.

KERMACK, W. O., and C. I. B. VOGE, 1924–1925. "The Action of Salts with Multivalent Cations on Colloidal Solutions of Gold and Gum Benzoin," *Proc. Roy. Soc. Edin.*, **45**, pp. 90–101.

KOLLER, P. C., 1929. "Experimental Studies on Pigment Formation," *Archiv für experimentelle Zellforschung*, **8**, pp. 490–498.

KOLLER, P. C., 1930. "On Pigment Formation in the D-Black Rabbit," *Journ. of Genet.*, **22**, 1, pp. 103–107.

LITTLE, C. C., 1913. "Inheritance of Colour in Mice," *Carnegie Institution*, No. 179, pp. 17–102.

LUDFORD, R. J., 1924. "Nuclear Activity during Melanosis: With Special Reference to Melanin Formation in a Melanotic Sarcoma," *Journ. Roy. Micr. Soc.*, pp. 13–28.

MAKAROV, P., 1929. "Studien über die Pigmentogenese. Über den Anteil der Mitochondrien an der Bildung des Melanins in den Zellen des Pigmentsepithels der Retina," *Russk. Arch. Anat.*, **8**, pp. 255–278. [In Russian.]

MIESCHER, G., 1923. "Die Pigmentgenese im Auge nebst Bemerkungen über die Natur des Pigmentkorns," *Arch. mikr. Anat.*, **97**, pp. 326–369.

MONCRIEFF, R. SCOTT, 1931. "The Chemical Effect of a Mendelian Factor for Flower Colour," *Nature*, **127**, No. 3217, p. 974.

PERCIVAL, G. H., and C. P. STEWART, 1930. "Melanogenesis: A Review," *Edin. Med. Journ.*, Sept., pp. 497–523.

PRYDE, J., 1928. *Recent Advances in Biochemistry.* London: J. & A. Churchill. 8vo. Pp. x + 379.

PRZIBRAM, H. (Unter Mitwirkung von DEMBOWSKI, J., und L. BRECHER), 1921. "Einwirkung der Tyrosinase auf Dopa—Zugleich: Uraschen tierischer Farbkleidung. IV," *Arch. f. Entw.-Mech.*, **48**, pp. 140–165.

Raper, H. S., 1926. "The Tyrosinase-Tyrosine Reaction. Production of Dihydroxyphenylalanine from Tyrosine," *Biochem. Journ.*, **21**, p. 735.

Ridgway, Robert, 1886. *Nomenclature of Colors.* Boston: Little, Brown & Company.

Roberts, J. A. Fraser, 1924. "Colour Inheritance in Sheep. I.—Black Colour and Badger-face Pattern in Welsh Mountain Sheep," *Journ. of Genet.*, **14**, pp. 367–374.

Roberts, J. A. Fraser, 1926. "Colour Inheritance in Sheep. II.—The Piebald Pattern of the Piebald Breed," *Journ. of Genet.*, **17**, pp. 77–84.

Roberts, J. A. Fraser, 1926. "Piebald (Spanish) Sheep and their Crosses with other Breeds," *Welsh Journ. of Agric.*, **2**, pp. 66–69.

Roberts, J. A. Fraser, 1927. "The Inheritance of Some Colours and Patterns in Sheep," *Journ. Text. Inst.*, **18**.

Roberts, J. A. Fraser, 1928. "Colour Inheritance in Sheep. III.—Face and Leg Colour," *Journ. of Genet.*, **19**, pp. 261–268.

Roberts, J. A. Fraser, and R. G. White, 1930. "Colour Inheritance in Sheep. IV.—White Colour, Recessive Black Colour, Recessive Brown Colour, Badger-face Pattern, and Reversed Badger-face Pattern," *Journ. of Genet.*, **22**, pp. 165–180.

Roberts, J. A. Fraser, and R. G. White, 1930. "Colour Inheritance in Sheep. V.—Dominant Black," *Journ. of Genet.*, **22**, pp. 181–190.

Teodoreanu, N. I., 1930. "Histo-Ethnologische Untersuchungen über die Struktur der Haut bei Mangalicza—und Lincolnshire—Schweinen," *Bull. Sect. Sci. Acad. Rouman.*, **13**, 19 pp.

Wassin, B., 1928. "The Genetics of the Sheep I.—Inheritance of Colour and Pattern," *Trans. Centr. Stat. Genet. Farm Animals*, No. 2, p. 67.

Wright, Sewall, 1917. "Color Inheritance in Mammals," *Journ. of Hered.*, **8**, pp. 224–235.

Wright, Sewall, 1917. "Color Inheritance in Mammals. II.—The Mouse," *Journ. of Hered.*, **8**, pp. 373–378.

Wright, Sewall, 1917. "Color Inheritance in Mammals. VI.—Cattle," *Journ. of Hered.*, **8**, pp. 521–527.

Wright, Sewall, 1917. "Color Inheritance in Mammals. VII.—The Horse," *Journ. of Hered.*, **8**, pp. 561–564.

Wright, Sewall, 1918. "Color Inheritance in Mammals. XI.—Man," *Journ. of Hered.*, **20**, pp. 227–240.

DESCRIPTION OF PLATES.

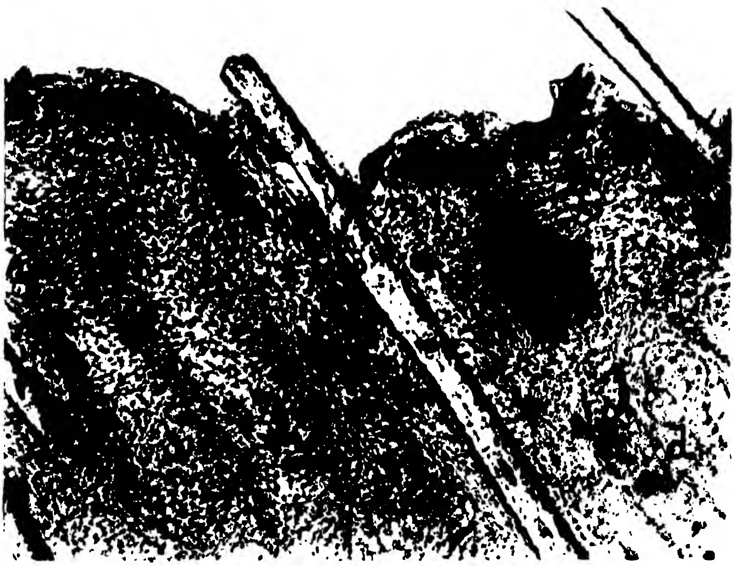
The sections were all treated as described in the text (page 225) by the Dopa technique.

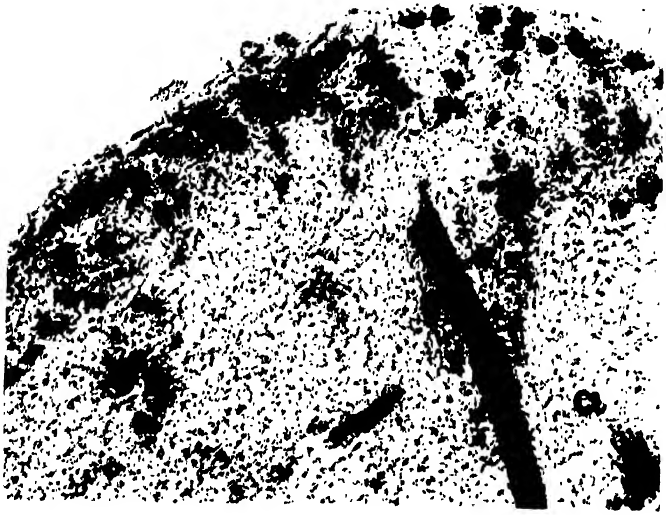
Plate I (*a*).—Microphotograph of section of Recessive black lamb's skin, 20 μ thick, × 100. Note the layer of dopa-positive cells in the Malpighian layer of the epidermis. Sections of fibres show massed preformed melanin, while dopa-positive cells are also seen in the root sheaths.

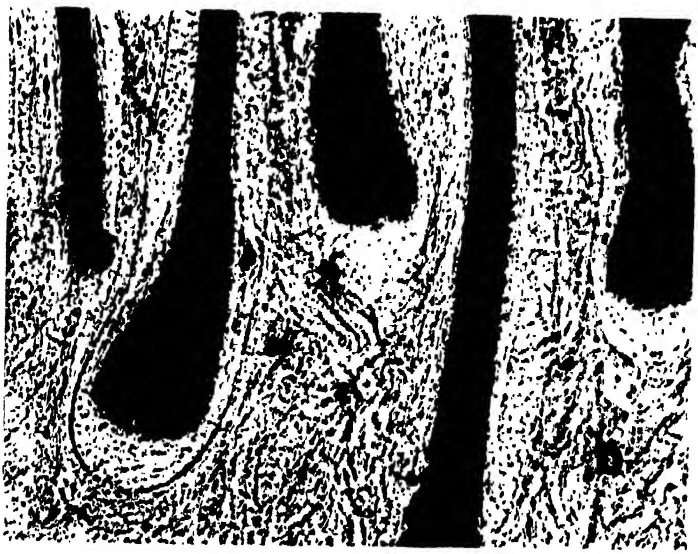

a


b


c.








a.

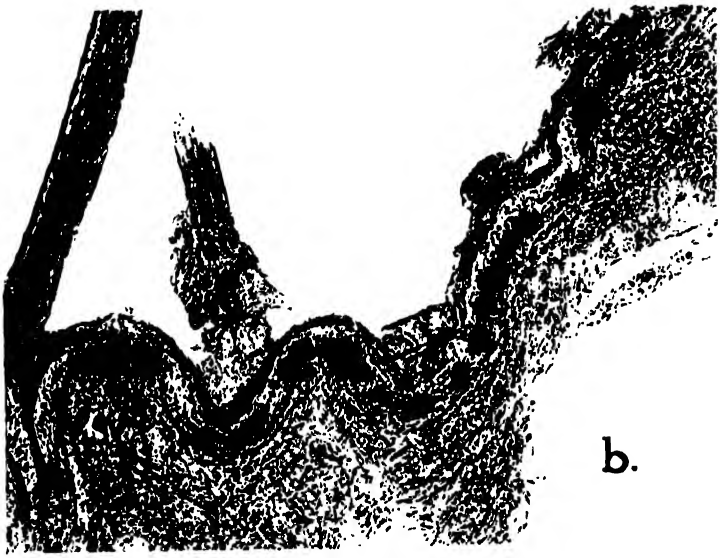

b.

PLATE I (*b*).—Showing a higher power view of the small area indicated in (*a*) × 1000. Granular pigment can be seen in the superficial layers of the epidermis.

PLATE II (*c*).—Showing a high-power (× 1000) view of a root sheath from Plate I (*a*)—Recessive black. Dendritic dopa-positive cells can be seen.

PLATE II (*d*).—Section from skin of white area of a Badger-faced lamb, × 100. Wool fibres are macroscopically colourless, but show a few pigmented cells microscopically. The reaction is markedly positive, numerous dopa-positive cells being visible in epidermis and root sheaths. (Compare with Plate IV (*a*).)

PLATE III (*a*).—Section from skin of black area of a Badger-faced lamb, × 100. Dopa-positive cells are numerous in the epidermis, but show no regularity in arrangement. No granular pigment was seen. Wool fibres show densely massed preformed melanin.

PLATE III (*b*).—Deeper field than shown in Plate III (*a*) from same animal. The fibre bulbs show a few dendritic dopa-positive cells.

PLATE IV (*a*).—Section from skin of a Dominant white lamb (homozygous), × 100. Absence of any reaction. (Compare with Plate II (*d*).)

PLATE IV (*b*).—Section from skin of a Recessive brown lamb. The wool fibres contain golden-brown granular pigment (not well shown in the photograph), while there is a single chain of dopa-positive cells in the Malpighian layer of the epidermis.

[For description of the reaction in other colour-types see text, pages 226 and 227.]

(*Issued separately May* 10, 1932.)

XI.—The Zeta Function of Jacobi. A seven-decimal table of $Z(u|m)$ at interval 0·01 for u and for values of $m(=k^2)$ from 0·1 to 1·0. By L. M. Milne-Thomson, M.A. *Communicated by* Professor Bevan B. Baker, M.A., D.Sc.

(MS. received December 4, 1931. Read May 2, 1932.)

The elliptic integral of the second kind

$$\int t^2[(A_1t^2+B_1)(A_2t^2+B_2)]^{-\frac{1}{2}}dt$$

can, by substitution,* be made to depend upon

$$E(u)=\int_0^u \mathrm{dn}^2\, z\, dz.$$

Writing $E(K)=E$, Jacobi's Zeta function is defined by

$$Z(u|m)=E(u)-uE/K,$$

where

$$m=k^2=1-k'^2=1-m_1.$$

$Z(u)$ is an odd function of u with period $2K$.

$$Z(u|0)=0, \qquad Z(u|1)=\tanh u.$$

Tables of E, K, $\pi/(2K)$, regarded as functions of m, have already been given.†

The tables of $Z(u)$ here given were formed by squaring the values of $\mathrm{dn}\, u$ from a seven-figure table which I had already calculated‡ at interval 0·01, and then integrating from 0 to u by means of the central difference formula §

$$100\int_0^{\frac{r}{100}} f(z)dz=\tfrac{1}{2}f_0+f_1+f_2+\ \dots\ +f_{r-1}+\tfrac{1}{2}f_r-\tfrac{1}{24}(\Delta f_r+\Delta f_{r-1}),$$

since for $f(z)=\mathrm{dn}^2 z$, $\frac{1}{2}(\Delta f_0+\Delta f_{-1})$ is zero and the third difference is negligible for the interval used. The summations made were actually those of $f_0+2f_1+2f_2+\ \cdots\ +2f_{r-1}+f_r$, and were found on an adding machine which printed the totals as new values were added. The sums so formed were multiplied by 0·5 on an electric arithmometer. The corrections for the

* Whittaker and Watson, *Modern Analysis*, 22·73.

† L. M. Milne-Thomson, *Proc. London Math. Soc.*, (2) **33** (1931).

‡ For five-figure tables of $\mathrm{sn}\, u$, $\mathrm{cn}\, u$, $\mathrm{dn}\, u$, see Milne-Thomson, *Die elliptischen Funktionen von Jacobi*, Berlin, Julius Springer, 1931.

§ Whittaker and Robinson, *Calculus of Observations*, ch. vii.

first difference were then applied as each value was obtained and the results, $E(u)$, were written down. With $E(u)$ still on the dials the number uE/K was then subtracted and the value of $Z(u)$ recorded. Both processes were continuous and easy to perform. The resulting tables of $E(u)$ and $Z(u)$ were then differenced and built up from the second difference on the adding machine.

As a check on the accuracy of the calculation, besides an examination of the differences, the value of E was calculated from the resulting table and subtracted from the known value of E. The errors found, in units of the seventh decimal, were as follows:—

m	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
error	+0·2	−0·1	−0·5	+0·1	0·0	+0·3	0·0	−0·2	−0·3

which is a satisfactory agreement.

The tables of $Z(u)$ have their direct application to the evaluation of the elliptic integrals of the second and third kinds.

For the elliptic integral of the second kind we have at once from the definition

$$\int_0^u \mathrm{dn}^2 z\, dz = E(u) = Z(u) + uE/K.$$

For the elliptic integral of the third kind we need

$$\int_0^u \frac{m \,\mathrm{sn}\, a \,\mathrm{cn}\, a \,\mathrm{dn}\, a \,\mathrm{sn}^2 z}{1 - m\,\mathrm{sn}^2 a \,\mathrm{sn}^2 z}\, dz = \Pi(u, a)$$

$$\Pi(u, a) = \tfrac{1}{2} \log \frac{\Theta(u-a)}{\Theta(u+a)} + uZ(a)$$

$$\Theta(u) = 1 - 2q \cos 2x + 2q^4 \cos 4x - \cdots$$

$$x = \pi u/(2K).$$

In practical applications to mathematical physics the parameter a is usually imaginary and the evaluation of $\Theta(u)$ for imaginary values of the argument is necessary. For such cases $\Theta(u)$ does not lend itself readily to compact tabulation and must be found from the q series which converges rapidly. Tables of q have already been calculated.*

For the complete elliptic integral of the third kind we have, however,

$$\Pi(K, a) = KZ(a),$$

so that the evaluation is direct.

For the evaluation of $Z(u)$ for imaginary arguments we can use the addition theorem

$$Z(u+v) = Z(u) + Z(v) - m \,\mathrm{sn}\, u \,\mathrm{sn}\, v \,\mathrm{sn}\,(u+v)$$

---

* L. M. Milne-Thomson, *Journal London Math. Soc.*, **5** (1930), p. 148.

together with Jacobi's imaginary transformation

$$iZ(iu|m) = Z(u|m_1) + u(E/K + E'/K' - 1) - \mathrm{dn}\,(u|m_1)\ \mathrm{sc}\,(u|m_1).$$

The addition theorem can also be used with advantage, when interpolating across the table for intermediate values of m, in those cases where u is large enough to make the differences unmanageable. This will be the case if u exceeds $\frac{1}{2}K$. Thus, for example,

$$Z(u) = Z(u-K) - m\ \mathrm{sn}\,(u-K)\ \mathrm{sn}\ u$$
$$Z(u) = Z(u-\tfrac{1}{2}K) + Z(\tfrac{1}{2}K) - m\,\mathrm{sn}\,(u-\tfrac{1}{2}K)\ \mathrm{sn}\,(\tfrac{1}{2}K)\ \mathrm{sn}\ u.$$

If five-figure tables of elliptic functions are used, the accuracy is limited to five figures.

A short table of E/K is appended.

m	E/K	E'/K'	m_1
0·1	0·9493 416	0·4285 242	0·9
0·2	·8972 125	·5221 013	0·8
0·3	·8433 234	·5982 907	0·7
0·4	·7872 725	·6660 082	0·6
0·5	·7284 733	·7284 733	0·5
m_1	E'/K'	E/K	m

The sign attached to the second difference or given at the head of the column is the actual sign of that difference, as found by giving the proper sign to the tabular entry to which the second difference relates.

[TABLES.

m	0·1		0·2		0·3		0·4	
u	$Z(u)$	Δ'' −	$Z(u)$	Δ'' −	$Z(u)$	Δ'' −	$Z(u)$	Δ'' −
0·00	0·0000 000	0	0·0000 000	0	0·0000 000	0	0·0000 000	0
·01	005 066	3	010 278	4	015 667	7	021 271	7
·02	010 129	3	020 552	8	031 327	11	042 535	17
·03	015 189	7	030 818	12	046 976	18	063 782	23
·04	020 242	7	041 072	16	062 607	25	085 006	33
·05	025 288	11	051 310	19	078 213	29	106 197	39
·06	030 323	11	061 529	25	093 790	36	127 349	48
·07	035 347	15	071 723	28	109 331	42	148 453	56
·08	040 356	15	081 889	31	124 830	48	169 501	64
·09	045 350	18	092 024	37	140 281	53	190 485	71
·10	050 320	20	102 122	39	155 679	60	211 398	79
·11	055 282	22	112 181	43	171 017	65	232 232	88
·12	060 216	24	122 197	48	186 290	71	252 978	94
·13	065 126	25	132 165	51	201 492	77	273 630	102
·14	070 011	28	142 082	56	216 617	82	294 180	111
·15	074 868	29	151 943	58	231 660	89	314 619	116
·16	079 696	32	161 746	63	246 614	94	334 942	126
·17	084 492	33	171 486	67	261 474	99	355 139	133
·18	089 255	35	181 159	69	276 235	106	375 203	139
·19	093 983	38	190 763	75	290 890	109	395 128	146
·20	098 673	38	200 292	76	305 436	117	414 907	156
·21	103 325	40	209 745	82	319 865	121	434 530	160
·22	107 937	44	219 116	85	334 173	126	453 993	168
·23	112 505	43	228 402	88	348 355	133	473 288	176
·24	117 030	46	237 600	92	362 404	136	492 407	182
·25	121 509	49	246 706	94	376 317	142	511 344	188
·26	125 939	48	255 718	99	390 088	148	530 093	196
·27	130 321	52	264 631	102	403 711	152	548 646	201
·28	134 651	53	273 442	105	417 182	156	566 998	209
·29	138 928	54	282 148	109	430 497	163	585 141	215
·30	143 151	56	290 745	111	443 649	166	603 069	220
·31	147 318	58	299 231	116	456 635	171	620 777	227
·32	151 427	60	307 601	117	469 450	176	638 258	233
·33	155 476	60	315 854	121	482 089	180	655 506	238
·34	159 465	63	323 986	123	494 548	184	672 516	245
·35	163 391	64	331 995	128	506 823	190	689 281	251
·36	167 253	65	339 876	130	518 908	193	705 795	253
·37	171 050	67	347 627	132	530 800	197	722 056	262
·38	174 780	68	355 246	136	542 495	201	738 055	266
·39	178 442	70	362 729	138	553 989	205	753 788	272
·40	182 034	71	370 074	140	565 278	210	769 249	275
·41	185 555	72	377 279	144	576 357	212	784 435	281
·42	189 004	74	384 340	145	587 224	217	799 340	285
·43	192 379	75	391 256	149	597 874	220	813 960	291
·44	195 679	76	398 023	151	608 304	223	828 289	294
·45	198 903	78	404 639	152	618 511	227	842 324	299
·46	202 049	78	411 103	156	628 491	230	856 060	302
·47	205 117	80	417 411	157	638 241	233	869 494	308
·48	208 105	81	423 562	160	647 758	236	882 620	310
·49	211 012	82	429 553	161	657 039	239	895 436	314
0·50	0·0213 837	82	0·0435 383	164	0·0666 081	243	0·0907 938	319

m	0·5		0·6		0·7		0·8	
u	$Z(u)$	Δ''	$Z(u)$	Δ''	$Z(u)$	Δ''	$Z(u)$	Δ''
		−		−		−		−
0·00	0·0000 000	0	0·0000 000	0	0·0000 000	0	0·0000 000	0
·01	0027 151	10	0033 397	12	0040 169	15	0047 787	16
·02	0054 292	20	0066 782	23	0080 323	27	0095 558	31
·03	0081 413	30	0100 144	37	0120 450	43	0143 298	49
·04	0108 504	40	0133 469	48	0160 534	55	0190 989	64
·05	0135 555	50	0166 746	59	0200 563	70	0238 616	79
·06	0162 556	59	0199 964	73	0240 522	84	0286 164	96
·07	0189 498	70	0233 109	82	0280 397	97	0333 616	111
·08	0216 370	80	0266 172	97	0320 175	111	0380 957	128
·09	0243 162	89	0299 138	106	0359 842	125	0428 170	141
·10	0269 865	99	0331 998	119	0399 384	139	0475 242	159
·11	0296 469	109	0364 739	130	0438 787	151	0522 155	174
·12	0322 964	117	0397 350	142	0478 039	166	0568 894	188
·13	0349 342	129	0429 819	153	0517 125	178	0615 445	204
·14	0375 591	137	0462 135	165	0556 033	192	0661 792	219
·15	0401 703	147	0494 286	175	0594 749	205	0707 920	233
·16	0427 668	155	0526 262	188	0633 260	217	0753 815	249
·17	0453 478	166	0558 050	197	0671 554	231	0799 461	262
·18	0479 122	174	0589 641	209	0709 617	243	0844 845	278
·19	0504 592	184	0621 023	219	0747 437	255	0889 951	290
·20	0529 878	192	0652 186	230	0785 002	267	0934 767	306
·21	0554 972	200	0683 119	241	0822 300	281	0979 277	318
·22	0579 866	210	0713 811	251	0859 317	291	1023 469	333
·23	0604 550	218	0744 252	260	0896 043	303	1067 328	345
·24	0629 016	227	0774 433	271	0932 466	315	1110 842	359
·25	0653 255	235	0804 343	282	0968 574	327	1153 997	371
·26	0677 259	243	0833 971	289	1004 355	337	1196 781	385
·27	0701 020	251	0863 310	301	1039 799	348	1239 180	395
·28	0724 530	259	0892 348	309	1074 895	360	1281 184	408
·29	0747 781	267	0921 077	318	1109 631	368	1322 780	421
·30	0770 765	275	0949 488	328	1143 999	381	1363 955	431
·31	0793 474	281	0977 571	335	1177 986	389	1404 699	443
·32	0815 902	289	1005 319	346	1211 584	400	1445 000	453
·33	0838 041	297	1032 721	352	1244 782	409	1484 848	465
·34	0859 883	303	1059 771	362	1277 571	418	1524 231	474
·35	0881 422	309	1086 459	369	1309 942	428	1563 140	485
·36	0902 652	318	1112 778	377	1341 885	436	1601 564	495
·37	0923 564	323	1138 720	385	1373 392	445	1639 493	504
·38	0944 153	329	1164 277	392	1404 454	453	1676 918	513
·39	0964 413	337	1189 442	398	1435 063	462	1713 830	522
·40	0984 336	340	1214 209	407	1465 210	468	1750 220	530
·41	1003 919	349	1238 569	412	1494 889	477	1786 080	540
·42	1023 153	352	1262 517	420	1524 091	483	1821 400	547
·43	1042 035	359	1286 045	425	1552 810	492	1856 173	554
·44	1060 558	363	1309 148	431	1581 037	496	1890 392	562
·45	1078 718	370	1331 820	438	1608 768	505	1924 049	569
·46	1096 508	374	1354 054	443	1635 994	510	1957 137	575
·47	1113 924	378	1375 845	448	1662 710	516	1989 650	582
·48	1130 962	383	1397 188	453	1688 910	522	2021 581	589
·49	1147 617	387	1418 078	459	1714 588	527	2052 923	593
0·50	0·1163 885	392	0·1438 509	463	0·1739 739	533	0·2083 672	600

m	0·9		1·0		m	0·1		0·2	
u	$Z(u)$	Δ''	$Z(u)$	Δ''	u	$Z(u)$	Δ''	$Z(u)$	Δ''
		−		−			−		−
0·00	0·0000 000	0	0·0000 000	0	0·50	0·0213 837	82	0·0435 383	164
·01	0057 145	19	0099 997	21	·51	216 580	85	441 049	166
·02	0114 271	35	0199 973	39	·52	219 238	84	446 549	167
·03	0171 362	55	0299 910	60	·53	221 812	87	451 882	168
·04	0228 398	71	0399 787	80	·54	224 299	86	457 047	172
·05	0285 363	90	0499 584	100	·55	226 700	88	462 040	172
·06	0342 238	107	0599 281	119	·56	229 013	89	466 861	175
·07	0399 006	126	0698 859	139	·57	231 237	88	471 507	174
·08	0455 648	140	0798 298	159	·58	233 373	91	475 979	178
·09	0512 150	165	0897 578	178	·59	235 418	91	480 273	178
·10	0568 487	176	0996 680	197	·60	237 372	91	484 389	179
·11	0624 648	194	1095 585	217	·61	239 235	92	488 326	181
·12	0680 615	213	1194 273	235	·62	241 006	93	492 082	182
·13	0736 369	228	1292 726	254	·63	242 684	93	495 656	183
·14	0791 895	247	1390 925	274	·64	244 269	94	499 047	184
·15	0847 174	262	1488 850	290	·65	245 760	95	502 254	185
·16	0902 191	278	1586 485	309	·66	247 156	94	505 276	185
·17	0956 930	296	1683 811	328	·67	248 458	96	508 113	186
·18	1011 373	311	1780 809	345	·68	249 664	95	510 764	188
·19	1065 505	326	1877 462	362	·69	250 775	96	513 227	187
·20	1119 311	343	1973 753	379	·70	251 790	97	515 503	189
·21	1172 774	358	2069 665	396	·71	252 708	96	517 590	188
·22	1225 879	372	2165 181	413	·72	253 530	97	519 489	189
·23	1278 612	387	2260 284	429	·73	254 255	97	521 199	190
·24	1330 958	403	2354 958	445	·74	254 883	98	522 719	189
·25	1382 901	415	2449 187	461	·75	255 413	96	524 050	191
·26	1434 429	430	2542 955	475	·76	255 847	98	525 190	189
·27	1485 527	444	2636 248	490	·77	256 183	98	526 141	190
·28	1536 181	457	2729 051	506	·78	256 421	97	526 902	190
·29	1586 378	470	2821 348	519	·79	256 562	97	527 473	190
·30	1636 105	483	2913 126	533	·80	256 606	98	527 854	190
·31	1685 349	494	3004 371	547	·81	256 552	98	528 045	190
·32	1734 099	508	3095 069	559	·82	256 400	96	528 046	188
·33	1782 341	518	3185 208	573	·83	256 152	98	527 859	190
·34	1830 065	530	3274 774	586	·84	255 806	96	527 482	188
·35	1877 259	542	3363 754	594	·85	255 364	97	526 917	187
·36	1923 911	551	3452 140	609	·86	254 825	96	526 165	188
·37	1970 012	563	3539 917	619	·87	254 190	96	525 225	187
·38	2015 550	572	3627 075	631	·88	253 459	96	524 098	186
·39	2060 516	581	3713 602	639	·89	252 632	95	522 785	185
·40	2104 901	592	3799 490	651	·90	251 710	95	521 287	185
·41	2148 694	599	3884 727	660	·91	250 693	94	519 604	183
·42	2191 888	609	3969 304	668	·92	249 582	94	517 738	183
·43	2234 473	617	4053 213	678	·93	248 377	94	515 689	181
·44	2276 441	624	4136 444	685	·94	247 078	92	513 459	181
·45	2317 785	633	4218 990	694	·95	245 687	92	511 048	179
·46	2358 496	639	4300 842	701	·96	244 204	92	508 458	179
·47	2398 568	645	4381 993	708	·97	242 629	91	505 689	176
·48	2437 995	653	4462 436	715	·98	240 963	91	502 744	176
·49	2476 769	659	4542 164	720	0·99	239 206	89	499 623	175
0·50	0·2514 884	664	0·4621 172	728	1·00	0·0237 360	88	0·0496 327	172

m	0·3		0·4		0·5		0·6	
u	Z(u)	Δ″ −	Z(u)	Δ″ −	Z(u)	Δ″ −	Z(u)	Δ″ −
0·50	0·0666 081	243	0·0907 938	319	0·1103 885	392	0·1438 509	463
·51	674 880	243	0920 121	320	1179 761	396	1458 477	468
·52	683 436	248	0931 984	326	1195 241	399	1477 977	471
·53	691 744	250	0943 521	327	1210 322	403	1497 006	476
·54	699 802	252	0954 731	330	1225 000	407	1515 559	480
·55	707 608	254	0965 611	334	1239 271	410	1533 632	484
·56	715 160	257	0976 157	336	1253 132	412	1551 221	486
·57	722 455	258	0986 367	339	1266 581	416	1568 324	489
·58	729 492	261	0996 238	341	1279 614	419	1584 938	494
·59	736 268	262	1005 768	344	1292 228	421	1601 058	495
·60	742 782	265	1014 954	345	1304 421	423	1616 683	498
·61	749 031	265	1023 795	347	1316 191	426	1631 810	501
·62	755 015	268	1032 289	350	1327 535	427	1646 436	502
·63	760 731	269	1040 433	350	1338 452	430	1660 560	504
·64	766 178	269	1048 227	353	1348 939	431	1674 180	507
·65	771 356	272	1055 668	354	1358 995	432	1687 293	507
·66	776 262	272	1062 755	355	1368 619	435	1699 899	509
·67	780 896	274	1069 487	356	1377 808	435	1711 996	510
·68	785 256	274	1075 863	358	1386 562	435	1723 583	510
·69	789 342	274	1081 881	357	1394 881	438	1734 660	512
·70	793 154	277	1087 542	360	1402 762	437	1745 225	512
·71	796 689	276	1092 843	358	1410 206	439	1755 278	513
·72	799 948	277	1097 786	361	1417 211	437	1764 818	512
·73	802 930	276	1102 368	360	1423 779	440	1773 846	512
·74	805 636	279	1106 590	360	1429 907	438	1782 362	513
·75	808 063	276	1110 452	361	1435 597	439	1790 365	511
·76	810 214	279	1113 953	360	1440 848	438	1797 857	512
·77	812 086	277	1117 094	361	1445 661	437	1804 837	511
·78	813 681	277	1119 874	359	1450 037	439	1811 306	509
·79	814 999	278	1122 295	360	1453 974	435	1817 266	509
·80	816 039	276	1124 356	359	1457 476	437	1822 717	507
·81	816 803	277	1126 058	358	1460 541	434	1827 661	506
·82	817 290	277	1127 402	357	1463 172	435	1832 099	506
·83	817 500	274	1128 389	358	1465 368	431	1836 031	502
·84	817 436	275	1129 018	354	1467 133	432	1839 461	501
·85	817 097	274	1129 293	356	1468 466	430	1842 390	499
·86	816 484	274	1129 212	352	1469 369	427	1844 820	498
·87	815 597	271	1128 779	353	1469 845	427	1846 752	495
·88	814 439	271	1127 993	350	1469 894	424	1848 189	492
·89	813 010	270	1126 857	349	1469 519	423	1849 134	491
·90	811 311	269	1125 372	348	1468 721	420	1849 588	487
·91	809 343	267	1123 539	345	1467 503	418	1849 555	485
·92	807 108	266	1121 361	344	1465 867	416	1849 037	482
·93	804 607	265	1118 839	341	1463 815	414	1848 037	480
·94	801 841	263	1115 976	341	1461 349	410	1846 557	475
·95	798 812	261	1112 772	337	1458 473	409	1844 602	474
·96	795 522	259	1109 231	336	1455 188	406	1842 173	470
·97	791 973	259	1105 354	333	1451 497	402	1839 274	466
·98	788 165	255	1101 144	331	1447 404	401	1835 909	463
0·99	784 102	255	1096 603	328	1442 910	396	1832 081	460
1·00	0·0779 784	251	0·1091 734	326	0·1438 020	395	0·1827 793	456

m	0·7		0·8		0·9		1·0	
u	$Z(u)$	Δ'' −	$Z(u)$	Δ'' −	$Z(u)$	Δ'' −	$Z(u)$	Δ'' −
0·50	0·1739 739	533	0·2083 672	600	0·2514 884	664	0·4621 172	728
·51	1764 357	536	2113 821	605	2552 335	669	4699 452	732
·52	1788 439	543	2143 365	608	2589 117	675	4777 000	737
·53	1811 978	545	2172 301	615	2625 224	679	4853 811	742
·54	1834 972	551	2200 622	618	2660 652	683	4929 880	747
·55	1857 415	554	2228 325	621	2695 397	688	5005 202	750
·56	1879 304	558	2255 407	627	2729 454	692	5079 774	753
·57	1900 635	560	2281 862	629	2762 819	693	5153 593	758
·58	1921 406	565	2307 688	632	2795 491	699	5226 654	759
·59	1941 612	566	2332 882	636	2827 464	699	5298 956	762
·60	1961 252	570	2357 440	637	2858 738	703	5370 496	765
·61	1980 322	572	2381 361	639	2889 309	704	5441 271	766
·62	1998 820	574	2404 643	643	2919 176	707	5511 280	767
·63	2016 744	575	2427 282	642	2948 330	707	5580 522	768
·64	2034 093	577	2449 279	646	2976 789	710	5648 996	770
·65	2050 865	579	2470 630	646	3004 532	709	5716 700	770
·66	2067 058	580	2491 335	647	3031 566	710	5783 634	769
·67	2082 671	581	2511 393	647	3057 890	710	5849 799	770
·68	2097 703	581	2530 804	648	3083 504	712	5915 194	769
·69	2112 154	582	2549 567	649	3108 406	709	5979 820	768
·70	2126 023	582	2567 681	647	3132 599	710	6043 678	768
·71	2139 310	583	2585 148	648	3156 082	709	6106 768	765
·72	2152 014	581	2601 967	648	3178 856	708	6169 093	764
·73	2164 137	583	2618 138	645	3200 922	707	6230 654	763
·74	2175 677	580	2633 664	647	3222 281	705	6291 452	760
·75	2186 637	581	2648 543	643	3242 935	703	6351 490	758
·76	2197 016	579	2662 779	644	3262 886	702	6410 770	755
·77	2206 816	579	2676 371	640	3282 135	698	6469 295	753
·78	2216 037	576	2689 323	640	3300 686	698	6527 067	749
·79	2224 682	577	2701 635	638	3318 539	694	6584 090	745
·80	2232 750	573	2713 309	634	3335 698	691	6640 368	743
·81	2240 245	573	2724 349	634	3352 166	689	6695 903	739
·82	2247 167	569	2734 755	630	3367 945	685	6750 699	735
·83	2253 520	569	2744 531	628	3383 039	681	6804 760	730
·84	2259 304	565	2753 679	625	3397 452	680	6858 091	727
·85	2264 523	564	2762 202	621	3411 185	674	6910 695	722
·86	2269 178	561	2770 104	619	3424 244	671	6962 577	718
·87	2273 272	557	2777 387	615	3436 632	666	7013 741	712
·88	2276 809	556	2784 055	612	3448 354	664	7064 193	708
·89	2279 790	552	2790 111	609	3459 412	658	7113 937	702
·90	2282 219	549	2795 558	604	3469 812	654	7162 979	698
·91	2284 099	545	2800 401	600	3479 558	649	7211 323	693
·92	2285 434	543	2804 644	598	3488 655	645	7258 974	686
·93	2286 226	539	2808 289	592	3497 107	641	7305 939	681
·94	2286 479	536	2811 342	588	3504 918	634	7352 223	676
·95	2286 196	531	2813 807	585	3512 095	631	7397 831	670
·96	2285 382	528	2815 687	579	3518 641	625	7442 769	664
·97	2284 040	524	2816 988	575	3524 562	619	7487 043	658
·98	2282 174	520	2817 714	571	3529 864	615	7530 659	652
0·99	2279 788	517	2817 869	566	3534 551	610	7573 623	645
1·00	0·2276 885	510	0·2817 458	561	0·3538 628	603	0·7615 942	641

m	0·1		0·2		0·3		0·4	
u	Z(u)	Δ″	Z(u)	Δ″	Z(u)	Δ″	Z(u)	Δ″
		—		—		—		—
1·00	0·0237 360	88	0·0496 327	172	0·0779 784	251	0·1091 734	326
·01	235 426	89	492 859	172	775 215	251	1086 539	324
·02	233 403	86	489 219	170	770 395	247	1081 020	320
·03	231 294	87	485 409	168	765 328	247	1075 181	317
·04	229 098	86	481 431	167	760 014	242	1069 025	316
·05	226 816	84	477 286	166	754 458	242	1062 553	312
·06	224 450	84	472 975	162	748 660	238	1055 769	308
·07	222 000	82	468 502	162	742 624	237	1048 677	307
·08	219 468	82	463 867	160	736 351	233	1041 278	303
·09	216 854	80	459 072	157	729 845	231	1033 576	299
·10	214 160	80	454 120	157	723 108	229	1025 575	296
·11	211 386	79	449 011	154	716 142	226	1017 278	294
·12	208 533	76	443 748	151	708 950	223	1008 687	289
·13	205 604	77	438 334	151	701 535	220	0999 807	287
·14	202 598	76	432 769	147	693 900	217	0990 640	283
·15	199 516	73	427 057	146	686 048	215	0981 190	278
·16	196 361	72	421 199	144	677 981	211	0971 462	277
·17	193 134	72	415 197	141	669 703	209	0961 457	271
·18	189 835	70	409 054	139	661 216	205	0951 181	269
·19	186 466	70	402 772	136	652 524	203	0940 636	264
·20	183 027	66	396 354	135	643 629	198	0929 827	261
·21	179 522	67	389 801	132	634 536	197	0918 757	257
·22	175 950	65	383 116	130	625 246	192	0907 430	254
·23	172 313	63	376 301	126	615 764	190	0895 849	248
·24	168 613	63	369 360	126	606 092	186	0884 020	245
·25	164 850	60	362 293	121	596 234	183	0871 946	242
·26	161 027	60	355 105	120	586 193	179	0859 630	237
·27	157 144	57	347 797	117	575 973	176	0847 077	233
·28	153 204	56	340 372	114	565 577	173	0834 291	229
·29	149 208	56	332 833	112	555 008	168	0821 276	224
·30	145 156	53	325 182	110	544 271	166	0808 037	222
·31	141 051	51	317 421	105	533 368	161	0794 576	216
·32	136 895	51	309 555	104	522 304	159	0780 899	211
·33	132 688	49	301 585	101	511 081	154	0767 011	209
·34	128 432	46	293 514	98	499 704	152	0752 914	204
·35	124 130	46	285 345	95	488 175	146	0738 613	198
·36	119 782	44	277 081	93	476 500	144	0724 114	196
·37	115 390	43	268 724	89	464 681	139	0709 419	190
·38	110 955	40	260 278	86	452 723	136	0694 534	187
·39	106 480	39	251 746	85	440 629	133	0679 462	181
·40	101 966	37	243 129	80	428 402	127	0664 209	177
·41	097 415	36	234 432	78	416 048	125	0648 779	173
·42	092 828	34	225 657	75	403 569	121	0633 176	168
·43	088 207	32	216 807	72	390 969	116	0617 405	165
·44	083 554	30	207 885	69	378 253	113	0601 469	158
·45	078 871	30	198 894	66	365 424	108	0585 375	155
·46	074 158	26	189 837	63	352 487	105	0569 126	151
·47	069 419	26	180 717	59	339 445	101	0552 726	144
·48	064 654	24	171 538	58	326 302	97	0536 182	142
·49	059 865	21	162 301	52	313 062	93	0519 496	136
1·50	0·0055 055	21	0·0153 012	52	0·0299 729	88	0·0502 674	132

m	0·5		0·6		0·7		0·8	
u	$Z(u)$	Δ''	$Z(u)$	Δ''	$Z(u)$	Δ''	$Z(u)$	Δ''
1·00	0·1438 020	395	0·1827 793	456	0·2276 885	510	0·2817 458	561
·01	1432 735	390	1823 049	452	2273 472	509	2816 486	556
·02	1427 060	387	1817 853	449	2269 550	503	2814 958	552
·03	1420 998	385	1812 208	444	2265 125	498	2812 878	546
·04	1414 551	381	1806 119	442	2260 202	495	2810 252	541
·05	1407 723	377	1799 588	436	2254 784	490	2807 085	537
·06	1400 518	374	1792 621	433	2248 876	485	2803 381	531
·07	1392 939	371	1785 221	428	2242 483	481	2799 146	525
·08	1384 989	366	1777 393	425	2235 609	476	2794 386	521
·09	1376 673	363	1769 140	420	2228 259	471	2789 105	516
·10	1367 994	358	1760 467	416	2220 438	466	2783 308	509
·11	1358 957	357	1751 378	411	2212 151	462	2777 002	505
·12	1349 563	350	1741 878	408	2203 402	456	2770 191	499
·13	1339 819	348	1731 970	402	2194 197	452	2762 881	494
·14	1329 727	343	1721 660	399	2184 540	446	2755 077	488
·15	1319 292	339	1710 951	393	2174 437	442	2746 785	482
·16	1308 518	335	1699 849	389	2163 892	437	2738 011	478
·17	1297 409	332	1688 358	384	2152 910	431	2728 759	471
·18	1285 968	326	1676 483	380	2141 497	426	2719 036	466
·19	1274 201	322	1664 228	375	2129 658	422	2708 847	462
·20	1262 112	319	1651 598	370	2117 397	416	2698 196	453
·21	1249 704	313	1638 598	366	2104 720	410	2687 092	450
·22	1236 983	310	1625 232	360	2091 633	406	2675 538	444
·23	1223 952	305	1611 506	356	2078 140	401	2663 540	438
·24	1210 616	300	1597 424	351	2064 246	395	2651 104	432
·25	1196 980	297	1582 991	346	2049 957	390	2638 236	427
·26	1183 047	291	1568 212	341	2035 278	385	2624 941	421
·27	1168 823	286	1553 092	337	2020 214	379	2611 225	416
·28	1154 313	284	1537 635	330	2004 771	375	2597 093	411
·29	1139 519	277	1521 848	327	1988 953	369	2582 550	403
·30	1124 448	273	1505 734	322	1972 760	363	2567 604	400
·31	1109 104	269	1489 298	315	1956 216	359	2552 258	393
·32	1093 491	263	1472 547	312	1939 307	353	2536 519	389
·33	1077 615	260	1455 484	306	1922 045	347	2520 391	381
·34	1061 479	254	1438 115	302	1904 436	344	2503 882	378
·35	1045 089	249	1420 444	296	1886 483	337	2486 995	371
·36	1028 450	246	1402 477	290	1868 193	332	2469 737	366
·37	1011 565	239	1384 220	288	1849 571	327	2452 113	361
·38	0994 441	236	1365 675	280	1830 622	322	2434 128	354
·39	0977 081	230	1346 850	276	1811 351	316	2415 789	351
·40	0959 491	225	1327 749	271	1791 764	311	2397 099	344
·41	0941 676	222	1308 377	266	1771 866	307	2378 065	340
·42	0923 639	215	1288 739	261	1751 661	300	2358 691	333
·43	0905 387	211	1268 840	256	1731 156	297	2336 984	329
·44	0886 924	207	1248 685	250	1710 354	289	2318 948	323
·45	0868 254	200	1228 280	247	1689 263	287	2298 589	318
·46	0849 384	197	1207 628	239	1667 885	279	2277 912	314
·47	0830 317	192	1186 737	236	1646 228	276	2256 921	307
·48	0811 058	186	1165 610	231	1624 295	269	2235 623	303
·49	0791 613	181	1144 252	225	1602 093	266	2214 022	298
1·50	0·0771 987	177	0·1122 669	221	0·1579 625	260	0·2192 123	292

m	0·9		1·0		m		0·1			0·2	
u	Z(u)	Δ'' −	Z(u)	Δ'' −	u		Z(u)	Δ''		Z(u)	Δ''
1·00	0·3538 628	603	0·7615 942	641	1·50		0·0055 055	−21		0·0153 012	−52
·01	3542 102	599	7657 620	633	·51		050 224	−18		143 671	−47
·02	3544 977	592	7698 665	627	·52		045 375	−16		134 283	−44
·03	3547 260	588	7739 083	620	·53		040 510	−15		124 851	−41
·04	3548 955	581	7778 881	615	·54		035 630	−13		115 378	−39
·05	3550 069	575	7818 064	608	·55		030 737	−12		105 866	−34
·06	3550 608	570	7856 639	602	·56		025 832	−9		096 320	−32
·07	3550 577	565	7894 612	594	·57		020 918	−7		086 742	−29
·08	3549 981	557	7931 991	589	·58		015 997	−6		077 135	−25
·09	3548 828	553	7968 781	581	·59		011 070	−5		067 503	−22
·10	3547 122	546	8004 990	575	·60		006 138	−1		057 849	−20
·11	3544 870	541	8040 624	569	·61	+	001 205	−1		048 175	−15
·12	3542 077	534	8075 689	561	·62	−	003 729	+1		038 486	−13
·13	3538 750	528	8110 193	556	·63		008 662	+3		028 784	−9
·14	3534 895	522	8144 141	548	·64		013 592	+6		019 073	−7
·15	3530 518	516	8177 541	542	·65	−	018 516	+6	+	009 355	−3
·16	3525 625	511	8210 399	535	·66		023 434	+9	−	000 366	0
·17	3520 221	504	8242 722	529	·67		028 343	+10		010 087	+5
·18	3514 313	498	8274 516	521	·68		033 242	+12		019 803	+4
·19	3507 907	492	8305 789	516	·69		038 129	+14		029 515	+11
·20	3501 009	485	8336 546	508	·70	−	043 002	+16	−	039 216	+13
·21	3493 626	481	8366 795	502	·71		047 859	+17		048 904	+16
·22	3485 762	473	8396 542	496	·72		052 699	+20		058 576	+19
·23	3477 425	468	8425 793	488	·73		057 519	+21		068 229	+23
·24	3468 620	462	8454 556	483	·74		062 318	+22		077 859	+25
·25	3459 353	456	8482 836	475	·75	−	067 095	+25	−	087 464	+29
·26	3449 630	449	8510 641	469	·76		071 847	+26		097 040	+32
·27	3439 458	444	8537 977	464	·77		076 573	+28		106 584	+36
·28	3428 842	438	8564 849	456	·78		081 271	+30		116 092	+37
·29	3417 788	432	8591 265	449	·79		085 939	+31		125 563	+42
·30	3406 302	426	8617 232	445	·80	−	090 576	+33	−	134 992	+45
·31	3394 390	420	8642 754	437	·81		095 180	+35		144 376	+47
·32	3382 058	414	8667 839	431	·82		099 749	+37		153 713	+51
·33	3369 312	409	8692 493	424	·83		104 281	+38		162 999	+54
·34	3356 157	402	8716 723	420	·84		108 775	+39		172 231	+57
·35	3342 600	398	8740 533	412	·85	−	113 230	+43	−	181 406	+60
·36	3328 645	390	8763 931	407	·86		117 642	+42		190 521	+63
·37	3314 300	387	8786 922	400	·87		122 012	+45		199 573	+66
·38	3299 568	379	8809 513	395	·88		126 337	+46		208 559	+70
·39	3284 457	374	8831 709	388	·89		130 616	+48		217 475	+71
·40	3268 972	370	8853 517	384	·90	−	134 847	+50	−	226 320	+76
·41	3253 117	362	8874 941	376	·91		139 028	+51		235 089	+77
·42	3236 900	359	8895 989	371	·92		143 158	+52		243 781	+82
·43	3220 324	352	8916 666	366	·93		147 236	+52		252 391	+84
·44	3203 396	347	8936 977	359	·94		151 260	+57		260 917	+86
·45	3186 121	342	8956 929	355	·95	−	155 227	+56	−	269 357	+90
·46	3168 504	336	8976 526	348	·96		159 138	+58		277 707	+93
·47	3150 551	332	8995 775	344	·97		162 991	+61		285 964	+96
·48	3132 266	325	9014 680	338	·98		166 783	+61		294 125	+97
·49	3113 656	322	9033 247	331	1·99		170 514	+63		302 189	+102
1·50	0·3094 724	315	0·9051 483	328	2·00		−0·0174 182	+65		−0·0310 151	+103

m		0·3			0·4			0·5			0·6	
u		$Z(u)$	Δ''		$Z(u)$	Δ''		$Z(u)$	Δ''		$Z(u)$	Δ''
												−
1·50		0·0299 729	−88		0·0502 674	−132		0·0771 987	−177		0·1122 669	221
·51		286 308	−85		485 720	−127		752 184	−172		1100 865	214
·52		272 802	−80		468 639	−123		732 209	−166		1078 847	211
·53		259 216	−77		451 435	−117		712 068	−163		1056 618	206
·54		245 553	−73		434 114	−113		691 764	−156		1034 183	199
·55		231 817	−68		416 680	−109		671 304	−151		1011 549	196
·56		218 013	−64		399 137	−104		650 693	−148		0988 719	190
·57		204 145	−60		381 490	−98		629 934	−142		0965 699	185
·58		190 217	−56		363 745	−95		609 033	−136		0942 494	180
·59		176 233	−52		345 905	−89		587 996	−133		0919 109	175
·60		162 197	−47		327 976	−85		566 826	−126		0895 549	171
·61		148 114	−44		309 962	−81		545 530	−123		0871 818	164
·62		133 987	−40		291 867	−74		524 111	−116		0847 923	161
·63		119 820	−34		273 698	−72		502 576	−113		0823 867	155
·64		105 619	−32		255 457	−65		480 928	−106		0799 656	151
·65		091 386	−26		237 151	−61		459 174	−102		0775 294	145
·66		077 127	−23		218 784	−56		437 318	−98		0750 787	140
·67		062 845	−19		200 361	−52		415 364	−91		0726 140	136
·68		048 544	−13		181 886	−47		393 319	−88		0701 357	130
·69		034 230	−11		163 364	−42		371 186	−81		0676 444	126
·70		019 905	−6		144 800	37		348 972	−78		0651 405	120
·71	+	005 574	−1		126 199	−32		326 680	−72		0626 246	117
·72	−	008 758	+3		107 566	−28		304 316	−66		0600 970	110
·73		023 087	+6		088 905	−22		281 886	−63		0575 584	106
·74		037 410	+11		070 222	−19		259 393	−57		0550 092	102
·75	−	051 722	+15		051 520	−13		236 843	−51		0524 498	96
·76		066 019	+20		032 805	−8		214 242	−48		0498 808	91
·77		080 296	+23	+	014 082	−4		191 593	−42		0473 027	87
·78		094 550	+28	−	004 645	+1		168 902	−37		0447 159	81
·79		108 776	+32		023 371	+6		146 174	−32		0421 210	78
·80	−	122 970	+36	−	042 091	+11		123 414	−26		0395 183	71
·81		137 128	+40		060 800	+16		100 628	−23		0369 085	67
·82		151 246	+45		079 493	+20		077 819	−17		0342 920	63
·83		165 319	+48		098 166	+25		054 993	−12		0316 692	58
·84		179 344	+53		116 814	+31		032 155	−7		0290 406	52
·85	−	193 316	+57	−	135 431	+34	+	009 310	−3		0264 068	47
·86		207 231	+61		154 014	+39	−	013 538	+4		0237 683	44
·87		221 085	+65		172 558	+45		036 382	+8		0211 254	38
·88		234 874	+69		191 057	+49		059 218	+13		0184 787	34
·89		248 594	+74		209 507	+54		082 041	+18		0158 286	28
·90	−	262 240	+77	−	227 903	+58	−	104 846	+22		0131 757	24
·91		275 809	+82		246 241	+64		127 629	+29		0105 204	19
·92		289 296	+85		264 515	+69		150 383	+33		0078 632	14
·93		302 698	+89		282 720	+72		173 104	+37		0052 046	10
·94		316 011	+95		300 853	+78		195 788	+44	+	0025 450	−4
·95	−	329 229	+97	−	318 908	+82	−	218 428	+47	−	0001 150	0
·96		342 350	+102		336 881	+87		241 021	+54		0027 750	+5
·97		355 369	+105		354 767	+93		263 560	+57		0054 345	+10
·98		368 283	+110		372 560	+96		286 042	+64		0080 930	+15
1·99		381 087	+113		390 257	+101		308 460	+67		0107 500	+19
2·00		−0·0393 778	+118		−0·0407 853	+104		−0·0330 811	+70		−0·0134 051	+24

m	0·7		0·8		0·9		1·0	
u	$Z(u)$	$\underline{\Delta''}$	$Z(u)$	$\underline{\Delta''}$	$Z(u)$	$\underline{\Delta''}$	$Z(u)$	$\underline{\Delta''}$
1·50	0·1579 625	260	0·2192 123	292	0·3094 724	315	0·9051 483	328
·51	1556 897	255	2169 932	288	3075 477	311	9069 391	322
·52	1533 914	249	2147 453	283	3055 919	305	9086 977	317
·53	1510 682	246	2124 691	277	3036 056	302	9104 246	311
·54	1487 204	239	2101 652	274	3015 891	295	9121 204	307
·55	1463 487	236	2078 339	267	2995 431	292	9137 855	301
·56	1439 534	230	2054 759	264	2974 679	286	9154 205	297
·57	1415 351	224	2030 915	257	2953 641	281	9170 258	292
·58	1390 944	221	2006 814	255	2932 322	277	9186 019	287
·59	1366 316	216	1982 458	248	2910 726	273	9201 493	281
·60	1341 472	210	1957 854	245	2888 857	267	9216 686	279
·61	1316 418	206	1933 005	239	2866 721	263	9231 600	272
·62	1291 158	200	1907 917	237	2844 322	259	9246 242	268
·63	1265 698	197	1882 592	227	2821 664	254	9260 616	264
·64	1240 041	191	1857 040	228	2798 752	249	9274 726	260
·65	1214 193	187	1831 260	221	2775 591	246	9288 576	254
·66	1188 158	182	1805 259	217	2752 184	241	9302 172	251
·67	1161 941	177	1779 041	212	2728 536	237	9315 517	246
·68	1135 547	172	1752 611	209	2704 651	233	9328 616	243
·69	1108 981	169	1725 972	203	2680 533	229	9341 472	237
·70	1082 246	162	1699 130	200	2656 186	223	9354 091	235
·71	1055 349	160	1672 088	195	2631 616	222	9366 475	229
·72	1028 292	153	1644 851	191	2606 824	216	9378 630	226
·73	1001 082	150	1617 423	187	2581 816	212	9390 559	222
·74	0973 722	144	1589 808	182	2556 596	209	9402 266	218
·75	0946 218	141	1562 011	179	2531 167	204	9413 755	214
·76	0918 573	136	1534 035	174	2505 534	202	9425 030	211
·77	0890 792	131	1505 885	170	2479 699	197	9436 094	206
·78	0862 880	127	1477 565	166	2453 667	193	9446 952	204
·79	0834 841	122	1449 079	163	2427 442	191	9457 606	200
·80	0806 680	118	1420 430	157	2401 026	185	9468 060	195
·81	0778 401	114	1391 624	155	2374 425	184	9478 319	194
·82	0750 008	109	1362 663	151	2347 640	178	9488 384	188
·83	0721 506	104	1333 551	145	2320 677	176	9498 261	187
·84	0692 900	101	1304 294	144	2293 538	172	9507 951	181
·85	0664 193	95	1274 893	138	2266 227	170	9517 460	181
·86	0635 391	93	1245 354	135	2238 746	165	9526 788	175
·87	0606 496	86	1215 680	131	2211 100	162	9535 941	173
·88	0577 515	83	1185 875	128	2183 292	159	9544 921	170
·89	0548 451	79	1155 942	123	2155 325	155	9553 731	166
·90	0519 308	74	1125 886	121	2127 203	154	9562 375	165
·91	0490 091	70	1095 709	116	2098 927	148	9570 854	160
·92	0460 804	66	1065 416	112	2070 503	147	9579 173	158
·93	0431 451	61	1035 011	110	2041 932	144	9587 334	155
·94	0402 037	57	1004 496	105	2013 217	139	9595 340	152
·95	0372 566	53	0973 876	103	1984 363	138	9603 194	150
·96	0343 042	49	0943 153	97	1955 371	134	9610 898	146
·97	0313 469	44	0912 333	96	1926 245	132	9618 456	144
·98	0283 852	40	0881 417	91	1896 987	128	9625 870	142
1·99	0254 195	36	0850 410	88	1867 601	126	9633 142	138
2·00	0·0224 502	32	0·0819 315	84	0·1838 089	122	0·9640 276	137

m		0·7			0·8		0·9		1·0	
u		$Z(u)$	Δ''		$Z(u)$	Δ''	$Z(u)$	Δ''	$Z(u)$	Δ''
								−		−
2·00		0·0224 502	− 32		0·0819 315	−84	0·1838 089	122	0·9640 276	137
·01		0194 777	− 28		788 136	−82	1808 455	121	9647 273	133
·02		0165 024	− 22		756 875	−77	1778 700	117	9654 137	132
·03		0135 249	− 20		725 537	−74	1748 828	115	9660 869	128
·04		0105 454	− 14		694 125	−71	1718 841	112	9667 473	127
·05		0075 645	− 12		662 642	−67	1688 742	109	9673 950	124
·06		0045 824	− 5		631 092	−65	1658 534	107	9680 303	122
·07	+	0015 998	− 3		599 477	−60	1628 219	105	9686 534	119
·08	−	0013 831	+ 2		567 802	−58	1597 799	101	9692 646	118
·09		0043 658	+ 6		536 069	−54	1567 278	99	9698 640	115
·10	−	0073 479	+ 11		504 282	−50	1536 658	97	9704 519	112
·11		0103 289	+ 14		472 445	−48	1505 941	94	9710 286	112
·12		0133 085	+ 19		440 560	−45	1475 130	91	9715 941	109
·13		0162 862	+ 22		408 630	−40	1444 228	91	9721 487	106
·14		0192 617	+ 28		376 660	−38	1413 235	86	9726 927	105
·15	−	0222 344	+ 32		344 652	−35	1382 156	85	9732 262	103
·16		0252 039	+ 35		312 609	−30	1350 992	82	9737 494	101
·17		0281 699	+ 40		280 536	−29	1319 746	80	9742 625	99
·18		0311 319	+ 44		248 434	−25	1288 420	78	9747 657	97
·19		0340 895	+ 48		216 307	−21	1257 016	76	9752 592	96
·20	−	0370 423	+ 53		184 159	−18	1225 536	72	9757 431	93
·21		0399 898	+ 56		151 993	−15	1193 984	72	9762 177	91
·22		0429 317	+ 62		119 812	−13	1162 360	69	9766 832	91
·23		0458 674	+ 65		087 618	− 8	1130 667	67	9771 396	88
·24		0487 966	+ 70		055 416	− 6	1098 907	64	9775 872	87
·25	−	0517 188	+ 73	+	023 208	− 1	1067 083	62	9780 261	85
·26		0546 337	+ 79	−	009 001	0	1035 197	61	9784 565	83
·27		0575 407	+ 83		041 210	+ 4	1003 250	58	9788 786	82
·28		0604 394	+ 86		073 415	+ 7	0971 245	57	9792 925	80
·29		0633 295	+ 92		105 613	+11	0939 183	53	9796 984	79
·30	−	0662 104	+ 95	−	137 800	+13	0907 068	53	9800 964	77
·31		0690 818	+100		169 974	+18	0874 900	50	9804 867	76
·32		0719 432	+104		202 130	+19	0842 682	48	9808 694	75
·33		0747 942	+110		234 267	+24	0810 416	46	9812 446	72
·34		0776 342	+112		266 380	+27	0778 104	44	9816 126	72
·35	−	0804 630	+118	−	298 466	+29	0745 748	43	9819 734	70
·36		0832 800	+122		330 523	+34	0713 349	39	9823 272	69
·37		0860 848	+127		362 546	+35	0680 911	39	9826 741	67
·38		0888 769	+130		394 534	+40	0648 434	37	9830 143	67
·39		0916 560	+136		426 482	+43	0615 920	34	9833 478	64
·40	−	0944 215	+141	−	458 387	+46	0583 372	32	9836 749	65
·41		0971 729	+143		490 246	+50	0550 792	32	9839 955	61
·42		0999 100	+150		522 055	+52	0518 180	28	9843 100	63
·43		1026 321	+153		553 812	+56	0485 540	27	9846 182	59
·44		1053 389	+159		585 513	+59	0452 873	25	9849 205	59
·45	−	1080 298	+163	−	617 155	+63	0420 181	23	9852 169	58
·46		1107 044	+167		648 734	+66	0387 466	22	9855 075	56
·47		1133 623	+172		680 247	+69	0354 729	20	9857 925	57
·48		1160 030	+177		711 691	+73	0321 972	17	9860 718	54
·49		1186 260	+182		743 062	+76	0289 198	16	9863 457	53
2·50		−0·1212 308	+187		−0·0774 357	+79	0·0256 408	14	0·9866 143	53

m		0·9		1·0		m		0·9		1·0	
u		$Z(u)$	Δ''	$Z(u)$	Δ'' –	u		$Z(u)$	Δ''	$Z(u)$	Δ'' –
2·50		0·0256 408	−14	0·9866 143	53	2·75		−0·0563 228	+32	0·9918 597	31
·51		0223 604	−12	9868 776	51	·76		0595 796	+33	9920 203	32
·52		0190 788	−11	9871 358	51	·77		0628 331	+35	9921 777	31
·53		0157 961	−9	9873 889	49	·78		0660 831	+37	9923 320	31
·54		0125 125	−7	9876 371	49	·79		0693 294	+40	9924 832	29
·55		0092 282	−4	9878 804	47	·80	−	0725 717	+40	9926 315	29
·56		0059 435	−4	9881 190	47	·81		0758 100	+44	9927 769	29
·57	+	0026 584	−1	9883 529	46	·82		0790 439	+44	9929 194	28
·58	−	0006 268	0	9885 822	45	·83		0822 734	+47	9930 591	27
·59		0039 120	+2	9888 070	44	·84		0854 982	+49	9931 961	27
·60	−	0071 970	+4	9890 274	43	·85	−	0887 181	+51	9933 304	27
·61		0104 816	+6	9892 435	42	·86		0919 329	+53	9934 620	25
·62		0137 656	+8	9894 554	42	·87		0951 424	+55	9935 911	26
·63		0170 488	+9	9896 631	41	·88		0983 464	+56	9937 176	25
·64		0203 311	+11	9898 667	39	·89		1015 448	+60	9938 416	24
·65	−	0236 123	+13	9900 664	40	·90	−	1047 372	+61	9939 632	24
·66		0268 922	+14	9902 621	38	·91		1079 235	+64	9940 824	24
·67		0301 707	+18	9904 540	37	·92		1111 034	+64	9941 992	23
·68		0334 474	+18	9906 422	38	·93		1142 769	+69	9943 137	22
·69		0367 223	+20	9908 266	35	·94		1174 435	+68	9944 260	22
·70	−	0399 952	+22	9910 075	37	·95	−	1206 033	+73	9945 361	22
·71		0432 659	+24	9911 847	34	·96		1237 558	+74	9946 440	21
·72		0465 342	+26	9913 585	34	·97		1269 009	+77	9947 498	22
·73		0497 999	+28	9915 289	33	·98		1300 383	+78	9948 534	19
·74		0530 628	+29	9916 960	34	2·99		1331 679	+81	9949 551	20
2·75	−	0·0563 228	+32	0·9918 597	31	3·00		−0·1362 804	+83	0·9950 548	21

(*Issued separately May* 10, 1932.)

XII.—Isohedral and Isogonal Generalizations of the Regular Polyhedra. By Professor D. M. Y. Sommerville, M.A., D.Sc., Victoria University College, Wellington, N.Z.

(MS. received November 18, 1931. Read January 11, 1932.)

0.1. In Max Brückner's discussion of isohedral (*gleichflächig*) and isogonal (*gleicheckig*) polyhedra (*Vielecke und Vielflache*, pp. 140 ff.) two faces of a polyhedron are defined to be equal when they are either directly or inversely congruent, and the dihedral angles at corresponding edges are equal; two vertices are equal when the spherical polygons, which they form on unit spheres with centres at the vertices, are either directly or inversely congruent, and the lengths of corresponding edges are equal. With these definitions he shows that all possible isogonal polyhedra are obtained by truncating the corners and edges of the regular n-sided prism, the octahedron and hexahedron, and the icosahedron and dodecahedron, in a manner similar to that in which the semi-regular polyhedra are obtained from the regular polyhedra. The semi-regular or Archimedean bodies are indeed just special varieties of the more general isogonal polyhedra. The isohedral polyhedra are the polar reciprocals of the isogonal polyhedra, and are obtained from the regular double-pyramid and the regular polyhedra by the reciprocal constructions, "pointing" the faces and edges.

0.2. An isogonal polyhedron, in the above sense, has always a circumscribed sphere, and an isohedral polyhedron has always an inscribed sphere.

Among these isogonal and isohedral polyhedra there occur generalizations of the regular tetrahedron, hexahedron, and octahedron, the tetrahedron (with opposite edges equal) being both isogonal and isohedral; there occur also isogonal icosahedra and isohedral dodecahedra, but not isohedral icosahedra or isogonal dodecahedra (except the regular ones).

0.3. In this paper the condition of equal corresponding dihedral angles, in the case of the isohedral polyhedra, and equal corresponding edges, in the case of isogonal polyhedra, is omitted. For the tetrahedron, hexahedron, and octahedron this does not lead to any further generalization (except for a greater degree of variability of the isogonal hexahedron). But for the icosahedron it gives an isohedral form, and for the dodecahedron

an isogonal form; these have neither a circumscribed nor an inscribed sphere unless they are regular.

0.4. We shall consider first a few general theorems.

(1) *If a polyhedron has a circumscribed sphere, every face has a circumscribed circle,* for a plane cuts a sphere in a circle.

Reciprocally—

(1′) *If a polyhedron has an inscribed sphere, all the planes which meet at any vertex touch a circular cone, or, equivalently, the spherical polygon surrounding each vertex has an inscribed circle.*

(2) *If an isohedral polyhedron has a circumscribed sphere it has also an inscribed sphere and the two spheres are concentric.*

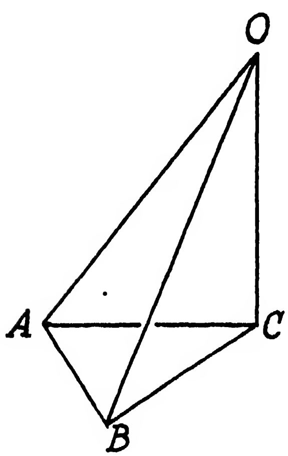


FIG. 1.

Let O be the centre of the circumscribed sphere, OC perpendicular to a face, and A, B two consecutive vertices of that face. Then OA = OB = . . ., whence CA = CB = . . ., and C is the centre of the circumscribed circle of the face. Then since the faces are all equal, CA is constant and therefore OC is constant.

Reciprocally—

(2′) *If an isogonal polyhedron has an inscribed sphere, it has also a circumscribed sphere, and the two spheres are concentric.*

The converses of (2) and (2′) are not necessarily true; for example, a semi-regular polyhedron, which is isogonal, has a circumscribed sphere but not in general an inscribed sphere.

A polyhedron may have an inscribed and a circumscribed sphere which are not concentric, *e.g.* a general tetrahedron, a frustum of a square pyramid of suitable height (if a, a' are the edges of the two squares and h the height of the frustum, then for an inscribed sphere $aa' = h^2$), a double-pyramid on a square base (if a is the edge of the square and h, h' the heights of the two pyramids, then for a circumscribed sphere $a^2 = 2hh'$); etc.

1. *The Tetrahedron.*—If a tetrahedron is isohedral, the faces are all directly congruent; the vertices are also directly congruent and the tetrahedron is isogonal. Pairs of opposite edges are equal (Brückner, No. 7 or 7′).

In a special case the faces are isosceles (Brückner, No. 4 or 4′), and as a still more special case we have the regular tetrahedron.

2.1. *Hexahedron, Isohedral.*—It is easily shown that in each face two

adjacent edges are equal. The faces are all directly congruent. There are two opposite vertices at which three equal edges c meet, at the others there are three unequal edges a, b, c. The dihedral angles at the edges c are all equal, and those at the edges a and b are constant throughout (Brückner, No. 6′).

In the general case when a, b, c are given, the plane angles of a face are determined. Take OA, OB, OC as coordinate-axes (oblique) and let

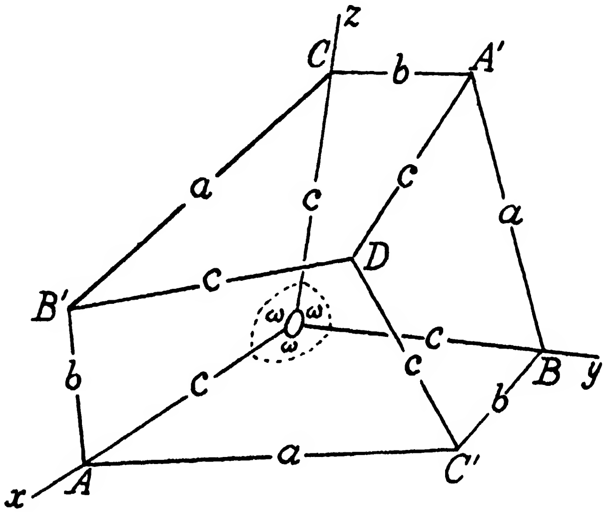


Fig. 2.

the plane angles at $O = \omega$. Let the coordinates of A′ be $(0, u, v)$, then $B' \equiv (v, 0, u)$ and $C' \equiv (u, v, 0)$. The vertex D, opposite the origin, will be determined as the intersection of the three planes B′AC′, C′BA′, and A′CB′. We find the equation of the plane B′AC′ to be

$$-uv(x-c)+u(u-c)y+v(v-c)z=0.$$

By symmetry the coordinates of D (which lies in this face) are all equal (t, t, t). Hence, substituting $x=y=z=t$, we have

$$t\{uv-u^2-v^2+c(u+v)\}=cuv.$$

Write $u+v=p, \quad uv=q, \quad u^2+v^2-uv=r\equiv p^2-3q$. Then

$$t=\frac{cq}{cp-r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

We have next to express that $DA'=c$. We have

$$\begin{aligned} c^2 &= t^2+(t-u)^2+(t-v)^2+2\{(t-u)(t-v)+t(2t-u-v)\}\cos\omega \\ &= 3t^2-2tp+r+q+2(3t^2-2tp+q)\cos \end{aligned}$$

Substituting the value of t we get

$$(c^2-r)(cp-r)^2=(1+2\cos\omega)qr(r-c^2).$$

Therefore either $c^2=r$ or $(cp-r)^2+qr(1+2\cos\omega)=0$.

Further, the angle $B'DC'=\omega$, therefore $B'C'=2c\sin\frac{1}{2}\omega$. We have

$$\begin{aligned}B'C'^2&=(u-v)^2+u^2+v^2+2\{-uv-(u-v)^2\}\cos\omega\\&=2r(1-\cos\omega)=4r\sin^2\tfrac{1}{2}\omega.\end{aligned}$$

Hence to satisfy both conditions

$$c^2=r=u^2+v^2-uv \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This hexahedron has always an inscribed sphere. If it has also a circumscribed sphere the centre is at the midpoint of OD: $S\equiv(\frac{1}{2}t,\ \frac{1}{2}t,\ \frac{1}{2}t)$. Let the radius $=R$. Then

$$R^2=OS^2=\tfrac{3}{4}t^2+\tfrac{3}{2}t^2\cos\omega=\tfrac{3}{4}t^2(1+2\cos\omega).$$

Also

$$\begin{aligned}R^2=SA^2&=(\tfrac{1}{2}t-c)^2+\tfrac{1}{2}t^2+2\{\tfrac{1}{4}t^2+t(\tfrac{1}{2}t-c)\}\cos\omega\\&=(\tfrac{3}{4}t^2-ct)(1+2\cos\omega)+c^2.\end{aligned}$$

Equating these we get, as the condition for a circumscribed sphere,

$$1+2\cos\omega=c/t=c(p-c)/q \quad . \quad . \quad . \quad . \quad . \quad (3)$$

2.11. In non-euclidean geometry the general conditions are the same, viz. three equal edges c at two opposite vertices. We may again take a system of coordinates, analogous to oblique cartesians with O as origin, in which the equation of the absolute is

$$x^2+y^2+z^2+w^2+2(yz+zx+xy)\cos\omega=0.$$

Let

$$\begin{gathered}A'\equiv(0,u,v,1),\ B'\equiv(v,0,u,1),\ C'\equiv(u,v,0,1);\\ A\equiv(\tan c,0,0,1),\ \text{etc.},\ D\equiv(t,\ t,\ t,\ 1).\end{gathered}$$

Then by the same method as before we find

$$t=\frac{q\tan c}{p\tan c-r} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1')$$

And expressing that $A'D=c$ and the angle $B'DC'=\omega$ we obtain the condition

$$r\cos^2 OA'=\sin^2 c \quad . \quad . \quad . \quad . \quad . \quad . \quad (2')$$

This may be expressed in a form more nearly resembling (2). In this system of coordinates, if L, M, N are the feet of the perpendiculars from the point P on the coordinate-planes, and L, M, N the angles which each

coordinate-axis makes with the opposite coordinate-plane, the coordinates of P are

$$x = \sin \mathrm{LP}/\sin \mathrm{L}, \quad y = \sin \mathrm{MP}/\sin \mathrm{M}, \quad z = \sin \mathrm{NP}/\sin \mathrm{N}, \quad w = \cos \mathrm{OP}.$$

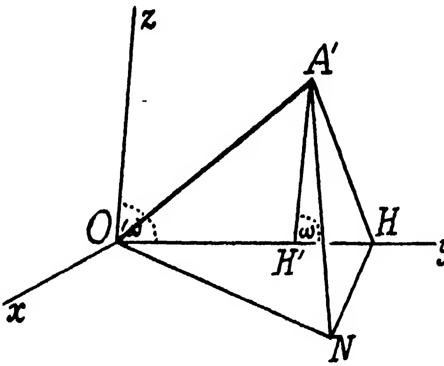


Fig. 3.

For the point A′ we have

$$z = \sin \mathrm{NA}'/\sin \mathrm{N}, \quad w = \cos \mathrm{OA}',$$

so that

$$v = \frac{z}{w} = \frac{\sin \mathrm{NA}'}{\sin \mathrm{N} \cos \mathrm{OA}'}.$$

Let $\mathrm{A'H} \perp \mathrm{O}y$ and $\angle y\mathrm{H'A'} = \omega$, and let δ be the dihedral angle between the coordinate-planes. Then $\sin \mathrm{N} = \sin \omega \sin \delta$. Also $\sin \mathrm{NA}' = \sin \mathrm{HA}' \sin \delta$ and $\sin \mathrm{HA}' = \sin \mathrm{H'A'} \sin \omega$. Therefore

$$v = \frac{\sin \mathrm{H'A'} \sin \omega \sin \delta}{\sin \omega \sin \delta \cos \mathrm{OA}'} = \frac{\sin \mathrm{H'A'}}{\cos \mathrm{OA}'}.$$

Write $\sin \mathrm{H'A'} = \mathrm{V}$, and similarly for U. Then

$$\mathrm{U} = u \cos \mathrm{OA}', \quad \mathrm{V} = v \cos \mathrm{OA}',$$

and the condition (2′) becomes

$$\mathrm{U}^2 + \mathrm{V}^2 - \mathrm{UV} = \sin^2 c \quad . \quad . \quad . \quad . \quad . \quad . \quad (2'')$$

2.12. Consider now the special case in which $a = b$ and therefore $u = v$. (2) then gives $c = u$, and the faces are equilateral, with $a = b = c$. The figure is a rhombohedron (Brückner, No. 3′). In the euclidean case, with a given rhombus as face there are two values of ω and therefore two different rhombohedra, provided, of course, that $3\omega < 360°$. The condition for a circumscribed sphere becomes $\omega = 90°$, and the hexahedron is a cube.

2.13. In non-euclidean geometry consider the face OBA′C with sides all equal, and let, as before, $\angle y\mathrm{H'A'} = \omega$. Then by (2′) it follows that $\mathrm{H'A'} = c = \mathrm{OB} = \mathrm{BA'}$. Therefore $\angle \mathrm{OBA'} = \omega$ and the face is regular. In non-euclidean geometry therefore there is no "rhombohedron," and the only hexahedron with equal symmetrical faces is the regular hexahedron.

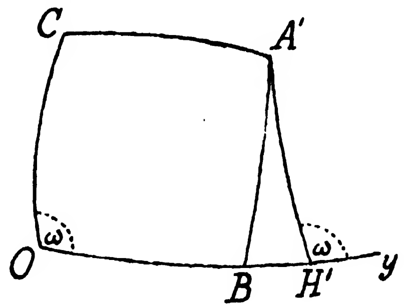
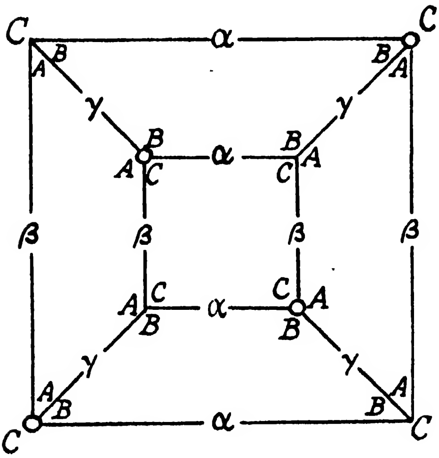


Fig. 4.

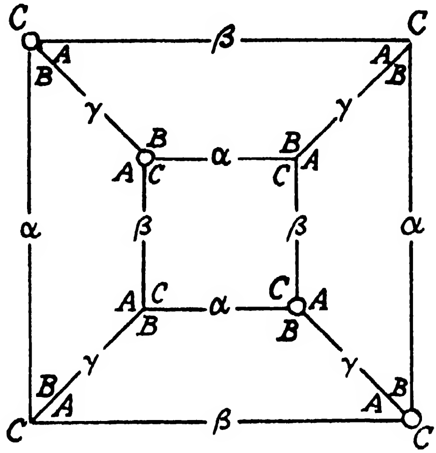
2.2. *Hexahedron, Isogonal.*—Let A, B, C be the face-angles and α, β, γ

the dihedral angles of each vertex. Two forms are possible (fig. 5, (i) and (ii)).

2.21. In (i) the faces are all equiangular, and therefore, in euclidean geometry, rectangles; the polyhedron is a rectangular parallelepiped (Brückner, No. 2). For an inscribed sphere the edges $a=b=c$ and it is



(i) Fig. 5. (ii)



regular. In non-euclidean geometry the angles A, B, C are in general different. If B = C, the two faces with angles A are regular (Brückner, No. 1).

2.22. In (ii) two opposite faces are equiangular, and therefore, in euclidean geometry, rectangles; the other faces are symmetrical trapezia of equal altitudes. In Brückner, No. 5, the two rectangles are congruent but oppositely turned; in this variety, omitting the equality of the edges, the sides a, b and a', b' of the two rectangles are connected by the relation $a+b=a'+b'$. This is an instance of a variable type of polyhedron which is isogonal in the weaker sense. It occurs always (in euclidean geometry) when each vertex has just three edges, *i.e.* when the polyhedron is trigonal, for each face may be replaced by a parallel face without altering the angles. (*Cf.* the isogonal dodecahedron 4.2.)

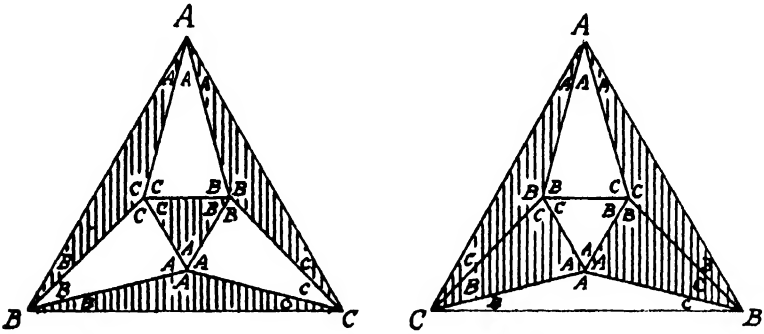
There is always a circumscribed sphere. If there is an inscribed sphere its centre is midway between the planes of the two rectangles. Taking the section perpendicular to the edges b, b', let d be the altitude of each trapezium, then $a+a'=2d$. Similarly $b+b'=2d$. Therefore $a=b'$, $a'=b$, and $a+b=2d$ (equivalent to $a^2+b^2=2c^2$, where c is the slant side of each trapezium). The two spheres are then concentric.

In non-euclidean geometry we find that the two opposite equiangular

faces are necessarily congruent; and the condition for an inscribed sphere is

$$\cos a + \cos b = 2 \cos c.$$

3.1. *Octahedron, Isohedral.*—Corresponding to the isogonal hexahedron there are two forms of the isohedral octahedron (fig. 6, (i) and (ii)).



(i) Fig. 6. (ii)

In (i) the vertices are all equiangular. For a convex polyhedron each of the face-angles must be $<90°$. Adjacent faces are mirror images of one another. The polyhedron is in three ways a double-pyramid on a rhombus as base (Brückner, No. 2′; No. 1′ when $b=c$; and the regular octahedron when $a=b=c$). It has always an inscribed sphere. If there is a circumscribed sphere the angles A′, B′, C′ of the spherical network are all right angles, and the polyhedron is regular.

In (ii) two opposite vertices are equiangular. There are two planes of symmetry (Brückner, No. 5′). With the same faces there are three different polyhedra, according as the equiangular vertices have the angles A, B, or C. There is also, with the same faces, one polyhedron of the type (i), and thus we have a set of four "isomeric" octahedra. One of these, of type (ii), may have a circumscribed sphere.

If there is a circumscribed sphere the angle A′ of the spherical network is $90°$ and $B'+C'=180°$ (see fig. 7). Then $b'+c'=180°$. Also $\sin \frac{1}{4}\pi = \sin \frac{1}{2}a' \sin B'$ and $\cos b' = \cot B'$. In the polyhedron

$$a = 2R \sin \tfrac{1}{2}a' = \sqrt{2}R \operatorname{cosec} B',$$
$$b = 2R \sin \tfrac{1}{2}b',$$
$$c = 2R \sin \tfrac{1}{2}c' = 2R \cos \tfrac{1}{2}b',$$
$$b^2 = 2R^2(1 - \cot B'), \quad c^2 = 2R^2(1 + \cot B');$$

therefore

$$b^2 + c^2 = 4R^2.$$

Eliminating R and B′ we find that

$$a^2(b^2+c^2)=b^4+c^4.$$

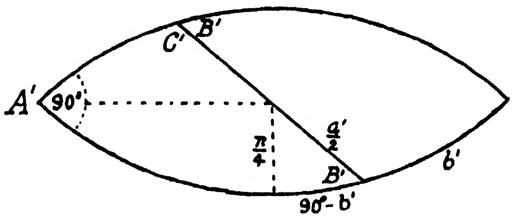


FIG. 7.

In non-euclidean geometry the condition for a circumscribed sphere is found in a similar way to be

$$\sin^2 \tfrac{1}{2}a(\sin^2 \tfrac{1}{2}b+\sin^2 \tfrac{1}{2}c)=\sin^4 \tfrac{1}{2}b+\sin^4 \tfrac{1}{2}c.$$

3.2. *Octahedron, Isogonal.*—At each vertex two adjacent dihedral angles are equal; the vertices are all directly congruent. Two opposite faces are then equiangular, angle D, and have equal dihedral angles γ.

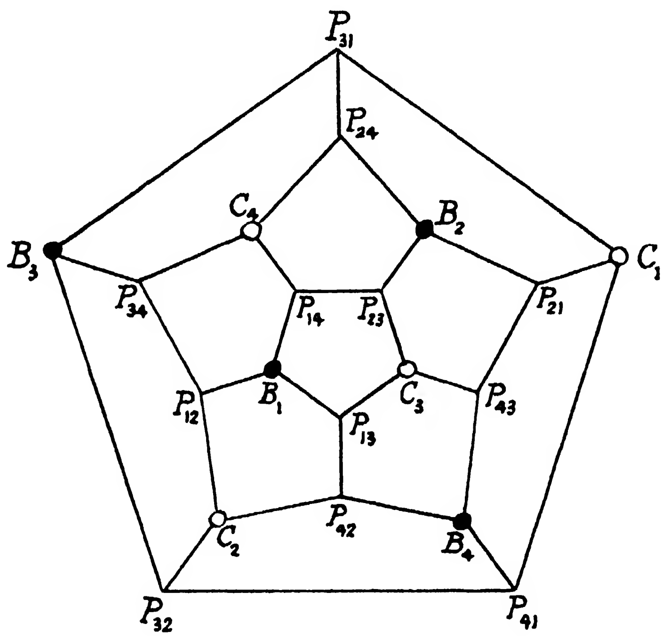


FIG. 8.

The other faces are all directly congruent, angles A, B, C, dihedral angles α, β, γ. The two equiangular faces are congruent and the line joining their centres is perpendicular to both of their planes (Brückner, No. 6; No. 3 if A = B).

The polyhedron has a circumscribed sphere. The simplest condition for an inscribed sphere is found by expressing that the spherical quadrilateral at any vertex has an inscribed circle. This is $C+D=A+B$. In euclidean geometry $D=\frac{\pi}{3}$ and $A+B+C=\pi$; hence in this case the condition is simply $C=\frac{\pi}{3}$.

4.1. *Pentagonal Dodecahedron, Isohedral.*—The faces are pentagons with two pairs of adjacent angles equal, *abbcc*. The faces are then all directly congruent. There are 4 vertices with three edges b, 4 with three edges c, and 12 with edges abc (Brückner, No. 19′; No. 14′ when $b=c$).

Denote the 4 b-vertices (fig. 8) by $B_1, \ldots, B_4$. B_1 is equidistant from 3 c-vertices; denote the remaining c-vertex by C_1, etc. The remaining 12 vertices are denoted by P_{ij}, this vertex being adjacent to B_i and C_j. Then we have 8 points forming a configuration isomorphic with the vertices of a cube, and with the twelve edges B_1C_2, etc., all equal. $B_1C_3B_2C_4$ form a skew equilateral quadrilateral. The 4 vertices B form a regular tetrahedron, and these have the same axes of symmetry, which are the lines joining the midpoints of opposite edges a. These three lines are mutually at right angles. Taking these as coordinate-axes the coordinates of the vertices are—

	B			C		
1	$-p$	p	p	q	$-q$	$-q$
2	p	$-p$	p	$-q$	q	$-q$
3	p	p	$-p$	$-q$	$-q$	q
4	$-p$	$-p$	$-p$	q	q	q

	P						
23	u	$-v$	w	u	v	$-w$	32
31	w	u	$-v$	$-w$	u	v	13
12	$-v$	w	u	v	$-w$	u	21
14	$-u$	v	w	$-u$	$-v$	$-w$	41
24	w	$-u$	v	$-w$	$-u$	$-v$	42
34	v	w	$-u$	$-v$	$-w$	$-u$	43

Expressing that the five points P_{43} P_{41} P_{21} B_4 C_1 are coplanar we obtain the relations

$$p=\frac{w(u^2-vw)}{u^2+v^2-uv+uw-2vw},$$

$$q=\frac{w(u^2-vw)}{u^2+v^2+uv-uw-2vw}.$$

Then the edges and diagonals of each pentagon face (fig. 9) are determined as follows:—

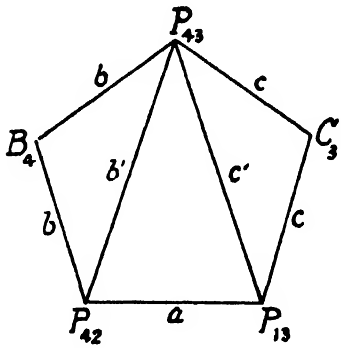


Fig. 9.

$$a^2 = 4(u^2 + v^2),$$
$$b^2 = (p-u)^2 + (p-v)^2 + (p-w)^2,$$
$$c^2 = (q+u)^2 + (q-v)^2 + (q-w)^2,$$
$$b'^2 = (v-w)^2 + (w-u)^2 + (u-v)^2,$$
$$c'^2 = (v-w)^2 + (w+u)^2 + (u+v)^2.$$

For a regular dodecahedron we find the conditions

$$u = 0, \qquad w = \tfrac{1}{2}(3 + \sqrt{5})v.$$

From the equality of the dihedral angles it is clear that the 3 faces at a vertex B all touch a sphere whose centre is O, and this sphere touches also all the faces at each of the B vertices, *i.e.* all 12 faces; hence there is always an inscribed sphere.

Condition for Circumscribed Sphere.—If there is a circumscribed sphere we have a spherical network in which the angles $B' = C' = \frac{2\pi}{3}$. Each face, such as ABDEC (fig. 10), is inscribed in a small circle. The centre S is the intersection of the bisectors of the angles at B and C. Then the angles

$$SDB = SBD = SBA = SAB = SAC = SCA$$
$$= SCE = SEC = 60°.$$

Also $A + D + E = 360°$, therefore

$$120° + 60° + SDE + 60° + SED = 360°$$

and

$$SDE = SED = 60°$$

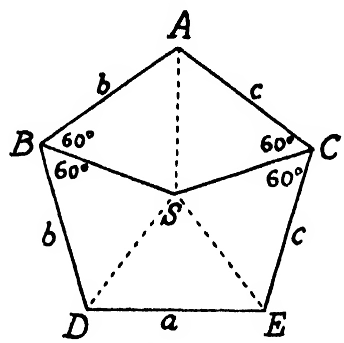


Fig. 10.

The pentagon is therefore equiangular. Further $\triangle SAC \equiv SAB$, therefore $b = c$, and $\triangle SDB \equiv SDE$, therefore $b = a$. The pentagons are therefore regular. Hence if the polyhedron has a circumscribed sphere it must be regular.

4.2. *Pentagonal Dodecahedron, Isogonal.*—Let α, β, γ be the dihedral angles at a vertex, A, B, C the face-angles. Then we find that there are 16 vertices for which $\alpha\beta\gamma$ are, say, clockwise, and 4 for which they are in the reverse order. There are three sorts of faces, the angles being

BAAAC, CBBBA, ACCCB.

Unless the polyhedron is regular it has neither a circumscribed nor an inscribed sphere. If there is an inscribed sphere, by § 0.4 there is a concentric circumscribed sphere. If there is a circumscribed sphere, the

spherical network has all its angles 120°, and the faces being cyclic pentagons are regular; hence the polyhedron is regular.

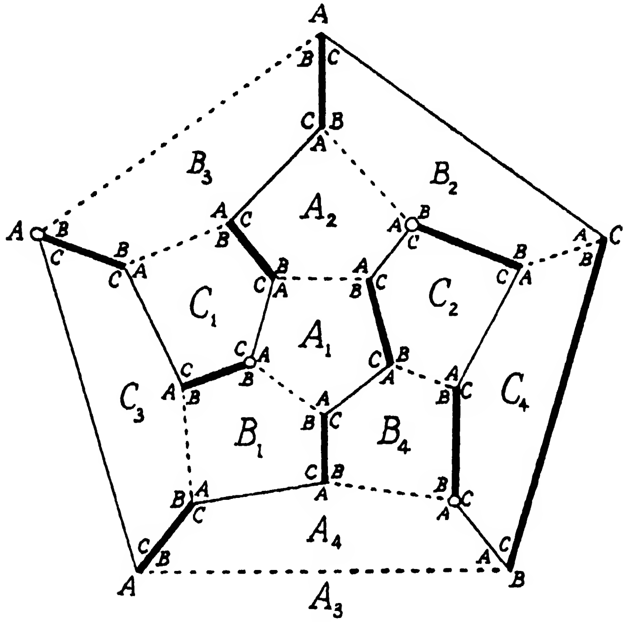


Fig. 11.

In euclidean geometry

$$3A + B + C = 3B + C + A = 3C + A + B = 540^\circ;$$

therefore $A = B = C = 108^\circ$, and therefore $\alpha = \beta = \gamma = \pi - \tan^{-1} 2$. The faces are therefore all equiangular and the dihedral angles are those of a regular dodecahedron. The polyhedron is therefore obtained from a regular dodecahedron by a process of truncation by planes parallel to the faces. It is therefore a variable form and is not one of Brückner's. If the lengths of three consecutive edges of one pentagon are chosen, the remaining two are determined. Then choosing the five edges which proceed from the 5 vertices, the edges of the second ring of pentagons are determined. If one more edge is chosen, we find that the last edge is determined twice. Four consecutive edges are connected by a fixed relation. Projecting perpendicular to XY (fig. 12), we have

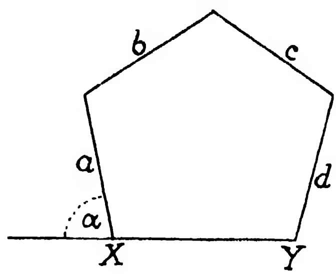


Fig. 12.

$$a \sin \alpha + b \sin 2\alpha = c \sin 2\alpha + d \sin \alpha,$$

i.e.

$$a - d = 2(c - b) \cos a.$$

We find that, having chosen 9 edges in this way, the equations determine the remaining 21 uniquely.

In non-euclidean geometry the angles A, B, C need not be equal, but the faces $A_1, \ldots, A_4$ are all congruent, and so also are $B_1, \ldots, B_4$ and $C_1, \ldots, C_4$.

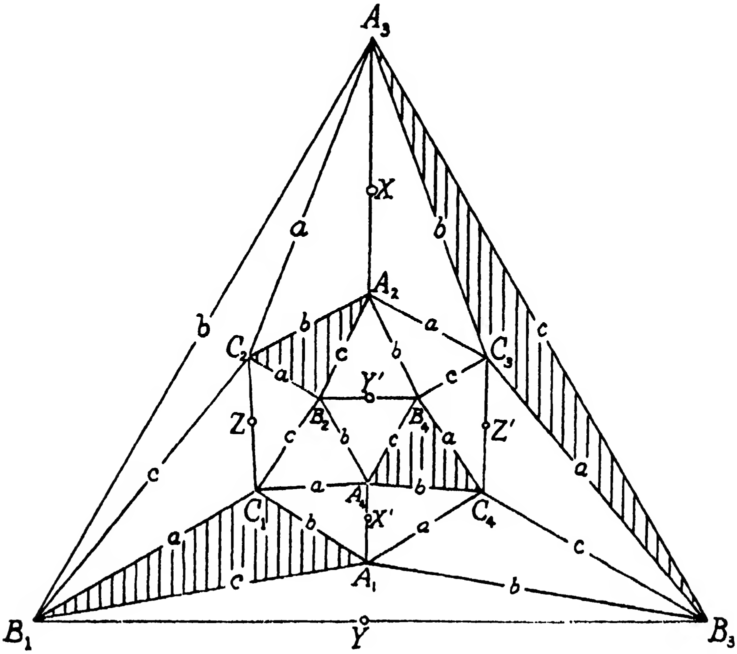


FIG. 13.

5.1. *Icosahedron, Isohedral.*—Four faces $A_1B_1C_1, \ldots, A_4B_4C_4$ (fig. 13) are, say, clockwise, the remaining 16 in the opposite sense. There are 4 sorts of vertices, $A_1, \ldots, B_1, \ldots, C_1, \ldots$, the plane-angles being

BAAAC, CBBBA, ACCCB.

Unless the polyhedron is regular it has neither an inscribed nor a circumscribed sphere. If it has a circumscribed sphere, by § 0.4, it has also a concentric inscribed sphere. If it has an inscribed sphere, it follows that all pairs of vertices, such as A_2 and A_3, which belong to adjacent faces, are equidistant from the centre; hence there is a circumscribed sphere. Then in the spherical network, the sums of the angles at the vertices

$$3A' + B' + C' = 3B' + C' + A' = 3C' + A' + B' = 360^\circ.$$

Hence $A' = B' = C' = 72^\circ$, and the faces are all regular.

Denote the midpoints of the edges A_2A_3, A_1A_4, B_3B_1, B_2B_4, C_1C_2, C_3C_4 by X, X′, Y, Y′, Z, Z′. The vertices A_1, A_2, A_3, A_4 are congruent, hence $XY' = XY = X'Y = X'Y'$. Similarly $YZ' = YZ = Y'Z = Y'Z'$ and $ZX' = ZX = Z'X = Z'X'$. Hence XX′, YY′, ZZ′ form three mutually perpendicular axes.

5.2. *Icosahedron, Isogonal.*—Corresponding to the case of the isohedral dodecahedron there must be two pairs of adjacent equal dihedral angles at

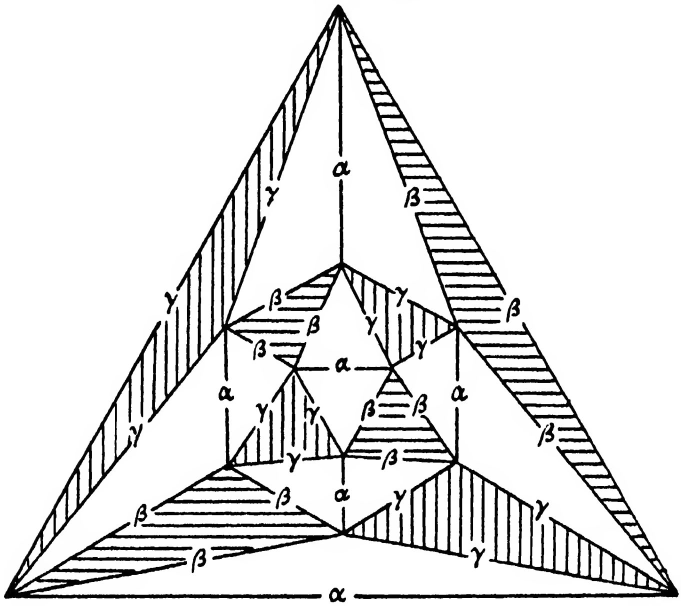


FIG. 14.

a vertex, and all the vertices are directly congruent. There are 4 equiangular faces with angles B′ and 4 equiangular faces with angles C′; the other 12 faces are congruent, with angles A, B, C.

The two sets of equiangular triangles may be unequal, one with side b, the other with side c (Brückner, No. 19), or they may be all equal (Brückner, No. 14).

There is a circumscribed sphere, but if there is an inscribed sphere the two sets of equiangular triangles are equidistant from the centre, therefore $b = c$, and then also $a = b$. Hence if the polyhedron has an inscribed sphere it must be regular.

(*Issued separately May* 19, 1932.)

XIII.—The Genetic Basis of Alkaptonuria.

By **Professor Lancelot Hogben**, M.A., D.Sc.; **R. L. Worrall**, M.B., Ch.B.; and **I. Zieve**, M.A., from the Department of Social Biology in the University of London. (With Six Plates.)

(MS. received January 19, 1932. Read May 2, 1932.)

1. Introduction.

In 1902 Garrod (33) published a short paper in which he put forward the hypothesis that alkaptonuria is determined by a single recessive Mendelian factor, or as we should now say, an autosomal recessive gene substitution. For two reasons this publication was a landmark in the history of human genetics. In the very year which witnessed the disclosure that Mendel's laws are applicable to animal breeding, it directed attention to the possibility of applying factorial analysis to clinical data and the possibility of interpreting *familial* conditions as recessive traits. In addition, it enunciated the principle which must remain the most powerful instrument for directing attention to the occurrence of recessive traits in a species whose matings cannot be regulated by the investigator to meet the more obvious requirements of Mendelian hypothesis. Garrod collected data concerning the incidence of first-cousin marriages among normal parents of alkaptonurics, and suggested the explanation that a person heterozygous for a rare recessive gene would be more likely to marry another heterozygote if he or she married a person who derived the gene from the same grandparent. From the genetical standpoint the salient facts which Garrod established are as follows:—

(i) That alkaptonuria is a congenital *familial* disorder, *i.e.* one which is generally manifest in more than one sib of the same fraternity.

(ii) That there is a striking excess of consanguineous unions among the parents of persons suffering from the disorder.

(iii) That with few exceptions neither parent of a person suffering from the disorder manifests it.

(iv) That alkaptonuria is much more common in males than in females.

Since the publication of Garrod's paper, considerable advances have been made in the theoretical analysis appropriate to the recognition of recessive gene substitutions in human beings. The literature of alkaptonuria has grown appreciably. Two reviews, one by Fromherz (1908) and one by King (1915), have summarised data subsequently collected with

results which confirm the conclusions stated above; but there has been no attempt to analyse the data by rigorous methods. This communication is an attempt to do so. As far as the authors have been able to achieve this task, the list of extant cases has been brought up to date. Before undertaking an examination of the completed list, some preliminary remarks concerning the nature of the disease and the theoretical requirements for a genetic interpretation of the facts elicited are necessary.

2. The Nature of the Condition.

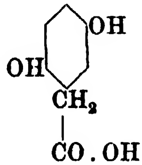
Alkaptonuria is a condition characterised by the fact that the urine rapidly becomes black on exposure to the air. Usually the darkening of the urine takes from two to eight hours. On account of its power of reducing Fehling's solution, alkapton is frequently detected in adults when the urine is examined for evidence of diabetes. In infants it is frequently noticed from a very early age on account of the staining of the napkins. With very few exceptions it makes its appearance in infancy and may be regarded in the overwhelming majority of cases as a congenital condition. Garrod states: "In the great majority of instances the anomaly dates from early childhood, but it has occasionally appeared as a temporary phenomenon during an illness or has apparently developed in later life." * In itself it is not necessarily associated with any particular disadvantage to the individual affected. It is, however, not infrequently associated with ochronosis, a condition which may involve joint affections as well as pigmentation of the skin, cartilages, and sclerotics. The ear-wax of alkaptonuric patients has been observed to be black in several cases. The sweat of the axilla and groin may assume a dark hue.

The condition of alkaptonuria is of clinical interest for the following reasons:—(1) It may be mistaken for diabetes; (2) it may give a positive Wassermann reaction; (3) it may be associated with dysuria.

A few instances of black urine are cited in old medical literature. The earliest appears to be due to Scribonius in 1584. The first clear account of an unmistakable case of alkaptonuria is due to Marcet writing in 1823. The word "alkapton" was compounded from two Arabic and Greek roots by Bödecker in 1857 to describe a reducing substance present in a specimen of urine which he received from Hasse to examine in connection with its peculiar behaviour towards alkaline copper solutions. Sixteen years later, Beale described a second case. In what also appears to have been alkapton urine Ebstein and Müller (1875) claim to have found pyrocatechin, and

* No such cases in which homogentisic acid appears in the urine have been certainly established in recent years.

in 1882 Maguire described the presence of protocatechuic acid. On this account some of the earlier cases of alkaptonuria in the literature have been referred to as pyrocatechinuria. In 1886, Marshall isolated from alkapton urine a crystalline substance which he called glycosuric acid. Volkow and Baumann (1899) subsequently named this substance *homogentisic acid*, and gave its empirical formula as $C_8H_8O_4$. Its constitution is that of hydroquinone acetic acid, *i.e.*:—



Homogentisic acid has since been synthesised by Neubauer and Flatow and by others. Subsequent to the work of Volkow and Baumann all investigators have found homogentisic acid in alkapton urine. Contrary findings in the earlier literature may be taken to have arisen from errors in method or impurities in reagents used. It is readily soluble in water, ether, and alcohol. It crystallises in colourless needles, with one molecule of water of crystallisation, having a melting-point of 147° C. Exposed to air, and more rapidly over sulphuric acid, the needles decompose to form a water-free acid with melting-point 156° C.

When voided, the urine of alkaptonuric patients is usually straw-coloured. Occasionally it is darker than normal on emission. It later becomes reddish brown to inky black, initially at the surface exposed to air. This change does not occur if air is excluded, or if the urine is strongly acid. Addition of an alkali immediately darkens it, the colour appearing first at the surface and slowly spreading throughout. Stirring hastens the change. King remarks: "The flash-like appearance of a chocolate-brown colour on slightly warming with Fehling's solution, once seen, will never be forgotten." Alkapton urine also reduces a cold ammoniacal solution of silver nitrate. A deep blue colour is produced when dilute ferric chloride is dropped into alkapton urine slowly. This quickly disappears and reappears with the next drop.

Alkapton urine is to be distinguished from that of a diabetic by the fact that it gives negative results with the bismuth, fermentation, phenylhydrazine, and polariscope tests. In the differential diagnosis, colouring or darkening of the urine from the use of drugs and phenol bodies must be excluded, as well as the condition of melanuria.

Homogentisic acid is certainly derived from proteins containing tyrosine and phenylalanine. Administration of either of these amino-acids in the

diet of an alkaptonuric increases its output in the urine. Some appear to be of endogenous origin, since the amount, though diminishing, does not disappear entirely during starvation. Concerning the relation of homogentisic acid to the fate of aromatic amino-acids in normal metabolism, two views have been held. One is that homogentisic acid is a normal stage in the breakdown of the aromatic protein complex. On this view the alkaptonuric patient may be regarded as lacking an enzyme necessary to break down homogentisic acid. Alternatively, it is possible that the breakdown of the aromatic protein complex follows a different path in the alkaptonuric patients. The latter view has been put forward by Dakin. In favour of the former, which Garrod supports, it is found that homogentisic acid does not appear in the urine of normal persons when it is included in their diet, unless taken in excessive quantities (Embden). The balance of evidence seems to favour the view that homogentisic acid is the final form in which the breakdown products of tyrosine and phenylalanine derived from the diet are excreted by an alkaptonuric patient. The data bearing upon this point are discussed in a recent communication by Braid and Hickman (1929). If alkaptonuria is primarily determined by a gene substitution, it illustrates how the manifestation of the presence of a gene may be restricted to the manufacture of a single chemical entity, which we may regard provisionally as the enzyme which catalyses the final breakdown of homogentisic acid in the human body.

3. The Theoretical Analysis of Single Recessive Gene Substitutions.

In the study of human genetics it is not possible to arrange that matings shall take place in such a way as to test the applicability of a given hypothesis in the simplest and most direct manner. To apply factorial analysis rigorously to the data provided by clinical pedigrees it is necessary to examine the consequences of different hypotheses in a corresponding system of mating, and to ascertain whether such consequences are realised. As a first approximation the theory of random mating may be utilised as the basis of such an investigation. A further complication arises from the fact that the mean size of the human family is small. Hence it is not always possible to treat selected matings on assumptions which would be valid for an infinite population.

The basic principle of random mating applied to a system of autosomal genotypes involving a single gene substitution is given by the equilibrium condition

$$l^2\mathrm{RR} : 2l(1-l)\mathrm{RD} : (1-l)^2\mathrm{DD} \quad . \quad . \quad . \quad . \qquad \text{(i)}$$

In this relation l^2 is the incidence of recessives in the population. From this it follows at once that the proportion of individuals with the constitution RR produced by matings RD × RD is:

$$(1-l)^2 \qquad \text{(ii)}$$

If a trait is determined by a sex-linked gene substitution the equilibrium condition in a system of random mating is given by:

$$\overbrace{\frac{s'}{2}\text{RY} : \frac{1-s'}{2}\text{DY}}^{\text{Males.}} : \overbrace{\frac{s'^2}{2}\text{RR} : s'(1-s')\text{RD} : \frac{(1-s')^2}{2}\text{DD}}^{\text{Females.}} \qquad \text{(iii)}$$

If the trait is recessive and sex-linked, the incidence of females in the population is therefore twice the square of the number of males, so that if the trait is rare (*i.e.* the incidence of males is a small fraction), affected females will be very much rarer than affected males.

For the recognition of recessive conditions, it follows from (ii) that affected individuals will almost always have parents who are both normal if the recessive trait is rare. Furthermore, matings between an affected and a normal individual will not usually give rise to any affected progeny. Genotypically such matings may be of two kinds, RR × RD and RR × DD. Only the first may have affected offspring. It follows from (i) that the proportion of matings between an affected parent and a normal parent having some affected offspring, when the size of the family is large, is:

$$\frac{2l}{1+l} \qquad \text{(iv)}$$

Since the human family is small, it will often happen that matings of the type RR × RD which might have affected offspring will not have them. The probability that an individual will be recessive is $\frac{1}{2}$ and the probability that an individual will be normal is $\frac{1}{2}$ if one parent of such an individual is recessive and the other a normal "carrier." The probability that all the sibs of an s-membered fraternity will be normal is therefore $(\frac{1}{2})^s$, so that only $1-(\frac{1}{2})^s$ of all s-membered fraternities which might contain affected members will actually contain them. If there are n_1 families of 1 sib, n_2 families of 2 sibs, . . . n_s of s sibs, we may therefore write for the proportion of matings of one affected and one normal parent with at least one recessive offspring:

$$\frac{2l}{1+l} \cdot \frac{\Sigma n_s(1-[\frac{1}{2}]^s)}{\Sigma n_s} \qquad \text{(v)}$$

This quantity will be smaller than (iv), and it is evident that, if the trait is very rare, the overwhelming majority of unions between an affected and normal parent will only produce normal offspring.

It is clear from this that recessive conditions will not be propagated by *direct transmission*. As Garrod was the first to realise, they can be recognised by two peculiarities, each susceptible of more precise treatment than had been devised at the time when his communication was published. These are (*a*) a high familial incidence; (*b*) an excess of consanguineous unions among the parents of recessives.

(a) *Familial Incidence.*—If p is the probability that the offspring of two heterozygous parents will be recessive, ($p=\frac{1}{4}$ and $q=\frac{3}{4}$) and $(p+q)=1$, the probability that all the members of a family of s individuals will be normal is q^s. Now a family must either be composed exclusively of normal sibs or else contain at least one recessive member. Therefore the probability that such a fraternity contains at least one recessive is $(1-q^s)$. The total number of sibs in n_s families of s is $s \cdot n_s$. To these observed families the corresponding number of normal sibs in families containing no recessives will be in the ratio $q^s : (1-q^s)$, so that the total theoretical population corresponding to r_s recessives in n_s families of s individuals of whom at least one is recessive must be:

$$s \cdot n_s + \frac{q^s}{1-q^s} \cdot s \cdot n_s = \frac{s \cdot n_s}{1-q^s} \quad . \quad . \quad . \quad . \quad . \quad \text{(vi)}$$

In general the observed data will include families of various sizes ranging from unity to c, the maximum size of the family. For the expected number of recessives (r_s) we may therefore write:

$$\frac{\sum_1^c r_s}{\sum_1^c \frac{s \cdot n_s}{1-q^s}} = p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(vii)}$$

Or, more conveniently,

$$\sum r_s = p \sum_1^c \frac{s \cdot n_s}{1-q^s} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(viii)}$$

The quantity on the right-hand side is now the expected number of affected offspring, and may be compared with the observed number of affected (Σt_s). The standard deviation of Σr_s as defined in (viii) is given by

$$\sigma^2 = \Sigma n_s k_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(ix)}$$

where

$$k_s = \frac{s(1-q)}{[1-q^s]^2}\{sq^s(q-1)+q(1-q^s)\}.$$

Tables of k_s and of $\frac{s}{1-q^s}$ for the calculation of (ix) and (viii) respectively are given by Hogben (40).

Where the data are not sufficiently extensive to permit the application of such precise analysis with confidence, it is useful to attach more definite significance to what is meant by saying that a disease is *familial.* As a criterion of significant familial incidence we might take the proportion of families in which only one individual is affected to all families in which at least one is affected. If the probability that an individual drawn at random from a community is affected be $1-m$ and the probability that an individual drawn at random from a community is not affected be m, for a group of n_s s-membered fraternities the number of families with at least one affected member will be:

$$n_s(1-m^s) \qquad \text{(x)}$$

Now the number of families in such a group with more than one affected member is:

$$n_s[1-m^s-s(1-m)m^{s-1}] \qquad \text{(xi)}$$

So the proportion of families having more than one affected in a group of families having at least one affected is:

$$\frac{\Sigma n_s[1-m^s-s(1-m)m^{s-1}]}{\Sigma n_s[1-m^s]} \qquad \text{(xii)}$$

If the probability that one sib will be affected is not related to the probability that another sib will be affected by any mechanism involved in the fact that (*a*) they share the same parents or (*b*) share a similar uterine or family environment, it is easily seen that (xii) rapidly approaches a limit of zero. If a trait is *familial* because it is determined by one or more recessive genes, the proportion of affected in a group of families having at least one affected sib is:

$$\frac{\Sigma n_s[1-q^s-spq^{s-1}]}{\Sigma n_s[1-q^s]} \qquad \text{(xiii)}$$

In this p is the expectation of recessives among the offspring of parents of a particular genotype which for reasons given may both be taken to be RD when the trait is rare. Now even when the number of genes involved is numerous, the ratio (xiii) will be significantly high compared with (xii) since the values of p tend to finite limits of $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, etc., as the trait becomes increasingly rare. So for a rare trait the familial incidence denoted by the ratio defined by (xii) or (xiii) will be readily recognisable even when many genes are involved. It is worth while illustrating this point numerically by reference to a group of families of four sibs. If the size of the family is uniform, (xii) becomes

$$1-\frac{s(1-m)m^{s-1}}{1-m^s}=F_s \qquad \text{(xiv)}$$

For traits which are not familial in the sense defined above, with the incidence specified below, this ratio assumes the following values when $s=4$:

$1-m$.	F_4.
1 : 1000	·00150
1 : 10,000	·00015
1 : 100,000	·000015

If on the other hand a trait is determined by n independent recessive gene substitutions, the limiting value at infinite dilution for (xiv) is obtained when m is replaced by $q=\frac{3}{4}, \frac{15}{16}$, etc. This gives:

n.	F_4.
1	0·383
2	0·094
3	0·023
4	0·006

From this it is clear that a trait which is rarer than one in a thousand will yield a family incidence defined by the proportion of families with more than one affected sib noticeably in excess of what would occur in a system of random mating, even when four recessive genes determine the trait.

(b) *Consanguineous Unions.*—The genetical theory of first-cousin marriages has been developed by Lenz (49) and by Dahlberg (20). Lenz assumes that the trait is so rare that only unions between normal (*i.e.* heterozygous) parents need be considered. His treatment gives the incidence of cousin marriages among heterozygous parents of recessive offspring as:

$$\frac{a}{a+16l(1-l)(1-a)} \quad . \quad . \quad . \quad . \quad . \quad (\text{xv})$$

where a is the incidence of first-cousin marriages in the general population and l^2 is the incidence of the recessive trait.

Dahlberg approaches the problem from a consideration of the distribution of the chromosomes in a single inbred line. This leads to the approximate relation

$$\frac{\frac{a}{16}(l+15l^2)}{\frac{a}{16}(l+15l^2)+(1-a)l^2} \quad . \quad . \quad . \quad . \quad . \quad (\text{xvi})$$

This tends to the same limit as (xv), viz.:

$$\frac{a}{a+16l}.$$

It should be noted that excess of cousin marriages among parents of affected will only be expected if the trait is determined by an autosomal

gene substitution. The considerations stated above do not apply to sex-linked conditions because a normal male parent having only one x chromosome cannot be a carrier. Hence consanguinity will not increase the probability that affected offspring will occur in unions of two normal parents or unions of a normal male and an affected female parent. It will increase the probability that affected individuals will occur among offspring of unions between normal female and male affected parents if the consanguinity occurs on the maternal side from which the male must derive his x chromosome. However, such unions will be rare if the trait is rare. The majority of affected individuals will be produced by unions of two normal parents when a recessive trait is sex linked as when it is determined by a recessive autosomal gene. For sex-linked conditions, using the symbolism of (ii) the actual ratio of affected individuals produced by unions of (i) two normal parents, (ii) a normal female and an affected male, (iii) an affected female and a normal male will be:

$$(1-s') : 2s' : s'.$$

4. Single Cases.

Sex-linked inheritance was discovered by Leonard Doncaster in 1905. When Garrod's original paper on the genetic basis of alkaptonuria appeared, the peculiarities of transmission involving recessive gene substitutions on the x chromosome were not known. Consequently no attempt was made to explain the preponderance of males among alkaptonuric patients. In the light of what we now know it is necessary to ask whether alkaptonuria is a recessive sex-linked trait or whether the condition exists in more than one form, of which one is sex linked. It is hence important to collect all the extant data bearing on the sex ratio.

Writing in 1923, Garrod states that 120 cases of alkaptonuria had then been recorded. The total number of cases up to date embodied in the ensuing tables amounts to 151.

In Table I are summarised "single cases," *i.e.* examples of the disease recorded without reference to the condition or relationship of the parents and the familial incidence. As regards Stange's case, it is stated that another member of the family was observed by the mother to have a similar condition. Of Graanboom's case, it is stated no other members of the same family show the peculiarity. Of Mitchell's patient, it is stated that the parents were probably related. Bilderback states that the family history of his patient was negative. J. Bauer states that there is no known case of alkaptonuria among the ancestors of his patient. H. Baar

TABLE I.—TABLE OF ALL ISOLATED CASES UP TO 1915 OF UNKNOWN FAMILY HISTORY.

Author.	Date.	Sex.	Age (Years).
1. Bödeker	1859	Male	44
2. Johnson	1864	Male	Unknown
3. Fürbringer	1875	Male	29
4. Ebstein and Müller	1875	Male	Infant
5. Fleischer	1875	Male	Unknown
6. Schmitt	1882	?	?
7. Maguire	1884	Female	27
8. Brune	1886	Male	(Child)
9. Geyger	1892	Male	?
10. Garnier and Voisin	1892	Male	?
11. Slosse	1895	Female	?
12. Stange	1896	Male	18
13. Moraczcewski	1896	Female	43
14. Deniges	1897	Male	50
15. Hirsch	1897	Female	17
16. Galloway	1898	?	?
17. Zimnicky	1900	Male	45
18. Mittelbach	1901	Male	44
19. Albrecht	1902	Male	47
20. Ysendyck	1906	Male	Infant
*21. Garrod and Clarke	1907	Female	3
22. Blum	1907	Male	34
23. Clemens	1907	Male	31
24. Abderhalden	1907	Male	?
25. Wagner	1908	Male	31
26. Graanboom	1908	?	Infant
27. Mitchell	1910	Male	46
28. Van Amstel	?	Female	42
29. Spencer	1910	Male	?
30. Ashburn	1911	Male	?
31. Triboulet and Bougalt	1912	Female	Infant
32. Söderburg	1913	Male	Infant
33. Gebhardt	1913	?	?
34. Steensma	1914	?	1
35. Scheltema	1914	Female	6
36. King	1915	Male	56
37. Gibson and Howard	1921	Male	44
38. Lereboullet	1920	Female	?
39. Bilderback	1922	Male	Infant
40. Baar	1925	Male	Infant
41. J. Bauer	1928	Male	54
42. Reinwein	1931	Male	54

* In a private communication Garrod informs us that this child was at the time the only child of two normal unrelated parents.

states that the relations of the child he examined are healthy and have no similar abnormality. These data were not regarded as sufficient to justify including these cases in Table II, which summarises those concerning which relevant family particulars are available.

TABLE II.

Author or Observer.	Date.	Condition of Parents.	Relationship of Parents.	Alkaptonurics. Female.	Alkaptonurics. Male.	Total known Number of Sibs.
1. Kirk	1886–9	Both normal	First cousins	0	3	4
2. Armstrong and Smith; Baker and Garrod.	1882–99	Both normal	Not related	1	1	3
3. Marshall and Futcher	1887–98	Both normal	Not related	0	2	?
4. Baumann and Embden	1891–3	Both normal	Not related	1	1	2
5. Ogden	1895	Both normal	First cousins	0	1	8
6. Noccioli and Dominici	1898	Both normal	Not related	1	0	11
7. Garrod (Völker's case)	1899–02	Both normal	First cousins	0	2	5
8. Garrod (Pavy's case)	1899	Both normal	First cousins	1	3	14
9. Stier	1898	Both normal	Not related	0	1	$2+x$
10. Winternitz . .	1899	Both normal	?	2	1	7
11. Meyer	1901	Both normal	First cousins	0	1	3
12. Langstein and Meyer	1903	?	?	0	1	6
13. Schumm	1904	Mother normal Father unstated	?	1	1	6
14. Zimper	1903	Both normal	?	0	2	10
15. Gerhardt	1904	Both normal	?	0	2	$3+x$
16. Bandel	1906	?	Not related	1	1	4
17. Grutterink . .	1907	Both normal	First cousins	1	0	?
18. Grutterink . .	1907	Both normal	Not related	0	2	?
19. Grutterink . .	1907	Both normal	Not related	2	2	?
20. Landois	1908	Both normal	?	0	1	3
21. Rocher and Bassot .	1909	?	?	1	0	2
22. Ravold and Warren .	1910	Both normal	Not related	1	0	2
23. Kolaczek	1911	Both normal	First cousins	3	0	11
24. Poulsen (Case IV) .	1911	Both normal	Not related	1	0	2
25. Poulsen (Case VI) .	1911	Both normal	First cousins	1	0	6
26. Poulsen (Case VII) .	1911	Both normal	First cousins	0	1	1
27. Poulsen (Cases VIII–IX)	1911	Mother normal Father doubtful	First cousins	0	2	7
28. Poulsen	1912	Both normal	?	0	1	4
29. Baldwin	1913	Both normal	Not related	1	0	5
30. Katsch	1918	Both normal	Not related	2	2	9
31. Ebstein	1918	Both normal	?	0	2	3
32. Debendetti . .	1920	Both normal	First cousins	0	4	6
33. Toenniessen . .	1922	Both normal	Not related	1	1	8
34. Kleinschmidt . .	1922	Both normal	Not related	0	1	2
35. Toenniessen . .	1922	Both normal	Not related	1	2	11
36. Cuthbert	1923	Both normal	First cousins	1	2	6
37. Lockwood . .	1924	?	?	2	1	7
38. Young	1924	Both normal	Not related	0	1	1
39. Bose and Ghosh . .	1929	?	Consanguine, of unknown degree	0	1	10
40. Sinha	1930	Apparently not alkaptonuric	Apparently not related	1	1	3
41. Bagnall	1929	Both normal	Not related	0	1	1
42. Braid and Hickman .	1929	Both normal	Not related	0	1	1
43. Rahmlow . .	1930	Both normal	?	0	1	5
44. Sachs	1931	Both normal	Not related	1	1	8
45. O. Bauer	1930	Both normal	?	0	1	2

5. Matings of Two Normal Parents.

In Table II are summarised forty-five fraternities of which there is some definite information concerning the condition and relationship of the parents and the family incidence of the disease. In most of these both parents are definitely stated to be normal. In the remainder they may be presumed to be normal. With reference to this table it is necessary to remark that some authors referred to in the earlier reviews by Garrod (33), Fromherz (25), and King (43) are not included in it. This is because it has sometimes happened that subsequent investigation by another author has revealed the occurrence in the same fraternity of an additional sib examined or born later. Some relevant information with reference to the earlier pedigrees is based on private communications made by the authors to Garrod, and stated in his review published in 1902.

An examination of the data contained in Table II prompts inquiry into three issues: (*a*) the familial nature of the disease; (*b*) the conformity of the numerical data to the requirements of genetic hypothesis; (*c*) the incidence of consanguineous unions among the parents of alkaptonurics. Before considering these in detail, it is necessary to bear in mind the great rarity of the disease. During the last half-century less than 150 cases have been recorded in medical literature. In view of the fact that this anomaly is readily noticed in infancy and peculiarly liable to be detected in medical examinations for life insurance, etc., it may be presumed that the number of recorded cases constitute a very appreciable fraction of those which have actually occurred. It is most improbable that the incidence of the disease is greater than one in a million, and possibly an estimate of one in ten million would correspond more closely to its occurrence.

(*a*) *Familial Incidence.*—In Table II there are 20 out of 45 sibships in which only 1 individual occurs. That is to say, there are 25 sibships in which more than 1 individual occurs. Thus the ratio of families with more than 1 affected sib to all fraternities containing affected sibs is $\frac{25}{45} = 0{\cdot}56$. As the most conservative estimate, on the evidence available, we might include the cases in Table I of which with one exception only one sib is known to be affected. This would make the ratio defined in (xii), (xiii), and (xiv) $\frac{26}{86} = 0{\cdot}302$.

It is evident from the considerations advanced in Section 3 and the extreme rarity of the condition that the occurrence of more that one sib in a fraternity is vastly higher than could be accounted for by pure chance.

(*b*) *Factorial Analysis.*—To submit these results to factorial analysis it is necessary to reject all those fraternities of which the condition of the

parents is uncertain. It is also advisable to eliminate those of which the total known number of sibs is not definitely stated. This reduces the number of fraternities to thirty-seven.

If both parents are definitely stated to be normal, and also have affected offspring, the hypothesis that the condition is determined by a single recessive gene substitution presupposes that each parent is heterozygous, *i.e.* carries the gene on one chromosome but not on the other. If both parents are heterozygous, the probability that any one of their offspring will be affected is $\frac{1}{4}$, and the probability that any one of their offspring will be normal is $\frac{3}{4}$. Since the human family is small, there will be many small families with two heterozygous parents having only normal offspring. These are not included in our census, hence it is not legitimate to divide the total number of individuals in the observed fraternity by four in order to obtain the expected number of affected individuals. The true expectation must be obtained by use of equation (viii), in which the value of q is assigned as $\frac{3}{4}$. The expectation in the last column but one on the right-hand side of Table III, which summarises the thirty-seven selected pedigrees, is calculated on this basis. The variance (σ^2) for each group of families is given by the column on the extreme right. The total expectation is 61·9 affected individuals. The standard deviation of this number is $\pm\sqrt{23{\cdot}53}$. The discrepancy between the expected number 61·9 and the observed number 66 is 4·1. This is less than the standard deviation (4·8) of the former, and is not significant.

TABLE III.

Known Size of Fraternity	Pedigree No. in Table II.	n_s.	Alkaptonuric Members. Observed.	Alkaptonuric Members. Expected.	$\sigma^2 = k_s n_s$.
1	17. 26, 38, 41, 42	5	5	5	0·00000
2	3, 4, 9, 21, 22, 24, 34, 45	8	10	9·14	0·97960
3	2, 11, 20, 31, 40	5	8	6·48	1·31485
4	1, 28	2	4	2·93	0·84010
5	7, 29, 43	3	4	4·92	1·77534
6	25, 32, 36	3	8	5·47	2·32785
7	10, 27	2	5	4·04	1·94048
8	5, 33, 44	3	5	6·67	3·51720
9	30	1	4	2·43	1·38020
10	14	1	2	2·65	1·59170
11	6, 23, 35	3	7	8·61	5·41590
14	8	1	4	3·56	2·44640
	Total .	37	66	61·90	23·52962

(*c*) *Incidence of Consanguinity.*—A familial trait determined by two independent dominant genes, however rare, does not involve any excess of consanguinity among parents of individuals who exhibit the trait. On the other hand, there will be a considerable excess of consanguineous unions among parents of individuals displaying a rare recessive trait. The data embodied in Table II may be summarised as follows:

TABLE IV.

Relationship of Parents.	No. of Fraternities.	Alkaptonurics.		Total.
		Female.	Male.	
First cousins	12	7	19	26
Consanguineous (degree unknown) .	1	0	1	1
Not ascertained	12	6	14	20
Unrelated	20	15	21	36
Total . .	45	28	55	83

It is thus seen that of 33 unions concerning which the relationship of the parents has been ascertained 12 were first-cousin marriages, 20 were unions of unrelated persons, and 1 was a consanguineous union of unknown degree. There are 27 alkaptonurics with consanguineous parents out of a total of 63 alkaptonurics concerning whose parents relationship has been ascertained. This would make the percentage of consanguineous marriages 43 per cent., or of first-cousin marriages among parents of alkaptonurics 42 per cent. To take the most conservative estimate, there are in Tables I and II 125 individuals, of whom 28 are known to have been offspring of consanguineous unions. This would still make the percentage of consanguineous unions among parents of alkaptonuric individuals higher than it is for any disease hitherto recorded. This fact is especially noteworthy, because alkaptonuria is probably more rare than any disease which has hitherto been made the subject of genetic analysis

The incidence of first-cousin marriages in European communities may be taken to be about 8 in 1000. Pearson gives 0·81 per cent. as the figures for patients in London hospitals. In France during the first decade of the present century the percentage of cousin marriages was 0·87. In Prussia during the last quarter of the nineteenth century it was 0·59. If we take provisionally the incidence of alkaptonuria as one in a million, applying the approximate formula based on equation (xv),

we have:

$$\frac{a}{a+16l}=\frac{\frac{8}{1000}}{\frac{8}{1000}+\frac{16}{1000}}=33\%.$$

If the incidence were one in ten million, the percentage of cousin marriages expected would be about 60 per cent. As far as we can form an estimate of the incidence of the disease on the one hand, and the incidence of consanguineous percentage, which varies in different social strata, it is clear that the frequency of first-cousin marriages is of the order of magnitude predictable from the Principle of Segregation in a system of random mating.

6. Direct Transmission.

It has been pointed out that if a trait is determined by a single recessive gene, it will rarely happen that unions of one affected and one normal parent will produce any affected offspring. It is therefore extremely unlikely that alkaptonuric parents will bear alkaptonuric offspring if the anomaly has its basis in a single recessive gene substitution. There seem to be 7 cases of fraternities exclusively composed of normal sibs having an alkaptonuric parent. These are summarised in Table V. It is to be noted that 2 out of these 7 are fraternities of single individuals, and if a fraternity consists of only one individual with an affected parent, the odds in favour of such an individual being affected or normal are equal, when the other parent is a carrier.

Table V.—Unions of One Affected Parent with no Affected Offspring.

Author.	Number of Sibs.
5. Ogden (1895)	3
23. Kolaczek (1911) (three fraternities) .	13, 6, 6
31. Ebstein (1918)	5
35. Toenniessen (1922)	1
37. Lockwood (1924)	1
Total number of fraternities: 7.	

Compared with the number of recorded cases in which transmission from parent to offspring does not occur, the extant observations upon direct transmission recorded in Table VI are formidably great. In Table VI there are 8 fraternities, of which 7 are known to have an affected parent. These 8 fraternities occur in 5 separate pedigrees. Of these, 4 are not

included in Table II. Osler's case is the third generation of the pedigree originally investigated by Marshall and Futcher. The last pedigree due to Pieter is peculiarly significant. There is no doubt whatever that this pedigree represents transmission by a simple autosomal gene which behaves

TABLE VI.—UNIONS OF ONE AFFECTED PARENT WITH SOME AFFECTED OFFSPRING.

Author.	Total Number of Sibs.	Known Affected Members.		
		Female.	Male.	Total.
3. Osler (1902)	2	0	1	1
46. Orsi (1899)	2	1	1	2
47. Fromherz (1908)	12	0	3	3
48. Toenniessen (1922)	8	3	1	4
49. Pieter (1925):				
(*a*) First generation	1	0	1	1
(*b*) Second ,,	1	0	1	1
(*c*) Third ,,	10	2	3	5
(*d*) Fourth ,,	10	1	5	6

as a dominant. Further, as will be seen from the explanatory notes on the pedigree plates given below, there is no reason to doubt that this was a genuine case of alkaptonuria. We are therefore compelled to draw one of two conclusions. The first is that there are genetically two types of alkaptonuria in human beings just as there are genetically four types of white plumage in fowls. The second is that the gene substitution which usually manifests itself as a recessive can, in a restricted range of variation, behave as a dominant in what might be called a different genetic configuration. The latter view is consonant with R. A. Fisher's hypothesis concerning the origin of dominance.*

If, then, as we are compelled by Pieter's pedigree to conclude, there exists a dominant type or mode of transmission, the remaining four cases

* In a private communication to the senior author, Professor J. B. S. Haldane calls attention to a system of genotypes which may provide a clue to such cases as the above which is illustrative of a difficulty frequently met with in human genetics (*e.g.* Leber's disease, night-blindness and retinitis pigmentosa). In *Primula sinensis* D and G are two genes which inhibit pigment formation in the corolla. The following phenotypes arise from different combinations of D, G, and their allelomorphs :—

DDGG, DdGG, DDGg, DdGg	White corolla.	ddGg, ddGG	Pink corolla.
ddgg	Red corolla.	DDgg	White with a pink centre.
		Ddgg	Pink with a red centre.

of direct transmission do not offer any formidable difficulties to genetic interpretation. Of the four, one is of doubtful significance. Fromherz states that the mother of this fraternity, who was not related to the father, apparently showed an intermittent alkaptonuria. He remarks, however, that the proof is not altogether free from objection. It is not known whether either of the parents of the affected parent in Orsi's pedigree were themselves affected, nor whether they were related. The affected parent in Toenniessen's pedigree is not known to be the offspring of normal individuals. It is stated that the affected parent was not related to his wife. The fact that one affected individual in the F_1 generation had two offspring, both normal, provides no conclusive evidence concerning the mode of inheritance in this pedigree. As regards Osler's case, the affected parent was the offspring of two normal individuals who were not related. Of the cases recorded in Table VI, there is no evidence to exclude the hypothesis that they illustrate the more rare dominant type. The one case of an affected parent who was the offspring of two normal parents may have arisen as a mutation. The absence of consanguinity in these pedigrees is worthy of comment, since a high incidence of consanguinity would be expected among unions of a normal parent and a parent displaying a rare recessive trait in cases of direct transmission.

7. The Sex Ratio.

A summary of all the data upon the abnormal sex ratio in alkaptonuria is given in Table VII. To the figures in Table VI have been added the original parents of Orsi's case (a female), Fromherz (a female), and Toenniessen (a male). This brings the total number of recorded cases up to 151, of which 100 are males and 46 are females.

Table VII.

	Males.	Females.	Unknown.	Total.
Table I	28	9	5	42
Table II	55	28	..	83
Table VI	17	9	..	26
Total	100	46	5	151
Expected	75·5±6·1	75·5±6·1	..	..

There is no special circumstance concerning the distribution of the sex ratio in the three groups of data to throw light on this anomaly. An

excess of males might be interpreted in four ways. The first is that the condition is sex linked. This is definitely excluded in 21 out of the 45 pedigrees in Table II by the occurrence of female offspring of two normal parents. Furthermore, with a trait so rare as alkaptonuria the likelihood that any females would occur is indefinitely small as shown by equation (iii). The subsidiary hypothesis that one form of alkaptonuria is sex linked and one form is not is excluded by the high incidence of cousin marriages in the group of fraternities with only male alkaptonurics. A second hypothesis which might account for the excess of males is that the manifestation of the double recessive constitution is subject to greater somatic variability in females than in males. If this were true, the observed family incidence should be lower than that predicted by Mendelian hypothesis for a gene substitution which is not significantly influenced by environment. Actually, the discrepancy recorded is a slight excess rather than a deficiency. A third hypothesis to account for the smaller number of females is that the condition may be semi-lethal in the female. There is no clinical evidence for this belief, and if it were true we should be led to the same deduction as that which follows from the preceding hypothesis. This, as stated, is controverted by the facts at present available.

There remains a possibility that the true explanation of the anomalous sex ratio is sociological rather than biological. Alkaptonuria is in general a harmless condition which is only in special circumstances brought to the notice of the medical profession. In several recorded cases it is specifically stated that the patients were diagnosed as alkaptonuric late in life when they were examined for life insurance. It cannot be doubted that males are more likely to present themselves for routine medical examinations which involve testing the urine. Another consideration which lends plausibility to the fourth suggestion is the fact that males are less disposed to reticence concerning micturition than females. Attractive as such an hypothesis might be at first sight, the data presented do not provide any conclusive proof for it. Indeed one circumstance might be quoted as unfavourable to such a view. This is, that out of 9 infants listed in Table I, 6 are known to be males and 1 only is definitely stated to be a female.

8. Summary.

1. The overwhelming majority of cases of alkaptonuria are consistent with the hypothesis that this disease is determined by a single autosomal recessive gene substitution.

2. The observed proportion of alkaptonurics in recorded fraternities with two normal parents and the incidence of consanguineous parentage conform quantitatively to the requirements of Mendelian hypothesis in a system of random mating.

3. There appears to be an alternative form of alkaptonuria which is dominant.

Grateful acknowledgment is made to Sir Archibald Garrod for advice regarding the sources consulted.

9. Bibliography.

(*Note.*—References of single cases omitted from this list are given in the publications of Fromherz (25) and King (43).)

1. Abderhalden, 1907, *Zeitschr. f. phys. Chem.*, vol. lii, p. 435.

2. Albrecht, 1902, *Zeitsch. f. Heilkunde*, p. 366.

3. van Amstel, 1909–10, *Med. Weekbi. Amst.*, vol. xvi, pp. 527, 540, 557, 564, 575, 587.

4. Armstrong, 1882, *Dublin Journ. of Med. Sci.*, vol. lxxiii, p. 53.

5. Baar, 1925, *Wiener med. Woch.*, vol. lxxv, p. 878.

6. Baldwin, 1913, *Amer. Journ. Med. Sci.*, vol. cxlv, p. 123.

7. Bagnall, 1929, *New England Journ. of Med.*, vol. cci, p. 422.

8. Bandel, 1906, *Münch. med. Woch.*, vol. liii, p. 1091.

9. Bauer, J., 1928, *Wiener med. Woch.*, vol. lxxviii, p. 839.

10. Bauer, O., 1928–30, *Mitteilungen aus dem Grenzgebieten der Med. u. Chir.*, vol. xli, p. 451.

11. Baumann and Kraske, 1891, *Münch. med. Woch.*, p. 1.

12. Baumann and Wolkow, 1891, *Zeitschr. f. phys. Chem.*, vol. xv, p. 228.

13. Bilderback, 1922, *Amer. Journ. of Dis. of Children* (Chicago), vol. xxiii, p. 259.

14. Blum, 1907, *Münch. med. Woch.*, p. 1055.

15. Bödeker, 1859, *Zeitschr. f. rat. Med.*, p. 130.

16. Bose and Ghosh, 1929, *Ind. Med. Gaz.*, vol. lxiv, p. 61.

17. Braid and Hickman, 1924, *Arch. of Dis. in Childhood*, vol. iv, p. 389.

18. Clemens, 1907, *Vortrag a. d. Cong. f. in. Med.* (Discussions).

19. Cuthbert, 1923, *Lancet*, No. I, p. 2593.

20. Dahlberg, 1930, *Journ. Genetics*, xiv; 1930, *Hereditas*, xiv.

21. Debendetti, 1920, *Policlin. Rom.* (sez. Prat.), vol. xxvii, p. 1379.

22. Deniges, 1897, *Journ. de Pharm. et de Chem.*, p. 50.

23. Ebstein and Müller, 1875, *Virchows Arch.*, p. 554.

24. Fleischer, 1875, *Berl. klin. Woch.*, vol. xii, pp. 529 and 547.

25. Fromherz, 1908, *Über Alkaptonurie*; Dissertation, Strassburg.

26. Fürbringer, 1875, *Berl. klin. Woch.*, vol. xii, pp. 313 and 330.

27. Futcher, 1898, *New York Med. Journ.*, vol. lxvii, p. 69.

28. Galloway, 1898, *Brit. Journ. of Dermatology*, p. 450.
29. Garnier and Voisin, 1892, *Arch. de Physiol.*, p. 225.
30. Garrod, 1899, *Trans. Med.-Chir. Soc.*, vol. lxxxii, p. 367.
31. Garrod, 1901, *Lancet*, No. II, p. 1484.
32. Garrod, 1902, *Trans. Med.-Chir. Soc.*, vol. lxxxv, p. 69.
33. Garrod, 1902, *Lancet*, vol. ii, p. 1616.
34. Garrod and Clarke, 1907, *Biochem. Journ.*, p. 217.
35. Gerhardt, 1904, *Münch. med. Woch.*, vol. li, p. 176.
36. Geyger, 1892, *Pharm. Zeitung*, p. 488.
37. Gibson and Howard, 1921, *Trans. of Assoc. of Amer. Physicians*, vol. xxxvi, p. 258.
38. Graanboom, 1908, *Ned. Tijdschr. v. Geneesk.*, p. 1741.
39. Grutterink, 1907, *Ned. Tijdschr. v. Geneesk*, No. II, p. 1117.
40. Hogben, 1931, *Journal of Genetics.*
41. Johnson, 1864, *Urine and Urinary Deposits.*
42. Katsch, 1918, *Münch. med. Woch.*, vol. lxv, No. II, p. 1337.
43. King, E. F., 1915, *Trans. of Amer. Urological Assoc.*, vol. ix, p. 459.
44. Kirk, 1886, *Brit. Med. Journ.*, vol. ii, p. 1017.
45. Kleinschmidt, 1922, *Frankfurter Zeitschr. f. Pathologie*, vol. xxviii, p. 73.
46. Kolaczek, 1911, *Beiträge z. klin. Chir.*, vol. lxxi, p. 254.
47. Landois, 1908, *Arch. f. pathol. Anat. u. Physiol. von Virchow*, vol. cxciii, p. 275.
48. Langstein and Meyer, 1903, *Deutsches Arch. f. klin. Med.*, vol. lxxviii, p. 161.
49. Lenz, 1919, *Münch. med. Woch.*, vol. lxvi.
50. Lereboullet, 1920, *Le progrès médical*, p. 452.
51. Lockwood, 1924, *Journ. Amer. Med. Assoc.*, vol. lxxxiii, p. 2091.
52. Maguire, 1884, *Brit. Med. Journ.*, p. 808.
53. Marshall, 1887, *Medical News* (Philadelphia), vol. l, p. 35.
54. Mitchell, 1910, *Med. Record*, p. 103.
55. Mittelbach, 1901, *Deutsches Arch. f. klin. Med.*, vol. lxx, p. 50.
56. Meyer, 1901, *Deutsches Arch. f. klin. Med.*, vol. lxx, p. 443.
57. Moraczewski, 1896, *Centralbl. f. in. Med.*, vol. xvii, p. 177.
58. Noccioli and Domenici, 1898, *Gaz. degli Ospedale*, vol. xix, p. 303.
59. Ogden, 1895, *Zeitschr. f. phys. Chem.*, vol. xx, p. 280.
60. Orsi, 1889, *Gaz. Med. Lomb.*, vol. xlviii, p. 115.
61. Osler, 1904, *Lancet*, No. I, p. 10.
62. Pavy, *vide* Garrod, 1902, *Lancet.*
63. Pieter, 1925, *Presse méd.* (Paris), vol. xxxiii, p. 1310.
64. Poulsen, 1910, *Beitr. z. path. Anat. u. z. allgemeine Pathol.*, vol. xlviii, p. 347.
65. Poulsen, 1912, *Münch. med. Woch.*, vol. lvii, No. I, p. 364.
66. Rahmlow, 1930, *Monatschr. f. Umfallheilkunde*, vol. xxxvii, p. 403.
67. Ravold and Warren, 1910, *Journ. Biol. Chem.*, Baltimore, vol. vii, p. 465.

68. Reinwein, 1931, *Deutsches Arch. f. klin. Med.*, vol. clxx, p. 327.
69. Rocher and Basset, 1909, *Gaz. heb. des Sciences*, vol. xxx, p. 472.
70. Sachs, 1931, *Deutsches Arch. f. klin. Med.*, vol. clxx, p. 344.
71. Schmitt, 1882, *Union Pharm. Lille*, p. 111.
72. Schumm, 1904, *Münch. med. Woch.*, p. 1599.
73. Sinha, 1930, *Ind. Med. Gaz.*, vol. lxv, p. 153.
74. Slosse, 1895, *Ann. d. l. Soc. Roy. d. Sci. méd. et nat.*, Bruxelles, p. 89.
75. Smith, 1882, *Dublin Journ. of Med. Sci.*, vol. lxxiii, p. 465.
76. Stange, 1896, *Virchows Arch.*, vol. cxlvi, p. 86.
77. Stier, 1898, *Berl. klin. Woch.*, vol. xxxv, p. 185.
78. Toenissen, 1922, *Zeitschr. Ind. Abstammungs u. Vererbungs Lehre*, vol. xxix, p. 26.
79. Wagner, 1908, *Zeitschr. f. klin. Med.*, vol. lxv, p. 119.
80. Winternitz, 1899, *Münch. med. Woch.*, p. 749.
81. Yysendyck, 1906, *Presse méd. belg.*, p. 1088.
82. Young, 1924, *Glasgow Med. Journ.*, vol. ci, p. 214.
83. Zimper, 1903, *Über Alkaptonurie*; Dissertation, Wurzberg.

10. Explanatory Clinical Notes on the Pedigree Charts.

The numbers of the pedigrees are the same as those given in Tables II and VI. Where the parents are known to be related, or stated to be unrelated, the fact is indicated on the chart. Where doubt exists concerning the presence or absence of the disorder in parents or sibs, a mark of interrogation is employed. Marks of interrogation are also employed within the larger circles to indicate that the number of healthy sibs so denoted is not definitely stated. Plain circles without an arrow or cross indicate that the sex of the individual is not stated. A number within a large circle indicates a series of healthy sibs. In the ensuing notes, the numbers in brackets refer to the Bibliography.

1. Kirk.

The parents were both normal and first cousins, according to Garrod. Of the sibs, II, 1 was ten years of age, II, 2 was eight years, II, 3 died of whooping-cough at three years, and II, 4 was an infant. The order of birth of II, 2 and II, 3 is not explicitly stated by Kirk. According to Garrod, "the only child of the second marriage of the father, not consanguineous, is a girl who does not exhibit the abnormality. . . ." History of jaundice and biliousness on both sides of the family. . . .

2. Armstrong and Smith.

According to Garrod, the parents were both normal and not related. It is not explicitly stated whether II, 1 was the first or second child. II, 2 was examined by Armstrong and Smith. She was then three years of age. The

mother had severe puerperal convulsions at her birth. The third child was examined by Baker and reported by Garrod. He was then fourteen years of age.

3. Marshall, Futcher, and Osler.

According to Garrod, the original parents (I, 1 and I, 2) were not related and not alkaptonuric. Marshall first reported the younger of the two alkaptonuric brothers (II, 2) when he was thirty-seven years of age. The elder consulted Futcher nine years later. He was then fifty-seven years of age. Osler reported on the sons of Marshall's case. Futcher's patient suffered from chronic arthritis, and is recorded as a case of (alkaptonuric) ochronosis. Marshall's case also was one of ochronosis. Both had pigmented sclerotics and ears. The condition of the two patients was discovered through their having been rejected in applying for life insurance, on the grounds of " diabetes."

4. Baumann and Embden.

The two alkaptonuric individuals (II, 1, II, 2) were born when the parents were unmarried. Garrod states: " As far as could be ascertained, the condition was not present in the children of the subsequent marriages which both parents contracted." Kraske and Baumann in 1891 recorded one of the cases (II, 1), a man of sixty-seven years, as being of a healthy family. This patient had suffered from an affection of both hip-joints for several years. Frequency of micturition for some months, scanty urine, with occasional passing of small stones. The urine, which was straw-coloured on emission, darkened to greenish black in twenty-four hours. The patient was diagnosed as carcinoma of the prostate. The sister (II, 2), aged sixty, also an alkaptonuric, suffered from cardiac disease and chronic rheumatism.

5. Ogden.

The patient (II, 7), a healthy man of forty-five years, did not notice any abnormality about his urine until informed by his medical adviser. Parents and sibs dead, and, so far as is known, none of these ever showed signs of alkaptonuria. The patient himself had occasional attacks of dyspepsia.

6. Noccioli and Domenici.

The alkaptonuric subject (II, 10), a woman of forty-five years, was a twin of an infant (II, 11), a female, according to Garrod, which did not survive long, but which was stated definitely not to have been alkaptonuric. None of the patient's relatives exhibited the condition. She was married, no pregnancies, no illnesses except the common ones of childhood, and an inguinal hernia for which she first sought medical attention. One day, shortly after operation for the hernia, it was notices that her urine darkened on exposure to air, a fact which had been remarked previous to the operation, but which had been attributed to the presence of menstrual blood. The parents had already noticed that the patient's urine, though pale-coloured when passed, darkened in a few

hours. Only one of the nine brothers was living; cause of death of the others unknown. It is stated that another reducing substance, in addition to homogentisic acid, was discovered in the urine. According to Fromherz, the alkaptonuria of case II, 10 may be taken as lifelong.

7. Garrod (Völcker's Case).

The subject (II, 4) was brought to hospital at the age of three months "on account of the peculiar appearance of his urine, which acquired a deep reddish-brown colour and stained the napkin deeply" (30). The stains became developed on exposure to air. The child, which was fairly well nourished, "exhibited no impairment of his general health." He was said to have suffered from "inflammation of the lungs" when two months old. Micturition, though scanty and unusually frequent, was apparently not painful. The mother was fairly strong, but the father (a drayman) was delicate and complained of chest trouble. Another child (II, 5), born in March 1901, was also alkaptonuric. Garrod states: "No other case of similar peculiarity of the urine has occurred in the family, and I have examined the urine of the elder children with negative results." The two elder unaffected children (II, 1–3) were two brothers and a sister.

8. Garrod (Pavy's Case).

This case was published by Garrod. Of the four alkaptonurics in a family of fourteen, II, 9 was examined at thirty years of age. The age of II, 13 was twenty-two years, and of II, 14 was eighteen years.

9. Stier.

The alkaptonuric subject (II, 2) was the eight-year-old son of a prosperous factory proprietor in Berlin. A brother (II, 1) one year older, as well as the parents, was free from the condition. Apparently there were other sibs who had died—"von den Geschwistern lebt nur ein." Fromherz mentions the case as one of lifelong alkaptonuria.

10. Winternitz's Case.

The patients (II, 2 and II, 3) were lifelong alkaptonurics. II, 7, a girl of six years, was noticed by the mother to exhibit the signs of the disorder since the age of five. II, 1 is the eldest. The position in the fraternity of the other sibs who were not affected (II, 4, 5, and 6) is not stated.

11. Meyer's Case.

The patient was three and a half years old in 1901 and had been previously examined in 1899, when, as stated by Garrod, she was the only child. Two brothers subsequently born, "völlig normalen Harn producieren." Of the patient it is stated that the external auditory meatus contained abundant black ear-wax. Patient otherwise healthy but suffered several times from intestinal catarrh in her first year. The parents noticed the staining of the child's napkins in infancy. Micturition frequent and abundant.

12. Langstein and Meyer.

The patient (II, 1), a male of fifty years, was apparently healthy till 1899, when his left knee-joint became red and swollen. The affection gradually spread to the left hip-joint. In 1902 he visited hospital for this chronic "rheumatic" complaint. The patient was a powerfully built, well-nourished man. He had never any gastric disturbances. There were no signs of gout. His mother is said to have died of diabetes. Two of the five remaining sibs died in childhood. The case is one of alkaptonuria of unknown duration.

13. O. Schumm.

One of the cases (II, 1) was a man of twenty years, a musician, of a healthy family. His condition of alkaptonuria was discovered in the course of an examination of the urine for albumen, which on a previous occasion had been found present. He was somewhat anæmic. The albuminuria always disappeared after a few days' rest. An adult sister (II, 2) was also found to be alkaptonuric, apparently without previous knowledge of the abnormality. Other sibs (three out of four, urine examined) free.

14. Zimper.

The patient (II, 3), aged twenty-nine years, consulted his medical adviser for troubles of digestion, when the urinary abnormality was discovered. The mother and father were healthy. Three of the sibs were dead. The brother (II, 8), sixteen years old, who, like the mother, had a defect of speech, was a lifelong alkaptonuric. "Eltern und die übrigen Geschwister sind frei." One of the two alkaptonurics (it is not clear which one) suffered from tapeworm. Fromherz describes both cases as lifelong alkaptonurics.

15. Gerhardt.

The case II, 3 was a boy of four years, a lifelong alkaptonuric, but otherwise healthy. The mother had noticed the abnormality through the staining of the napkins. An infant brother (II, 2), who died from an infectious disease, is also recorded as alkaptonuric. As regards other members of the family, Gerhardt states: "Die Eltern und übrigen Kinder darunter ein Zwillingsbruder des Verstorbenen sind frei von Alkaptonurie" (35).

16. Bandel.

The two cases II, 2 (male aged five years) and II, 4 (female aged one and a half years) were lifelong alkaptonurics. The ear-wax of both was black. The parents and all the sibs were healthy.

17. Grutterink.

As far as the patient knew, the condition was not shared by any member of the family. She had learnt from her mother that "as a very small child" her swaddling clothes showed black stains on exposure to air.

18. Grutterink.

The cases II, 1 and II, 2 were two males aged twenty-seven and twenty-one years respectively. That the condition was lifelong in these cases is evidenced by the fact that the mother had noticed staining of the swaddling clothes of the two in infancy.

19. Grutterink.

Four alkaptonurics in one fraternity. II, 1, female of nineteen years of age; II, 2, female, twelve years; II, 3, male, six years of age; and II, 4, male, one year.

20. Landois.

The patient (II, 1), aged forty-seven, had alkaptonuria apparently only since the age of forty-one years, although there was a transient darkening of the urine for a few days at thirty-one years, when he began to suffer from arthritis deformans. A post-mortem disclosed ochronosis, with pigmentation of the cartilages. The other two sibs were "healthy." The patient did not know of any similar abnormality in his family. This case also dealt with by Minkowski, Gross, and Allard.

21. Rocher and Basset.

This case (II, 2), a girl of three years, entered hospital for a fractured thigh. There the staining of the child's linen and the darkening of her urine were observed. The parents stated that they had noticed this abnormality since she was two years old. The child was quite healthy, breast-fed till fourteen months, never any serious gastro-intestinal disturbances. The urine took about eight hours to darken. The parents were healthy. A brother five years old (II, 1) suffered from time to time with (?) "kidney disease," but had not signs of alkaptonuria—"pas plus, du rests, qu'aucun autre membre de la famille."

22. Ravold and Warren.

The parents of patient III, 1, a female aged nineteen, had noticed the darkening of her urine and the staining of her clothing since birth. When fresh, the urine was clear, but darkened finally to black on standing. The patient was of German parentage: "her genealogy is accurately known for four generations. There is no evidence of a consanguineous marriage having occurred in the family during that time." The sib III, 2 died at birth. There was no record of the condition of alkaptonuria in any other members of the family.

(It may be mentioned that a woman in the home village of the patient was found on inquiry to have also had the abnormality, which disappeared, however, after the birth of her first child.)

23. Kolaczek.

The three cases III, 6, III, 8, and III, 10 were three females aged forty-four, thirty-five, and thirty years respectively. All showed signs of ochronosis (pigmentation of ears and sclerotics) in addition to alkaptonuria. Case III, 6

was healthy till 1909, when pain and swelling in the joints developed. Later there was a fistula of the knee-joint. No injections into the joint were given, and no medicine internally. At operation on the knee, a black pigmentation of the cartilage, ligaments, tendons, and joint capsule was observed. The patient had noticed that her urine had a " reddish " colour and that the pigmented markings of her ears appeared first ten or twenty years previously, perhaps after her first pregnancy. Case III, 8 was never ill as a child. Pain in the lumbar region developed two years before examination. The patient first noticed blue-grey markings of the external ears eight years previously (not as a girl). She had noticed that her urine darkened on standing to a reddish brown. Case III, 10 was always healthy, but stuttered somewhat. She had noticed that the pigmentation of her ears, which appeared first six years ago, became darker after each childbirth. Her husband had noticed the darkening of her urine.

With regard to the family history, there is no information whether the individuals in I were alkaptonuric or not. II, 1 and II, 2 (cousins) were not alkaptonuric. III, 7 (male, thirty-seven years), III, 9 (male, thirty-three years), and III, 11 (female, twenty-eight years) did not show signs of alkaptonuria or ochronosis. Of the individuals in IV, the eldest three children of the fraternity of thirteen sibs, as well as those marked with a query in the pedigree plate, were dead, and there was no information about them. The remaining individuals in IV were free from alkaptonuria and ochronosis.

24. Poulsen.

The patient (II, 1), a female aged fifty-six years, was diagnosed in 1905 as having rheumatism and alkaptonuria. She had been healthy, except for the common diseases of childhood. Her joint trouble began in her fortieth year, with pain, redness, swelling and limitation of movement of several joints. She then noticed a sediment in her urine, but could not remember when her urine became dark. It was observed to be always a brownish colour, but clearest immediately after being passed. It darkened rapidly on standing to a brownish black. Sometimes there was a sediment. The patient had never had chronic ulceration and had not used carbolic lotion.

Family history: The mother was stated not to have been alkaptonuric. There was no information about the father. Concerning II, 2, Poulsen states: "ihre Schwester (II, 2) hat Gelenkgicht; neder bei dieser noch bei anderen Mitgliedern der Familie hat man jemals abnorme Pigmentierungen oder dunklen Harn bemerkt."

25. Poulsen.

The patient (II, 1), an unmarried woman aged forty-nine years, was healthy in her youth, except for the common diseases of childhood. Her urine became rapidly brownish black on standing, and apparently this abnormality existed since childhood. During the previous twenty-five years arthritis deformans had gradually developed, and also pigmentation of the sclerotics and ears. As is frequently the case with such pigmentation of the ears, it was accompanied

by a stiffening and hardening of the ear cartilages. The patient had noticed that her ear-wax was very dark and that the sweat of axillæ had a greenish colour. Giving the family history, Poulsen states: "Kein Mitgleid der Familie soll abnorme Pigmentierungen gehabt oder bemerkt haben, dass der Harn beim Stehen dunkel geworden wäre." Of the sibs, three were dead and two sisters living. The case is one of (alkaptonuric) ochronosis.

26. Poulsen.

The patient, a man aged thirty-five years, was healthy till three years previous to examination, when he entered hospital for an operation for appendicitis. Here his condition of alkaptonuria was discovered, and appeared to have been present, according to the patient's statements, for at least fifteen years, probably longer. His ears were pigmented—a blue-grey colour; ear-wax was black, axillary sweat of a greenish tint. As in other cases of (alkaptonuric) ochronosis, there was no pigmentation of the nails. This case has been reported also by Cronvall.

27. Poulsen.

Case II, 1, a male aged sixty-eight years, was one of alkaptonuria (for at least forty years) and ochronosis (pigmentation of sclerotics for same time). The date of commencement of other pigmentation—of the ears, backs of hands, face, and body—is unknown. Carbolic lotion had never been used. The patient had had gout for forty years and arthritis deformans in most of the larger joints. A brother (II, 2), sixty-one years old, was also alkaptonuric, and when thirty-eight years old his urine was said to contain "sugar." Case II, 2 showed signs of ochronosis, pigmented ears, and sclerotics. Arthritis of right shoulder and right knee-joint. He had never used carbolic lotion. Both cases were reported by Hammarsten (*vide* 33).

Family history: "Der Vater hatte braune Flecken an beiden Sclerae." It was not known whether the father had other pigmentation, or what was the condition of his urine. He died of cancer. A brother of II, 1 and II, 2 died of pulmonary tuberculosis when fifty-six years old. In his last year of life, the skin of his face and hands was of a brown colour, so that it was thought he might have Addison's disease. Whether he had (alkaptonuric) ochronosis is problematical. II, 4 and II, 5 were living and free from alkaptonuria. II, 6 and II, 7 died in childhood. Members of the family generally exhibited a somewhat dark-coloured skin—"was sie einer vor vier Generationem erfolgten Beimischung von Zigeunerblut zuschreibt." The children of II, 3, II, 4, and II, 5 (numbers, ages, and sexes unknown) were all free from alkaptonuria.

28. Poulsen.

Case II, 1 was a man twenty-three years old, who showed signs of ochronosis as well as being alkaptonuric. Two sisters (II, 2 and II, 3) died early, and there is no information about them or the parents. II, 4, a male, was free from the abnormality.

29. Baldwin.

The patient II, 1 was a woman aged thirty-one years, who complained (1910) of vague, rheumatoid pains in her back and neck. Ten years before, she was told by a physician that she had diabetes. The mother noticed the darkness of the patient's urine, which stained linen, when she was one year old. The sclerotics had a greyish tint, and there was one small dark spot in the cartilage of the left ear, indicating ochronosis. There was a family history of diabetes, extending to the mother (not alkaptonuric), two sisters, two nephews, and a niece.

30. Katsch.

Cases II, 1 (male, sixteen years), II, 6 (male, five years), and II, 7 (female) were rachitic and poorly developed as children. II, 1 and II, 6 were apparently permanent alkaptonurics. II, 7 died at sixteen months in a rachitic state. According to the mother, the urine of II, 7 darkened after emission for a time, but later the peculiarity was lost; it was not found at the clinic examination. II, 8 was a twin of a non-alkaptonuric female (infant) and was poorly developed.

31. Ebstein.

The patients II, 1 (male, fifty-two years) and II, 2 (male, fifty years) both had alkaptonuria and ochronosis. Both had rickets in childhood. From the history of II, 2, who suffered from dysuria, his condition of alkaptonuria had existed for at most five years. The urine of II, 2 was clearer on emission than that of II, 1, but it also darkened on standing. II, 2 had not observed brown stains of his laundry.

32. Debendetti.

The four cases III, 1, III, 2, III, 3, III, 6 were males aged thirty-eight, thirty, twenty-two (position in family uncertain, died at that age), and thirteen years respectively. III, 1, in addition to alkaptonuria, had ochronosis (pigmentation of ears and sclerotics), and arthritis of several joints. He had two children (not shown in pedigree plate), one died in infancy, the other "in età di dodici anni, soffre di artrite coxofemorale." III, 2 was alkaptonuric from birth. He had arthritis of the right knee, with deformation, but was otherwise healthy. He had two children (not shown in pedigree plate), who were not alkaptonuric. III, 3 died from violent causes. His mother noticed that he displayed signs of alkaptonuria from birth. He had no joint affections. III, 6 was found to be alkaptonuric while in hospital with bronchitis. He had had no illnesses of note, and did not know of his abnormality. Two sisters (III, 4 and III, 5) were not alkaptonuric, and had neither ochronosis nor joint affections. III, 4 had three children—one rachitic, none alkaptonuric. III, 5 had one child, not alkaptonuric. No member of the fourth generation is shown in the pedigree plate, which was taken from Cuthbert's paper (19). The supplementary information in these notes is taken from Debendetti's original paper.

33 and 35. Toenniessen.

These pedigrees are reproduced from Toeniessen's paper, which is not concerned with clinical data of the cases.

34. Kleinschmidt.

The patient (II, 1), a man of sixty-six, showed signs of alkaptonuria since earliest youth, in addition to those of ochronosis. There was no other family history of alkaptonuria. The sib II, 2 was free. It is not stated whether there were other sibs.

36. Cuthbert.

The patient (III, 1), a boy of fifteen, showed signs of alkaptonuria from birth. His urine is described as "pink"; it caused black stains of his underclothes. III, 3 (male) was ten years old and III, 4 seven years. None of the children exhibited any structural malformation. The parents lived on a farm. The mother had had no miscarriages or stillbirths.

37. Lockwood.

The case II, 1 was a male aged fifty-six, who was referred by a physician for insulin treatment: "As long as he could remember he had noticed that the urine frequently stained his underclothes a dark colour." He stated that "two of his sisters complained of the same thing." The patient had diphtheria at ten years of age and malaria at twelve. There was no history of venereal disease. (Wassermann reaction negative.) The mother died of diabetes.

38. Young.

The case was an infant of eight months, an only child. "No staining of the napkins was observed until the fourteenth day, when a pink colouration was recognised. During the next two and a half months there was no staining, but the pink colouration returned at the age of three months, and deepened gradually until the napkins became quite black." No peculiarity of the eyes or ears was observed. A specimen of the urine became quite black in the course of twenty-four hours. With regard to the family history, Young states: "There is no record of the abnormality in any of the child's antecedents, within the past generation at least."

39. Bose and Ghosh.

The patient was a man aged fifty-five, a high-caste Hindu, who did not eat meat, fish, or eggs. His clothes were stated to have been stained chocolate-brown by his urine since birth. The urine was clear and straw-coloured when passed, "but on being exposed to the air it gradually darkened in colour." The patient complained of vague, fleeting articular pains in nearly all the joints of his body for twelve years. Frequency of micturition and thirst for the same period. It is stated that his parents were relations, but the exact relationship

he was not aware of. Of his five brothers, two among them were said to be diabetic. This case is unique in that the patient's urine contained glucose as well as homogentisic acid.

40. Sinha.

The case II, 1, a man of thirty-five, was one of alkaptonuria and ochronosis. He suffered at times from dysuria. He was not a vegetarian. A sister (II, 3), twin of a non-alkaptonuric female (II, 2), four years younger than himself, died when seventeen years old from "cough, etc."—fever and wasting. II, 3 was also alkaptonuric. No other cases of alkaptonuria known on either side of the family.

41. Bagnall.

The case was a male infant aged three weeks when first examined. There is no information as to whether there were any sibs or not. Bagnall states: "The mother consulted me because she thought there was blood on the napkins. . . . The child's skin was stained black where soiled by the urine. . . . There were no colour changes in the conjunctivæ or ears, and no colour change in the urine in the stopped test-tube. . . . So far as can be determined, there has been no period when the child has not been excreting homogentisic acid in the urine."

42. Braid and Hickman.

The case was a male infant of four months, "the first child of healthy parents who are not consanguineous, and who do not know of the existence of the anomaly under discussion in any of their antecedents." His mother stated that the child's urine had been very dark at intervals of several days since birth. She also stated, with conviction, that the child had attacks of abdominal pain at those times when the urine passed was not so dark. . . . "The urine varied in colour, sometimes dark when passed, sometimes dark on standing, but at no time, except during starvation, was alkapton absent. By 'dark' is meant a pinkish, smoky brown colour, which gave place rarely to a deep chocolate colour."

43. Rahmlow.

The patient, a male of fifty-seven years, had both alkaptonuria and ochronosis (pigmentation of the ears and sclerotics). He had suffered from arthritis since twenty-three years of age. He did not know of any other case of the abnormality in his family.

44. Sachs.

The case II, 1, a female aged thirty-six, was an alkaptonuric who had had pain in the back for four years, localised in the sacral region. There was no pigmentation of the ears, nose, or sclerotics, but the axillary sweat had a bluish colour. The age of II, 2, a male alkaptonuric, was unknown.

45. O. Bauer.

Patient II, 1, a man aged fifty, had both alkaptonuria and ochronosis (pigmentation of the nose and ears), also urinary concretions in the bladder and

prostate. The urine had a reddish tinge when passed, becoming black in a few hours. The patient had noticed stains on his underclothes from the urine, but did not know when this first appeared. He had "rheumatic pains" in both legs a year previously.

Family history: the abnormality unknown in any of the patient's relatives. A brother (II, 2) had three idiot (non-alkaptonuric) children.

46. Orsi.

The patient II, 1 was a male of twenty years. Age of II, 2 unknown. It is not known for certain whether there were any other sibs. II, 1 had rickets as a child, and later shifting, sharp pains in the joints. The three affected members of the family are said by Orsi to have had pyrocatechinuria. Pyrocatechin is non-rotatory, a di-phenol, and an isomer of resorcin. Its constitution is $C_6H_4(OH)_2$.

47 Fromherz.

The patient II, 4 was an alkaptonuric male of sixteen years. His urine darkened only very slowly, and there was no discoloration of the washing linen. II, 3, also alkaptonuric, died as a child from unknown causes. II, 10 was a weakly male alkaptonuric child aged four. II, 11, a twin of II, 10, died when one year old. The mother was uncertain whether the child had showed signs of alkaptonuria or not. The mother "showed apparently intermittently the appearance of alkaptonuria. The proof, however, is not altogether free from objections."

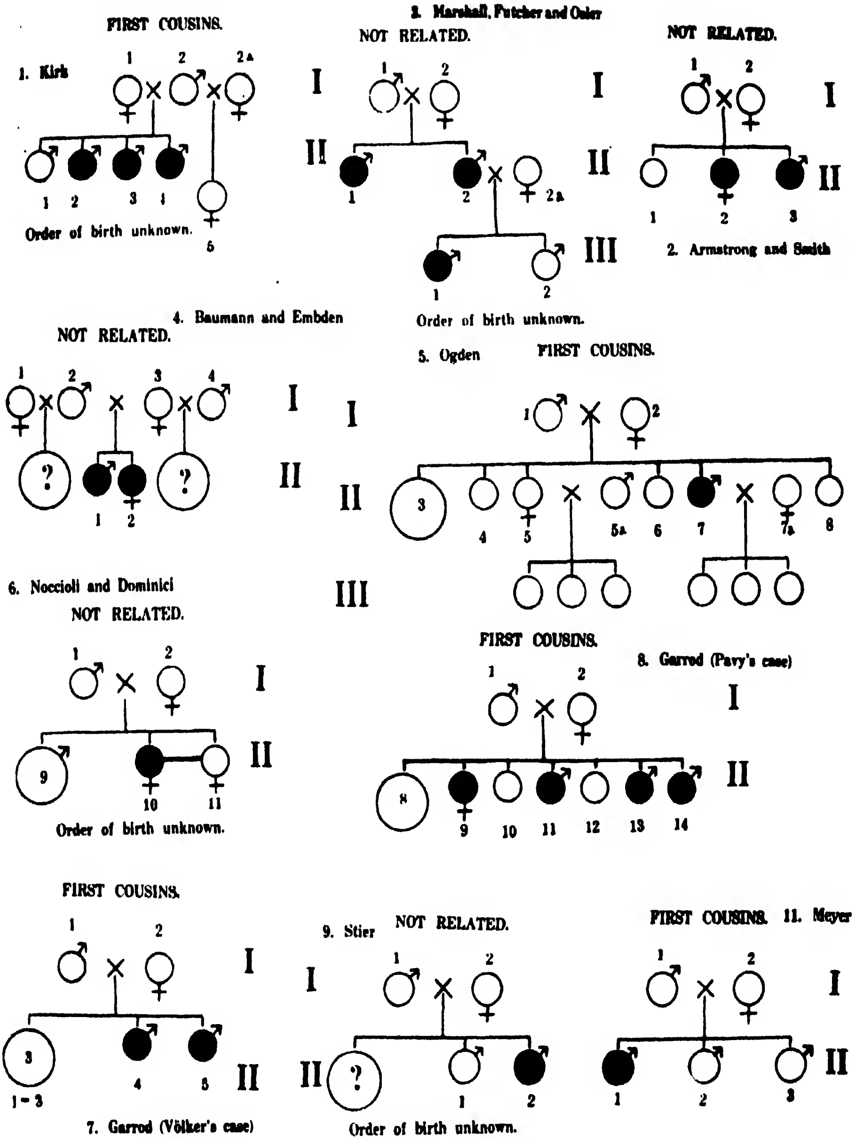
48. Toenniessen.

These pedigrees are reproduced from Toenniessen's paper, which is not concerned with clinical data of the cases.

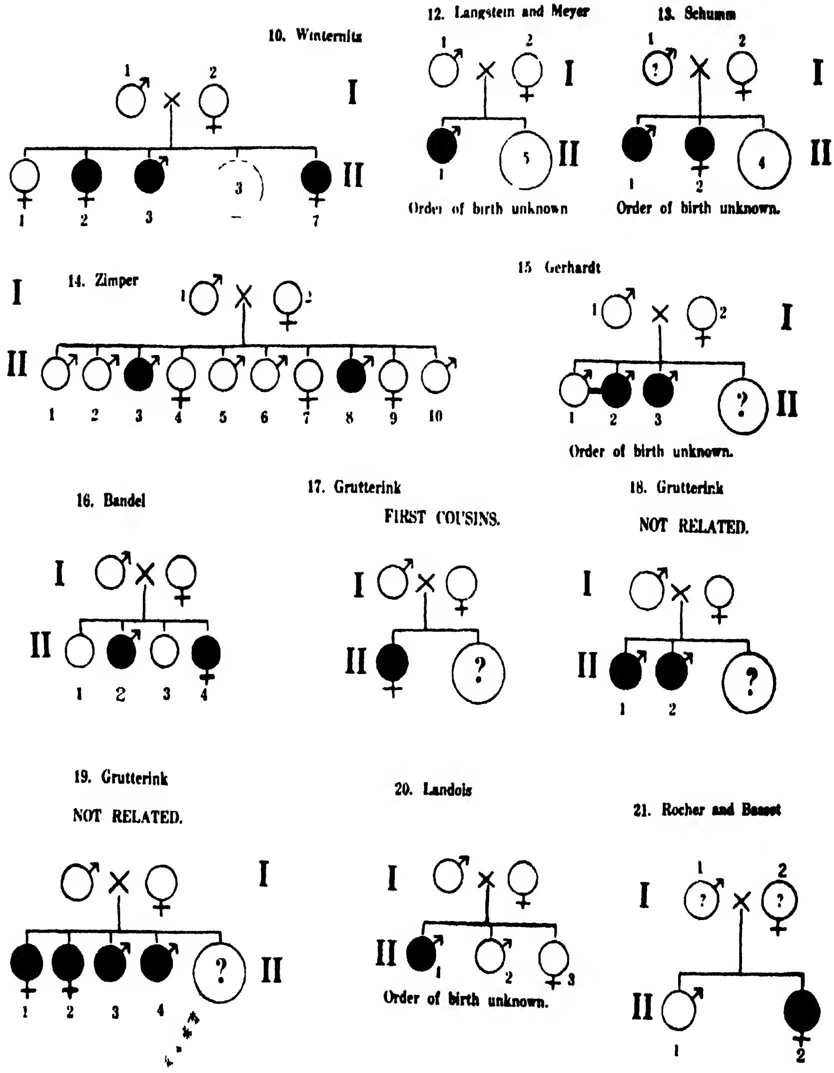
49. Pieter.

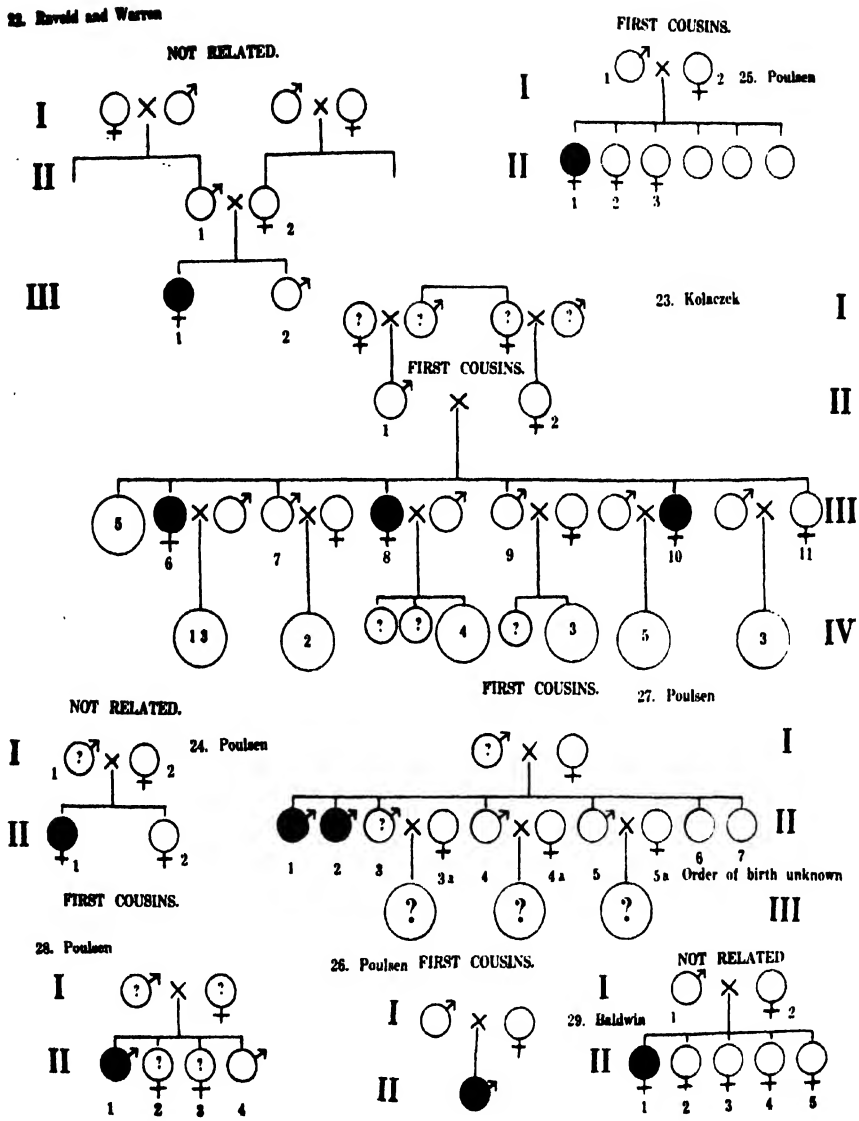
This important case first came to the notice of Dr Lara of the Hôpital National, Rep. Dominicaine. The patient III, 3 was a Spanish Indian cross-bred of forty-seven years, father of six children. He had been operated on successfully for calculi in the bladder and prostate, when it was noticed that his urine blackened on exposure. Nothing is known of other pathological conditions in his family other than this trait. The family lived in the country, sober agriculturalists free from hepatic disorders. The analysis of the urine is given as follows:—

Volume in twenty-four hours . .	850 c.cm.
Density.	1·018
Reaction	Acid.
Colour (when fresh)	Amber.
Odour	Normal.
Sediment	Appreciable.

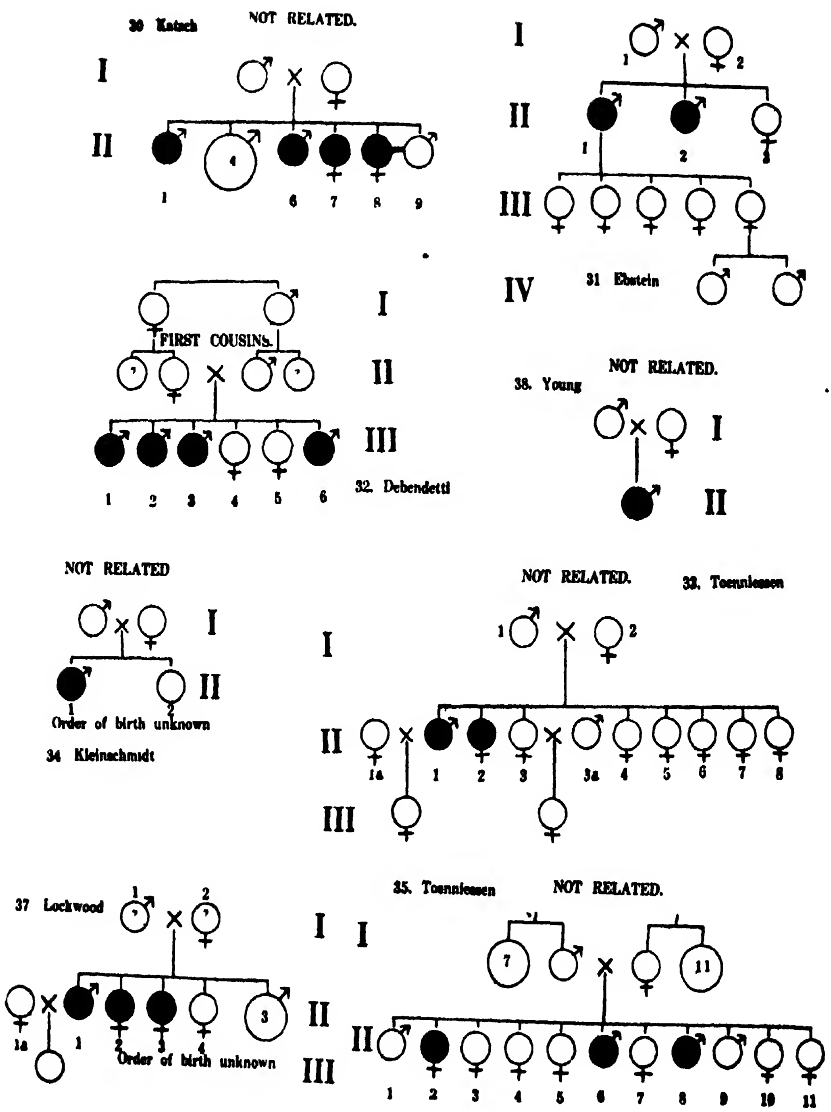


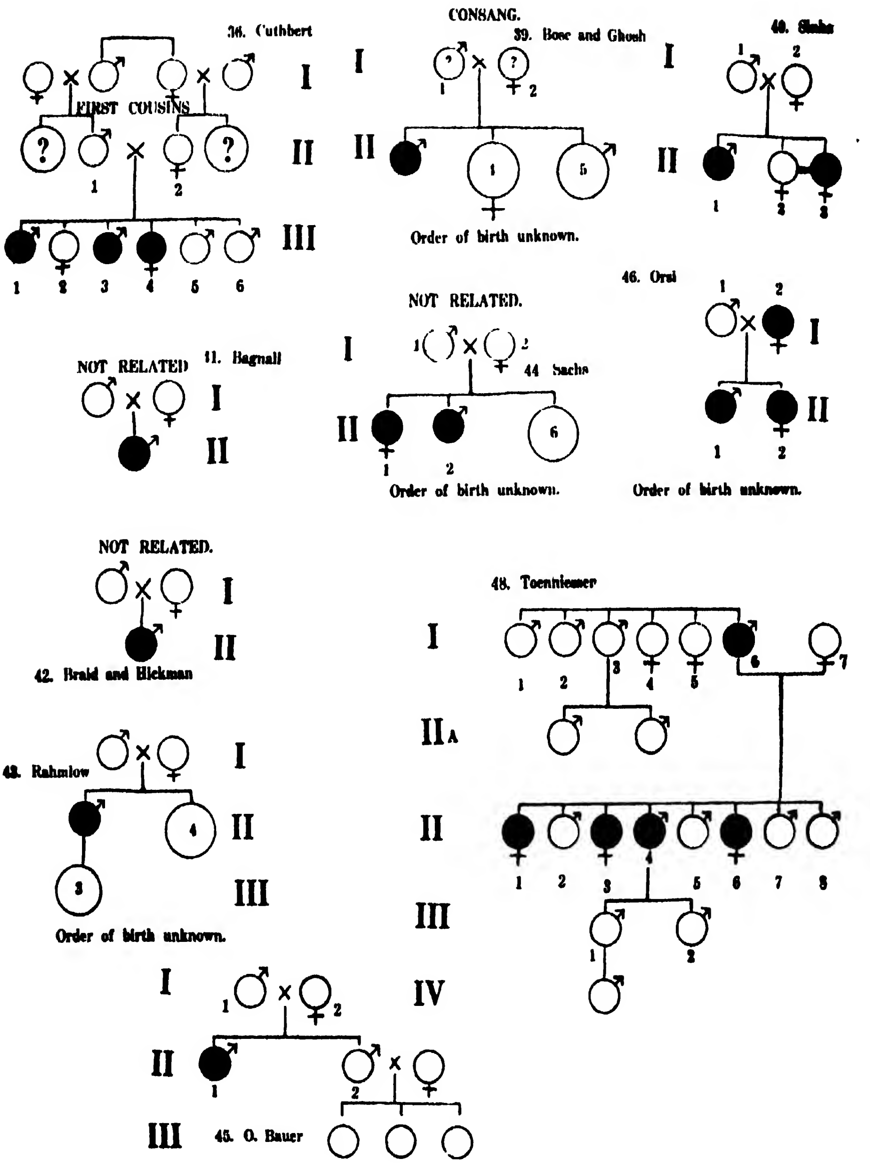
FIRST COUSINS.
1. Kirk
Order of birth unknown.
3. Marshall, Futcher and Osler
NOT RELATED.
Order of birth unknown.
NOT RELATED.
2. Armstrong and Smith
4. Baumann and Embden
NOT RELATED.
5. Ogden
FIRST COUSINS.
6. Nocciolî and Dominici
NOT RELATED.
Order of birth unknown.
FIRST COUSINS.
8. Garrod (Pavy's case)
FIRST COUSINS.
7. Garrod (Völker's case)
9. Stier
NOT RELATED.
Order of birth unknown.
FIRST COUSINS.
11. Meyer


10. Winternitz
12. Langstein and Meyer
Order of birth unknown
13. Schumm
Order of birth unknown.
14. Zimper
15. Gerhardt
Order of birth unknown.
16. Bandel
17. Grutterink
FIRST COUSINS.
18. Grutterink
NOT RELATED.
19. Grutterink
NOT RELATED.
20. Landois
Order of birth unknown.
21. Rocher and Basset

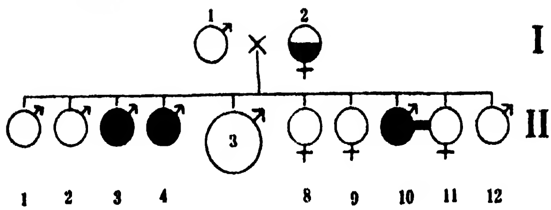


22. Ravald and Warren
NOT RELATED.
FIRST COUSINS.
25. Poulsen
23. Kolaczek
FIRST COUSINS.
NOT RELATED.
24. Poulsen
FIRST COUSINS.
27. Poulsen
Order of birth unknown
FIRST COUSINS.
28. Poulsen
26. Poulsen FIRST COUSINS.
NOT RELATED
29. Baldwin

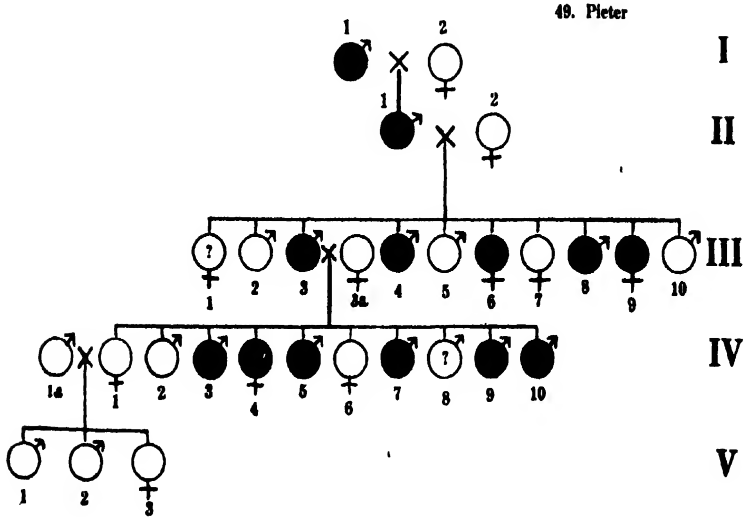

30 Katsch
NOT RELATED.
31 Ebstein
FIRST COUSINS.
32. Debendetti
NOT RELATED.
38. Young
NOT RELATED
Order of birth unknown
34 Kleinschmidt
NOT RELATED.
33. Toennlessen
37 Lockwood
Order of birth unknown
35. Toennlessen
NOT RELATED.


36. Cuthbert
FIRST COUSINS
CONSANG.
39. Bose and Ghosh
Order of birth unknown.
NOT RELATED.
41. Bagnall
NOT RELATED.
44 Sachs
Order of birth unknown.
46. Orsi
Order of birth unknown.
NOT RELATED.
42. Braid and Hickman
43. Rahmlow
Order of birth unknown.
45. O. Bauer

47. Fromherz



49. Pieter



Urea	19 gr. 85 per litre.
Chlorides	6 gr.
Phosphoric acid	1 gr. 60.
Uric acid	0 gr. 45.
Albumen, glucose, indican, pus; Blood and bile components	Absent.

In addition a few crystals of calcium oxalate and desquamated cells from the bladder and ureter.

Concerning the blackening and reducing properties Pieter gives the following information :—

" La réduction du Fehling Benedickt se fait en rouge sale. L'épreuve de Nylander, au sous nitrate de bismuth, est négative, de même que la fermentation en présence de la levure de bière. La réaction avec la phénylhydrazine ne décèle pas le lactose, les pentoses ou l'acide glyconurique. Quelques gouttes de perchlorure de fer, très dilué, versées dans un tube à essai contenant l'urine, donne une coloration bleue intense qui disparaît aussitôt pour reparaître en répétant l'expérience. Ces urines brunissent spontanément quand on les abandonne quelques heures au contact de l'air, la coloration commençant par la surface. Quelques gouttes d'ammoniaque ou de soude produisent cette même coloration presque instantanément. L'eau oxygénée et la chaleur l'accélèrent, tandis que les acides la retardent. Avec le réactif de Millon elles donnent un précipité jaune qui rougit par la chaleur. Cinq ou six gouttes de la solution normale d'azotate d'argent versées dans un peu de ces urines alcalinisées au préalable avec du carbonate de soude donnent instantanément un precipité noir très intense. Une plaque photographique de 400 H et D impressionée 4,000 fois la pose normale et développée dans: urines du malade 100 c.cm.; sulfite de Na anhydre, 1 gr.; carbonate de Na anhydre, 2 gr.; bromure de potasse 0 gr. 20, a révélé après trois heures d'immersion à la temperature de 26° C. une image faible, mais assez nette. . . ." Cela tient sans doute à la nature chimique de l'acide homogentisique. " . . . Dans la famille que nous étudions les enfants atteints tachent le linge dès le premier jour de leur naissance et les changements critiques normaux ou maladifs qui ont lieu pendant la vie ne causent aucune modification au trouble alkaptonurique."

The pedigree chart is reproduced as it appears in Pieter's paper (1925).

(*Issued separately May* 20, 1932.)

XIV.—The Fæcal Pellets of the Anomura. By Hilary B. Moore, B.Sc., The Marine Biological Station, Port Erin, I.O.M. (With Two Plates.) *Communicated by* Professor J. Graham Kerr, F.R.S.

(MS. received March 2, 1932. Read May 2, 1932.)

As far back as 1678 Martin Lister, in his *Historiæ Animalium Tractatus*, has noted that, in the case of certain Molluscs, the fæces are of a regular form, and that their shape differs according to the species; he deduces from this that the intestine must show a corresponding difference in the various species.

With a view to determining the origin of various fæcal pellets found in marine deposits and elsewhere, and also as an aid to determining the relationships of allied species and genera, descriptions and figures have now been published of the pellets of a number of the commoner British marine Mollusca (Moore, 3, 4). These show a wide range of form, from the simplest type of rod-like pellet to ribbon-like, or bead-like forms, and rods sculptured with complex transverse, longitudinal, or spiral patterns. Although in some cases the pellets of allied species cannot be differentiated, in others the pellets show a clearly characteristic form for each species of a genus, and may also give an indication of the relationships between allied genera.

In the pellets so far described, however, there has been little internal differentiation, except in the case of the Trochidæ, where the coarse and the fine material of which the pellets are composed are kept separate from one another in definite tracts of the pellets.

Among most of the Crustacea the fæcal pellets are of a very simple type, namely a rod which is either plain or very slightly modified. Among the Brachyura they are nearly always simple rods of rather loose consistency, and more or less wavy, but with no trace of internal differentiation other than a thin layer of mucous material coating the outside. The same type seems to be typical of the Astacura. Among lower forms of Crustacea the pellets of three Euphausids have been described, all of which were in the form of a rough rod with a thin clear outer layer; and this type is common also to all those shrimps and Mysids which have so far been examined. Among the Copepoda the pellets may be of this type, or else rounded at one end or drawn out at the end to a thin point. Among the Cirripedia, the pellets of *Balanus balanoides* and *B. crenatus* are also simple rods.

In all the Anomura so far examined the pellets are rod-shaped, and where there is any external sculpturing, this is of a comparatively simple type. A series of examples is given from the Galatheidea, Paguridea, and Thalassinidea, but unfortunately no Hippidea were available.

The methods used in collecting and preparing the pellets have been described in a previous paper (Moore, 4). In most cases only preserved material has been available, so that the pellets have had to be obtained by dissection of the guts of the animals in question. The pellets so obtained are apparently identical with those shed by the animal, since they receive their final form at the posterior end of the stomach, and are not further modified in their passage through the intestine to the anus. This fact has been demonstrated in the case of several species of Galathea and Porcellana by comparison of shed and unshed pellets, and in the case of Upogebia by comparison of pellets from the anterior and the posterior ends of the intestine. The method of thus obtaining pellets from preserved material has therefore been assumed to be legitimate throughout the group.

Owing to the great difficulty of preparing entire sections of material composed largely of sand, it is not in general possible to show a single section which illustrates all the points described in the anatomy of a given pellet. The details of the various systems of longitudinal canals in the pellets can only be built up from the examination of a large number of sections, since the process of cutting may produce in any single section either the occlusion of a true cavity or the introduction of an artificial one by the displacement of a sand grain. For this reason the sections photographed cannot be expected to show as much detail as do the diagrams accompanying them.

Very little variation is found in the pellets of individual specimens of the same species, so long as they are feeding well enough to keep a full gut. Pellets have been examined from a considerable number of specimens of *Galathea squamifera* and *G. dispersa*, and the only marked variation found has been in the proportions of the ventral cap, which is described later, and which consists of the finer material taken in with the food, whose proportion in the pellet will naturally depend on the nature of the food eaten. Otherwise the general form of the pellet remains the same in all examples of the same species.

The simplest type of pellet is found in the Paguridea, although a single example of a similar simple pellet has also been found in the Galatheidea.

Paguridea.

Parapagurus pilosimanus.

Locality.—Off Valparaiso, H.M.S. *Challenger.* One specimen only (preserved). Plate II, fig. 1.

The pellet is in the form of a rod, slightly wavy, and with a smooth surface and no surface sculpturing. In section it appears circular, and is composed of fine material, usually spread homogeneously throughout. There may be some coarser material present, and if so it is frequently localised in streaks, which do not, however, occupy any constant position in the pellet. When such coarse material is present there is always a thin outer layer of fine material in which no coarse particles are found. The pellet never shows any trace of longitudinal canals.

Eupagurus sp.

Locality.—Port Erin, I.O.M. Several specimens (preserved).

The pellets examined were not well preserved, but agreed in general with those of the above species; the external layer of fine material was not observed here, however. Although no canals have been found in this group, there is in Eupagurus a series of so-called "valves" in the posterior end of the stomach and in the beginning of the intestine, which are very similar in appearance to the structures in the stomach of Galathea which give rise to the canals in that species.

Galatheidea.

Two further additions occur in the structure of the pellets in this group, namely, the "ventral cap" and the "longitudinal canals"; the former being peculiar to the group, while the latter occur in an even more complex form in the Thalassinidea. In the stomach of Galathea there are a number of comb-like sieves, one of whose functions is to keep the coarse and the fine materials of the food in different tracts. As the food passes from the posterior end of the stomach into the intestine, the main mass of fairly coarse material occupies the upper region where it solidifies to form the body of the pellet, while the fine material is laid down as a thick ribbon along the ventral side of the pellet. The two portions adhere but remain distinct, and the ventral cap so formed may be dissected away from the rest of the pellet with the aid of a needle.

There is also a system of processes from the stomach wall, arising anteriorly as inwardly projecting longitudinal flanges, but becoming free

from the stomach wall posteriorly and pointing as fingers or bristles into the posterior end of the stomach. As the food material passes these it is presumably in a semi-fluid condition, and flows round them. If, as seems likely, they have an absorptive function, the food as it passes them will tend to solidify by the loss of some of its moisture. Whether this is a correct picture or not, the material as it enters the intestine is found to be in the form of a rod, and this is sufficiently solid as it draws away from the processes to retain their cast in the shape of a series of longitudinal canals running throughout the length of the pellet. The shape and arrangement of the processes is reflected by the cavities in the pellet; and canals which are circular or crescentic in section can be traced to processes of corresponding shape in the stomach of the animal. The details of these canals, as well as the structure of the ventral cap, where this is present, will be discussed in connection with the various species.

Æglea lævis.

Locality.—Rio Grande do Sul, Brazil. Two specimens (preserved). Plate I, fig. 1.

This species presents the simplest type of pellet so far found in the group. The pellet is circular in section, but slightly flattened on the ventral side, the whole of which is occupied by the ventral cap which also turns up the sides for a short distance. This cap may be almost flat, but in most cases it exhibits longitudinal grooving very similar to that seen in Upogebia, and in a modified form in Galathea and Porcellana. The sculpturing consists of a flat central area enclosed on either side by a slightly raised rounded ridge, this in turn being marked off on the outside by a rounded groove from a slightly wider rounded ridge. Somewhat similar sculpturing is found on the part of the main body of the pellet which underlies this cap.

The cap itself is composed of very fine grade material, but the main body of the pellet is considerably coarser. In the central region of the pellet are four large longitudinal canals which appear to be crescentic in section. This crescentic shape occurs also in other forms, especially those with only a small number of canals. The exact details of the canal system are difficult to observe owing to the scantiness of the material available and the comparatively coarse grade of the sand of which it is composed. The latter tends to obscure any smaller canals which might be present, but the four large canals at least are quite clear in all the section.

Galathea dispersa.

Localities.—Ouani Bay, Jersey; Ladram Bay, Devon; Port Erin, I.O.M. (preserved material and shed pellets). Numerous specimens. Plate I, fig. 2; Plate II, fig. 2.

These were all typical *G. dispersa,* and none of the *nexa* form were available.

The pellet, as in the previous species, is in the form of a rod with a distinct ventral cap. In section it is considerably wider on the dorsal than on the ventral side. A pair of large and prominent canals, crescentic in section and facing upwards, form the centres of two rounded ventral ridges which are separated by a fairly deep groove. The ventral cap of fine material covers these ridges and is of a similar contour. The cap in this species is thin, and is covered at its edges, and sometimes also almost to the centre, by an overlapping layer of the coarse material.

The external features of the pellet are somewhat similar to those of *Porcellana platycheles* in Plate II, fig. 4. This shows both the details of sculpturing and the difference of texture between the ventral cap and the rest of the pellet.

The canals form two bilateral series, an inner and an outer. The outer consists of an arc of six canals on each side of the pellet, of which the large crescentic-section canals described above are the ventral members. With the exception of the latter, the canals of the outer arc are all more or less circular in section, and the more ventrally situated ones are the largest. Within these is an inner arc consisting of four canals on each side, all circular in section and smaller than those of the outer arc, except the ventral pair which are crescentic but smaller than the corresponding canals of the outer arc. The ventral crescentic-section canals of each series are normally directed upwards and slightly outwards, but their direction may vary 90° in either direction in individual pellets.

The method by which the pellets are formed in the stomach, by the flowing of semi-liquid food material round the processes which form the canals, has already been described. Evidence of the process is found in the neighbourhood of the canals (Plate II, fig. 2), where there is frequently a line of demarcation between the different flows which have met but not mixed, and at whose junction a slight crack may indicate their incomplete fusion. Most of the canals described here as circular in section seem to have been the centre of junction of about four different currents of material, and as a result their outline tends to be more or less square, with corners at the points of junction.

Galathea strigosa.

Locality.—Port Erin, I.O.M. (preserved material and shed pellets). Numerous specimens.

The pellets are apparently identical with those of the preceding species. The material is somewhat coarser, and as a consequence the details of the canals are rather more difficult to observe, but all those described above may be found in this species also.

Galathea squamifera.

Localities.—Ouani Bay, Jersey; Ladram Bay, Devon; Port Erin, I.O.M. (preserved material and shed pellets). Numerous specimens. Plate I, fig. 3.

As in *G. strigosa*, the material is usually coarse so that the details of the canals are difficult to see, but their arrangement seems to be the same as that described for *G. dispersa*. The smaller canals are very difficult to see, and even the outer crescents are often obscured by pressure. The external features of the pellet are similar to those of *G. dispersa*. The ridges covering the outer crescentic canals are prominent, although not always so much so as in the specimen figured in section in Plate I, fig. 3.

The ventral cap is considerably thicker than in either of the preceding species, and the two ventral ridges of the main body of the pellet which project into it are of a different shape. They are smaller and much more prominent, and the crescentic canals frequently lie above instead of in them. The mid-ventral groove between them is correspondingly wider and of a different shape.

From an animal 4 cm. long from the tip of the rostrum to the tip of the extended telson the pellets average 1·3 mm. diameter.

Galathea intermedia.

Locality.—Port Erin, I.O.M. (preserved material and shed pellets). Numerous specimens.

The pellets are very small and have not yielded good sections, but the larger crescentic canals of the outer arc and some of the other canals of both the outer and inner arcs can be seen. The ventral cap is very thick, but the inner ventral ridges of coarse material do not project into it as sharply as they do in *G. squamifera*, and here the outer crescentic canals are contained within the ridges.

Galacantha rostrata.

Locality.—West of Valparaiso. H.M.S. *Challenger*. One specimen (preserved). Plate I, fig. 4.

The pellet is a rod, more or less square in section, but with the ventral side wider than the dorsal. The ventral cap is in the form of a flat ribbon occupying most, but not all, of the ventral surface. The material has not been sufficient to supply accurate details with regard to the canals, so that the following account must be taken as provisional only. There are four large canals showing clearly in the central area, and very similar to those of *Æglea lævis*, except that here they appear to be circular in section. Outside these there seems to be an outer arc of three or four smaller canals on either side, but their exact number and position is not clear. Unless this is an incomplete picture, the pellets of this species show an interesting intermediate form, with resemblances both to Galathea and to Æglea.

Munida bamffica.

Localities.—Millport; Port Erin, I.O.M. (preserved material). Several specimens.

Unlike all the other members of this tribe which have so far been examined, the pellets of this species are simple rods, circular in section, and with no trace either of a cap of fine material or of canals. The pellets are composed of extremely fine homogeneous material, and there is no differentiated outer layer as in Parapagurus. The stomach of this species is quite different from that of Galathea, and the processes which produce the canals in the pellets of that species are not present in Munida. It is suggested that the difference is correlated with a meat diet in Munida, as distinct from the detrital diet of Galathea. A similar diet is known in the case of Eupagurus, and that of Parapagurus may well be the same, although in its case there must be a considerable admixture of gritty material in the food.

Porcellana platycheles.

Localities.—Plymouth; Ladram Bay, Devon; Ouani Bay, Jersey; Port Erin, I.O.M. (preserved material and shed pellets). Numerous specimens. Plate I, fig. 5; Plate II, figs. 3 and 4.

The pellet is more flattened than any so far described, being about twice as wide as it is deep. The dorsal side is raised into a broad ridge, and the ventral side into two ridges with a shallow groove between them. The ventral cap covering these ventral ridges is of more or less uniform thickness, and as in the case of Galathea is overlapped at either side by a thin layer of the coarse material forming the body of the pellet.

There is a pair of large crescentic-section canals lying above the paired ventral ridges, in a position similar to those of Galathea, and as in that genus, they are part of an outer arc of six canals round the margin of the

pellet on either side. Here the mid-lateral canals of either side are crescentic also, while all the other canals of the pellet are either circular in section or else of the square type described in *Galathea dispersa.* Both sets of crescentic canals are large, but the lateral ones are frequently partially occluded by pressure.

The inner system of canals, corresponding to the inner arcs in Galathea, consists here of six pairs of canals, which is two pairs less than in Galathea, and they differ also in being arranged in a cluster instead of in an arc as in that species.

An animal with a carapace 1·0 cm. in diameter produces pellets with an average diameter of 1·0 mm.

Porcellana longicornis.

Localities.—Plymouth; Ladram Bay, Devon; Ouani Bay, Jersey; Port Erin, I.O.M. (preserved material and shed pellets). Numerous specimens. Plate I, fig. 6.

The pellet is of a very similar shape to that of the preceding species, but with the dorsal ridge rather less elevated. It is more elevated from the surface of the pellet in this species, and does not appear to be overlapped at the sides by any coarse material. The pellets are very small, so that it is possible that some canals may have been overlooked in the sections, but the latter are sufficiently good to render this doubtful. The canal system differs considerably from that of *P. platycheles.* There are ventral and lateral pairs of crescentic canals as in that species, together with an additional pair on the dorsal side. These, together with a single pair of circular-section canals in the ventro-lateral regions, form an outer arc with four pairs of canals instead of the six of the preceding species. The inner system also contains only two pairs of canals instead of six.

From an animal with a carapace 0·6 cms. in diameter, the pellets average 0·48 mms. in diameter.

THALASSINIDEA.

No ventral cap has been found in the pellets of any member of this tribe, and the pellets are always approximately circular in section.

Calocaris macandreæ.

Locality.—Millport. One specimen only (preserved).

The only specimen available had an almost empty gut, but such fæcal matter as was present in it showed indications of a rod-shaped pellet with no ventral cap, and with some canals in the central region. A transverse

section of the posterior end of the stomach shows that there are four processes, crescentic in section, and probably producing a system of canals similar to the four central canals of Axius, although not so large. Unless there are also other canals in the pellet forming an outer system, this is the simplest type of pellet known from the Thalassinidea.

Axius stirhynchus.

Locality.—Channel Islands One specimen (preserved). Plate I, fig. 7.

The material is scanty, and this description may have to be modified when more specimens can be examined. The pellet is circular in section, and has two systems of canals, an inner and an outer. All the canals appear in section as rather thin crescents, and frequently with the tips of the crescents enlarged. The central system contains four large canals facing outwards, while the outer system contains eight similar but rather smaller canals facing inwards.

Callianassa subterranea.

Locality.—Plymouth. One specimen (preserved). Plate I, fig. 8.

Although only a single specimen was available, it has yielded good sections in which the pattern of the canal system is quite clear. The pellet itself is circular in section and the canals, as seen in transverse section, are arranged in an inner and an outer circle as are those of Axius. Here, however, all the canals are circular in section and rather small. The inner circle contains six canals, while the outer one contains fourteen, and these are situated about an eighth of the pellet's diameter from the surface, which is deeper than the similar circle in Upogebia.

Callianassa californiensis.

Locality.—Departure Bay, Vancouver Island. One specimen (preserved).

The pellets contain a considerable quantity of large particles, so that it is not possible to prepare good sections of them, but there are indications of both an outer and an inner system of canals, and their arrangement is probably the same as those of *C. subterranea.*

Upogebia (Gebiopsis) deltaura.

Locality.—Plymouth. Three specimens (preserved). Plate I, fig. 9; Plate II, fig. 5.

Upogebia is the only genus of this tribe in which a pellet has been found with surface sculpturing similar to that in the Galatheidea. The pellet is roughly circular in section, and on one side (presumed, by analogy with the

Galatheidea, to be the ventral side) there is a pair of slight ridges with a single ridge between. The grooves separating these are only shallow and may even be absent, since the extent and shape of the ridges varies considerably in individual pellets. The rest of the surface is thrown into slight undulations, but these again may be absent in some specimens, or else visible only in the dorsal region.

The canal system contains an outer ring of twenty-six canals, which usually lie in arcs of thirteen on either side with slight gaps between the ends of the arcs. Running dorso-ventrally on either side of the mid-line is a row of nine canals of which the end members of each row are also the end members of the outer arcs. In the spaces between these dorso-ventral rows and the outer arcs are five more canals on either side, in the form of a "W" facing horizontally inwards. There are thus in all fifty canals in this pellet. All of them are circular in section.

Upogebia stellata.

Locality.—Plymouth. One specimen (preserved).

This material has not yielded quite such good sections as the preceding species but the pellets seem to be of the same type. The canal systems appear to be the same in the two species, and although in the one specimen of *U. stellata* examined the surface undulations were more marked than is usual in *U. deltaura*, this may not be typical of the species.

Thalassina anomala.

Locality.—West Borneo. One specimen (preserved). Plate II, fig. 6.

The material was well preserved and the pellets are so large—nearly a centimetre in diameter—that the details of the canal system can be readily made out in them. The pellets themselves are elliptical in section, being slightly flattened from side to side. A narrow wedge of material, not differing from that of which the rest of the pellet is composed, runs from one side to the centre and contains no canals. The whole of the rest of the pellet is honeycombed by a very large number of small canals, circular in section and arranged in a regular series of intersecting arcs. The fact that clefts in the pellets run along these rows, as shown in Plate II, fig. 6, is probably due to shrinkage of the pellet during the process of embedding, as they were not in evidence in the fresh material. The figure shows the position of such canals as are visible in a single pellet, but no attempt has been made to construct an ideal section showing all the canals that there are. At a rough estimate there must, in the pellet of this species, be about seven hundred longitudinal canals.

General Discussion.

The series of types of pellets here described presents a very wide range, from the simplest form with only four canals to the most elaborate with some seven hundred, and includes certain divergent elaborations such as the production of crescentic-section canals, and of modified ventral caps. But in all there is clearly a similarity of basic pattern which links each type to the next. The first problem arising from the examination of such a series is that of the significance, if any, of such a complicated mechanism for fæcal production in the physiology of the animals in question. Since this aspect of the study of fæcal pellets has not so far been directly examined, it is not legitimate to put forward more than one or two surmises and to express the hope that the subject may be pursued more fully in the future.

The explanation of the formation of the ventral cap of fine material in the pellets of the Galatheidea lies probably in the fact that the animal keeps the coarse and the fine constituents of its food in separate tracts of the stomach in order to subject them to different types of treatment. And since the removal of water and the consequent consolidation of the food-mass form part of these processes in the stomach, the two types of material will not come together until the pellet is partially formed and is in such a state that the two constituents will retain their individuality.

It is possible that the longitudinal canals may have some functional significance, although their small bore would render the transfer of any liquid through them a matter of difficulty. On the other hand they may be the purely mechanical product of a contrivance which was evolved in order to obtain an increased area of stomach wall in contact with the food material. In an animal such as Thalassina the increase of surface so obtained must be very great indeed.

Another point raised by the obvious relation between the pellet patterns of the various members of this group is the bearing that this has on the origin and affinities of the different families. Since the type of pellet with an internal canal system has not so far been found in any other group of the Crustacea, and since there is such a similarity between the pellets of the Galatheidea and the Thalassinidea, there is here an added argument for the inclusion of the latter tribe within the Anomura from which they were long excluded. The fact that the pellet of Æglea is of so simple a type, but yet one which can easily be related to that of Galathea and Porcellana, is of interest not only because it indicates the relationship between these genera, but also because it provides a comparison between a marine and a fresh-water species.

While it is possible that the mechanism for the production of these canals has been independently evolved in the various tribes, it seems far more likely that they have come from a common stock in which it was present in some form. And if this is tho case, the fact that such a mechanism is found only in those animals which are mud or detritus eaters, and not in any of the meat eaters which have been studied, suggests a similar diet for the common ancestor, whatever it may have been.

The geological value of a knowledge of such pellets as these cannot be more than touched on here. It has been shown that, in the deep waters of the Clyde (Moore, 2), the simple type of pellets formed by Maldanid worms will remain more or less unchanged in the bottom deposits for periods of fifty to a hundred years, and there is no reason to suppose that they will break down after longer periods. On the contrary it is probable that they will undergo a process of mineralisation, Takahashi and Yagi (5) having traced the process of glauconisation in similar pellets from fairly deep water and compared them with similar bodies found in some oil shales. Galliher (1) has published figures and analyses of similar bodies from the miocene brown shales of California, and those from that locality which I have examined are, except for the changes accompanying the process of fossilisation, almost identical with the Maldanid pellets from the Clyde.

If simple pellets such as these can be preserved there is no reason why more complicated ones should not also become fossilised. And although it is not to be expected that they will be found as frequently as the simple oval pellets which are the dominant form in so many marine deposits, yet it is to be hoped that a search of suitable deposits will yield at any rate some specimens. If such pellets can be found and identified, they may throw a most interesting light on the habits of the animal which produced them.

I should like to express my great indebtedness to Dr Calman and the authorities of the Natural History section of the British Museum for supplying me from their specimens with the pellets of all the non-British specimens described here; also to Dr Allen for the material from Plymouth; and to Professor Graham Kerr for the interest he has taken in the work.

REFERENCES.

(1) Galliher, E. W., 1929, "Collophane from Miocene Brown Shales of California," *Bull. Amer. Assoc. Petroleum Geologists*, vol. xv, No. 3.

(2) Moore, H. B., 1931, "The Muds of the Clyde Sea Area.—III. Chemical

and Physical Conditions, Rate and Nature of Sedimentation, and Fauna," *Journ. Mar. Biol. Assoc.*, N.S., vol. xvii, No. 2.

(3) Moore, H. B., 1931, "The Specific Identification of Fæcal Pellets," *Journ. Mar. Biol. Assoc.*, N.S., vol. xvii, No. 2.

(4) Moore, H. B., 1931, "The Systematic Value of a Study of Molluscan Fæces," *Proc. Malac. Soc.*, vol. xix, Part VI.

(5) Takahashi, J., and Yagi, T., 1929, "Peculiar Mud-grains and their Relation to the Origin of Glauconite," *Econ. Geol.*, vol xxiv, pp. 838–852.

DESCRIPTION OF PLATES.

Plate I.

Fig. 1.	*Æglea lævis.*	Transverse section.
,, 2.	*Galathea dispersa.*	,, ,,
,, 3.	*G. squamifera.*	,, ,,
,, 4.	*Galacantha rostrata.*	,, ,,
,, 5.	*Porcellana platycheles.*	,, ,,
,, 6.	*P. longicornis.*	,, ,,
,, 7.	*Axius stirhynchus.*	,, ,,
,, 8.	*Callianassa subterranea.*	,, ,,
,, 9.	*Upogebia deltaura.*	,, ,,

Plate II.

Fig. 1.	*Parapagurus pilosimanus.*	Transverse section.
,, 2.	*Galathea dispersa.*	,, ,,
,, 3.	*Porcellana platycheles.*	,, ,,
,, 4.	*P. platycheles.*	Dorsal and ventral surfaces.
,, 5.	*Upogebia deltaura.*	Transverse section.
,, 6.	*Thalassina anomala.*	End view.

(*Issued separately June* 28, 1932.)

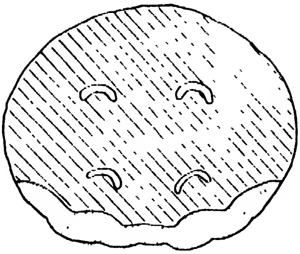


FIG. 1.

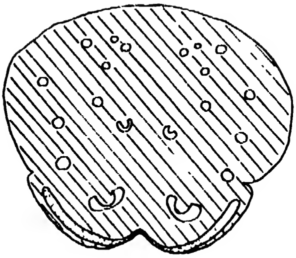


FIG. 2.

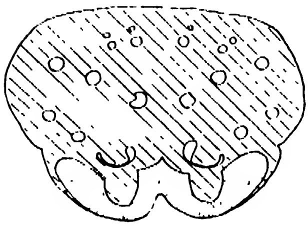


FIG. 3.

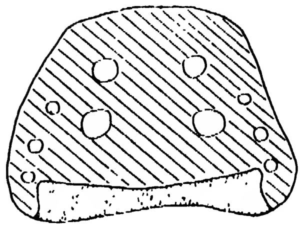


FIG. 4.

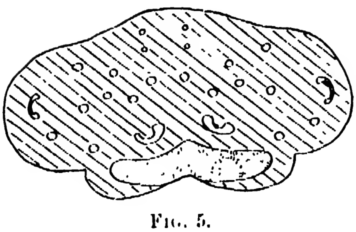


FIG. 5.

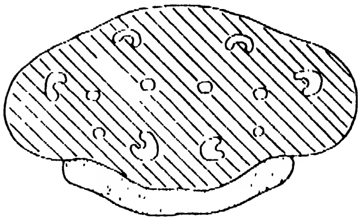


FIG. 6.

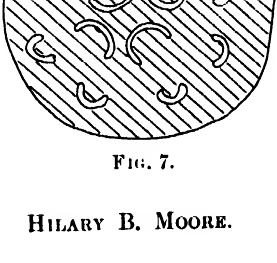


FIG. 7.

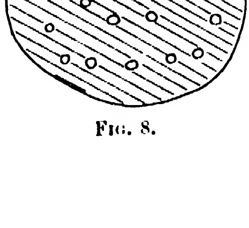


FIG. 8.

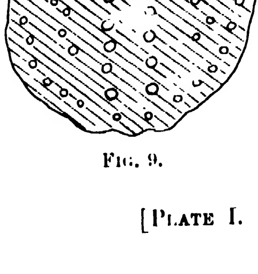


FIG. 9.

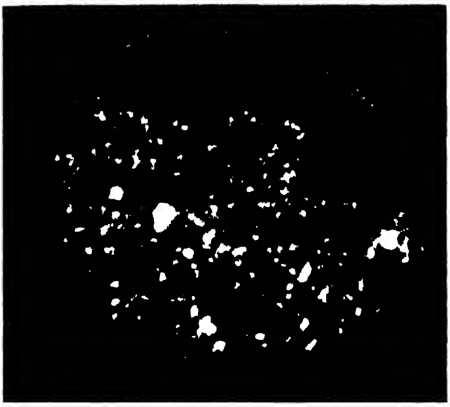


FIG. 1.

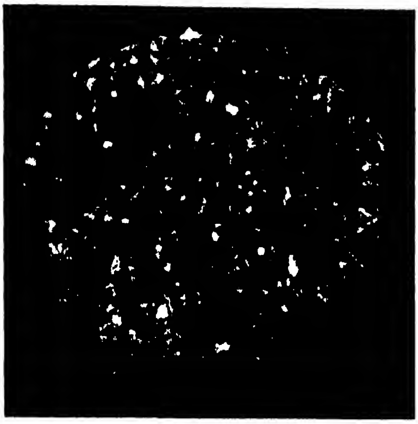


FIG. 2.

FIG. 3.

FIG. 4.

FIG. 5.

FIG. 6.

XV.—A Study of the Tyrosinase of Potato Tubers.

By **Ian M. Robertson**, B.Sc., Ph.D., Chemistry Department, Edinburgh and East of Scotland College of Agriculture. *Communicated by* Dr ALEXANDER LAUDER.

(MS. received April 21, 1932. Read June 6, 1932.)

THE object of the present investigation was to obtain chemical tests which would enable a rapid and accurate identification of a variety to be made from an examination of the tuber. It is well known that certain oxidising enzymes, which can be detected and estimated by simple colour reactions (12), exist in the potato tuber. Since such tests can be applied directly to the plant tissue (8), it was decided to make further investigation along this line.

PRELIMINARY INVESTIGATION AND LABORATORY TECHNIQUE.

A preliminary investigation showed that the enzyme tyrosinase was evenly distributed throughout the tuber, and could be detected in the tissue by the formation of coloured compounds from tyrosine and certain monohydric phenols (2). Tyrosine is present in potato tubers, and when these are cut and exposed to the air the action of the enzyme in the tissue produces a red-brown decomposition product (12), which later changes to melanin. The rate of development of the red-brown colour was found to depend on variety, but was unsuitable as a means of differentiation, since the amino-acid appeared to concentrate in the surface irregularities, giving the section an irregular appearance. The reaction also proceeded very slowly.

With *p*-cresol as substrate, tyrosinase produces a brilliant orange-red colour, and tuber sections treated with *p*-cresol solution develop in a short time a uniformly even colour, which lends itself readily to rapid analysis. The reaction tyrosinase-*p*-cresol was accordingly studied in detail in a large number of varieties.

The intensity of the colour produced on a section of tuber treated with a standard cresol solution was estimated with a tintometer and Lovibond's colour standards, the intensities of which are given numerically. The procedure adopted was to cut, at right angles to the long axis of a tuber, a section about 0·75 cm. thick, treat it with a few drops of reagent, and allow it to drain for 30 seconds on filter-paper. The section was then placed in the tintometer and the intensity of the developing colour measured at

definite time-intervals by comparison with a combination of Lovibond's red and yellow colour standards over a standard white background. A graph of colour intensity against time was thus obtained for the section. Since the velocity of an enzymic action is dependent on temperature, the tintometer was surrounded by a box and the temperature maintained at 18° C. by means of an electric hotplate.

An aqueous solution of *p*-cresol was used in the preliminary work, but, as the colour development was not very rapid, more concentrated solutions were tried, using ethyl alcohol and sodium hydroxide as solvents. Ethyl alcohol solutions containing 0·5 per cent., 1 per cent., 2 per cent., 4 per cent., and 8 per cent. *p*-cresol were prepared and applied to tuber sections, the colour being estimated at definite time-intervals. The graphs obtained by plotting colour intensity against time were of the logarithmic type, and showed that the velocity of colour formation increased with substrate concentration. The alcohol, however, evaporated on the tuber section, leaving an irregularly coloured surface. Furthermore, alcohol is frequently employed to precipitate oxidising enzymes in plant tissue, so that the course of the reaction was liable to be influenced by the alcohol present.

Various sodium hydroxide solutions containing 0·5 per cent., 1 per cent., 2 per cent., and 4 per cent. *p*-cresol were employed. The most suitable, and that which was used as standard throughout the investigation, contained 2 per cent. cresol in half the equivalent quantity NaOH (half the amount of NaOH required to neutralise the cresol to form the sodium salt). The graphs of intensity of red colour against time for this solution had a comparatively long linear period, from which the average velocity constant was calculated according to the formula $k=\frac{dr}{dt}$,

where k = velocity constant for the red-colour component,

r = intensity of red colour after time t.

It has already been mentioned that the colour produced by the tyrosinase-cresol reaction was represented by a combination of red and yellow standard colours. To prove whether or not the two colours proceeded from the same reaction, sections of tubers were treated with a 0·5 per cent. cresol solution in $\frac{1}{2}$ equivalent NaOH, and the colour estimated at definite time-intervals until the reaction had ceased. The intensities of the red and yellow components were then considered separately. Both gave graphs which had a short linear period at the commencement, followed by a logarithmic portion. The red colour was considered first, and it was assumed that the final intensity (r_∞) represented the original substrate concentration. The concentration of unchanged substrate after time t was

therefore given by the difference between r_∞ and the red-colour intensity, r_t, after time t.

Values of the constant k_r obtained from the formula

$$k_r = \frac{1}{t} \log \frac{r_\infty}{r_\infty - r_t}$$

proved to be approximately constant throughout the reaction, showing that the course of the reaction followed the mono-molecular law.

The values for the intensity of the yellow colour were treated in a similar manner. A correction for the original yellow colour of the flesh was deducted from each reading, and the velocity constant k_y obtained from the corrected readings. On comparison it was found that the two constants k_r and k_y were in close agreement (Table I), indicating that the red and yellow colours proceeded from the same reaction.

TABLE I.—REACTION VELOCITY OF THE TYROSINASE-p-CRESOL REACTION.

Time-interval in minutes t . .	5	10	15	20	25	30	35	45	55
Constant of red colour k_r . .	0·072	0·066	0·066	0·070	0·074	0·074	0·073	0·070	0·071
Constant of yellow colour k_y . .	0·069	0·067	0·063	0·068	0·082	0·077	0·098	...	...

Hence a study of the development of one of the colours provided sufficient information to determine the velocity of reaction. The red colour was chosen in preference to the yellow, because it was more rapidly formed and was more easily measured. Moreover, when using the yellow colour a correction had to be applied for the yellow already present in the flesh of the tubers.

The effect of temperature on the velocity of reaction was next investigated. The velocity constant k_t was obtained for the formation of the red colour at different temperatures, and used to calculate the temperature coefficient, *i.e.* $\frac{k_{t+10}}{k_t}$.

TABLE II.—TEMPERATURE COEFFICIENT OF THE TYROSINASE-p-CRESOL REACTION.

Variety.	Temp. Range.	Temp. Coefficient.
Tinwald Perfection	8°–18° C.	2·28
	13°–23° C.	1·86
	18°–28° C.	1·52
Edzell Blue . .	13°–23° C.	1·82
Rhoderick Dhu . .	13°–23° C.	1·89

Thus a rise of 10° C. produces approximately a twofold increase in reaction velocity.

Under existing laboratory conditions the tintometer could most easily be maintained at a temperature of 18° C., and this was taken as standard for subsequent observations.

With the standard cresol reagent at 18° C., the course of the reaction over a period of 15 minutes was represented by a straight line. From this, the reaction constant for the formation of the red colour was obtained, $k = \frac{dr}{dt}$. The slope of the graph represented by k did not necessarily determine the course of the reaction, because the graphs from the different varieties had different intercepts on the r axis. In order to determine the course of the reaction for any one variety, the value for k was supplemented by the value of the red colour at time 10 minutes (r_{10}). Estimates of the standard error for r_{10} were made by the method detailed by Fisher (4). The effect of the following factors on the reaction were then studied in detail: (*a*) maturity and tuber weight, (*b*) environment, (*c*) disease, etc., (*d*) season, and (*e*) storage.

It was found that the reaction is related to the variety, and could be used as a means of distinguishing between the different varieties. It is not affected by tuber weight, environment, season, or storage; the enzyme activity decreases with the maturity of the tuber, and the reaction is also affected by the presence of certain diseases and injury (7).

Discussion of Results.

From a consideration of the graphs for the rate of reaction, it was seen that the decomposition of *p*-cresol by tyrosinase, in agreement with other enzymic decompositions, did not follow one law throughout. The greater part was logarithmic, but the initial and final stages were linear. For the present purpose it was most convenient to observe the initial linear period, and by choice of a suitable cresol reagent it was found possible to extend this part of the curve over a period of at least 15 minutes after commencement of the reaction.

The form of this part of the curve, where the substrate was in excess, suggested that the reaction velocity was proportional to the concentration of enzyme. In the case of the potato tuber such a proportionality could not be assumed in view of the fact that other substances were present which might interfere with the course of the reaction. For instance, peroxidase and a homologue of catechol (9) have been identified in the potato tuber, and it has been shown that peroxidase retards (11) while

o-dihydric phenols accelerate (10) the action of tyrosinase. Hence tubers which were very reactive towards *p*-cresol might owe their reactivity to a high concentration of enzyme or to the presence of an accelerator. With less reactive varieties it was possible that a relatively high concentration of enzyme was masked by an active inhibitor.

A possible explanation of the dependence of the enzyme activity on maturity of the plant may be given. It has been observed (15) that normal tuber formation commences about the period of maximum growth of the haulm. Assuming this to be the case, the young and very reactive tubers would be formed at or about the period of greatest activity of the plant. The diminution of enzyme activity would then accompany the decreasing growth-rate of the plant until finally, when growth ceased at complete maturity, a minimum enzyme activity was observed. Up to this stage the tuber has been dependent on the growing plant for sustenance; with haulm maturity it commences a separate existence and assumes its own characteristic properties. It was about this period that a rise in tyrosinase activity to a constant value was noted (7). If it is the case that the minimum tyrosinase activity coincides with complete maturity of the plant, the occurrence of the minimum should vary with environment and variety. Such a variation was actually observed.

After removal from long storage at low temperature or in a pit, tubers showed a temporary decrease in enzyme activity. A temporary increase in acidity has also been observed in this period (13). These changes may be due to the temporary accumulation of acid decomposition products of respiration.

Previous work on the chemical composition of potato tubers—for example the estimation of dry matter and total nitrogen (3, 14, 16), the amount, distribution, and various physical properties of the starch (5, 6), the solanine content (1) and acidity (13)—has shown that these properties depend more on environmental conditions than on variety. The observation of the tyrosinase-cresol reaction, however, provides a basis for the classification of tubers according to variety.

Summary.

1. A reagent (*p*-cresol) has been used to determine the activity of tyrosinase in the potato tuber.

2. The course of the reaction tyrosinase-*p*-cresol has been shown to be mono-molecular, with a short linear period at the commencement. From observations of this period, constants for the enzyme activities have been obtained and a temperature coefficient deduced.

3. The enzyme activity has been shown to depend upon stage of maturity of tuber, variety of tuber, and disease infection, but to be independent of tuber weight, environment, season, and storage conditions.

4. The probable significance of the results has been discussed.

The writer wishes to thank the Department of Agriculture for Scotland for a Grant which enabled the work to be carried out, and also Dr A. Lauder and Dr A. M. Smith of the Edinburgh and East of Scotland College of Agriculture for their interest and assistance throughout the course of the work.

REFERENCES.

(1) Bömer, A., and Mattis, H., 1924, *Z. Unters. Nahr. Genussm.* **47**, 97.
(2) Chodat, R., 1910, *Aberhalden's Biochem. Arbeitsmethoden*, **3**, 42.
(3) Coudon, H., and Boussard, R., 1897, *Ann. Sci. Agron.*, Serie 2, **3**, 250.
(4) Fisher, R. A., *Statistical Methods for Research Workers*, London, 1925.
(5) Johnson, T., and Boyle, C., 1919, *J. Dept. Agric. Tech. Instruct.*, Ireland, **19**, 2.
(6) Kavčič, J., 1930, *Kolloidchem. Beih.*, **30**, 406.
(7) Lauder, A., and Robertson, I. M., 1931, *Scot. J. Agric.*, **14**, 47.
(8) M'Intosh, T. P., 1928, *Scot. J. Agric.*, **9**, 304.
(9) Onslow, M. W., 1919, *Biochem. J.*, **13**, 1.
(10) Onslow, M. W., and Robinson, M. E., 1928, *Biochem. J.*, **22**, 1326.
(11) Pugh, C., 1929, *Biochem. J.*, **23**, 456.
(12) Raper, H. S., 1928, *Physiol. Reviews*, **18**, 245.
(13) Robertson, I. M., and Smith, A. M., 1931, *Biochem. J.*, **25**, 763.
(14) *Royal Society Food (War) Committee*, 1919.
(15) Wellensiek, I. J., 1929, *Mededielugen van de Landbouwhoogeschool Wageningen*, **33**, No. 11.
(16) Willeman, J. J., and West, R. M., 1925, *Minnesota Studies in Plant Science*, **5**, 211.

(*Issued separately June* 28, 1932.)

XVI.—Pressure Effects in the Secondary Spectrum of Hydrogen.

By **W. G. Guthrie,** M.A., B.A.. Carnegie Fellow in the University of St Andrews. *Communicated by* Professor H. S. ALLEN, F.R.S.

(MS. received March 1, 1932. Revised MS. received May 16, 1932. Read May 2, 1932.)

THIS paper is an account of an experimental investigation which was made on the behaviour of the secondary spectrum of hydrogen as affected by the pressure of the luminous gas. It was conjectured that the classification of the lines of the spectrum according to their behaviour in this respect should be of value in the problem of accounting for their origin in terms of the quantum theory. There can be no doubt, on the evidence given below, that the method is of some importance. For this reason it has been considered worth while bringing the results already obtained to the notice of workers in this field by publishing tables of intensities.

The task of photographing the spectrum of hydrogen at pressures comparable with atmospheric is one of some difficulty. In the first place, the intensity of the arc at these higher pressures is very small and consequently long exposures are necessary to bring out the fainter lines. In practice exposures up to 10 or 12 hours' duration were used. Then there are the problems associated with any lengthy photographic process—how to ensure constancy of temperature, so as to avoid temperature shift of the lines; how to avoid pressure variations over the period of the experiment; and, most troublesome of all, how to run the arc for a length of time in such a manner as to avoid sputtering and glowing of the electrodes.

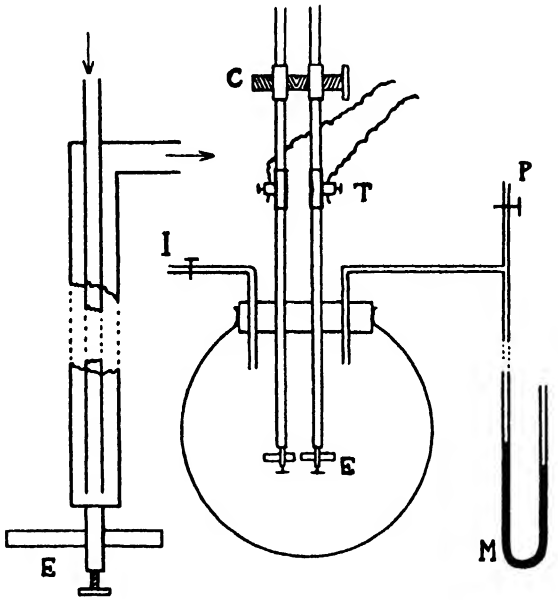
THE APPARATUS.

(1) *The Spectrograph.*—The lines of the secondary spectrum of hydrogen are very numerous in the visible region—some three or four thousand are given in the tables of Gale, Monk, and Lee.* In order to photograph the spectrum satisfactorily, it is therefore necessary to use an instrument of high dispersion. The spectrograph was similar to the Hilger E 1 model, except for the fact that a glass prism was used intead of the quartz prism originally supplied; this had the effect of providing increased dispersion in the visible region of the spectrum. In the present work two ranges of the visible region were photographed: (1) Violet to Green (approximately from

* H. G. Gale, G. S. Monk, and K. O. Lee, *Astrophys. Journal*, 1928, 67, 89.

$H\gamma$ to $H\beta$), and (2) Green to Red (from $H\beta$ to $H\alpha$). The dispersion obtained with the spectrograph increased along the spectrum from 20 A.U. per mm. of plate at $H\alpha$ to 5 A.U. per mm. at $H\gamma$. There is consequently a considerable difference in the concentration of lines on the photographic plates for the two ranges.

(2) *The Source.*—The source of light employed was the arc obtained by the discharge of electricity between two metallic electrodes in an atmosphere of hydrogen. The discharge tube and some of the fittings are represented



diagrammatically in the figure. The design of the apparatus is due to Dr I. Sandeman. The electrodes used (E) were rods of pure tungsten of circular cross-section and about 4 mm. in diameter. These were screwed to the ends of hollow brass tubes in which, during an exposure, a stream of cold water was kept running. The discharge tube was a spherical bowl of transparent glass fitted with a wide air-tight rubber stopper, through which were introduced the two brass tubes mounted parallel about 4 cm. apart, and also two glass tubes which served as inlet and outlet for the gas in the tube. For the purpose of striking the arc and regulating the distance between the electrodes, a screw was fitted between the upper ends of the brass tubes, by means of which it was found possible, on account of the slight flexibility of the rubber, to control the discharge gap over a sufficient range (about 0 to 4 mm.); this control is shown at C in the diagram. The

gas inlet tube (I) communicated through a drying tube containing phosphorus pentoxide with the hydrogen reservoir, a steel cylinder containing hydrogen under pressure. The outlet tube was connected through a glass T-piece to an open U-tube manometer (M) containing mercury, and (at P) to a Hyvac Cenco oil-pump. The whole apparatus was rendered air-tight by sealing up all the junctions between its parts with molten vacuum wax.

The current for running the arc was taken from the electric supply mains at 460 volts, the connections to the electrodes being made by terminals (shown at T) attached to the brass tubes. Included in the circuit were fixed and variable resistances for governing the intensity of the discharge, an ammeter for measuring the current carried by the arc, and a voltmeter to measure the voltage across the terminals. An important advantage in mounting the electrodes on the rubber support and not rigidly was that, by merely rubbing them together, they could be cleaned of any slight deposit which might accumulate during an exposure, without dismantling the discharge tube. Such a deposit has to be avoided, as it tends to glow brightly when the arc is running and might therefore cause fogging of the photographic plate.

When the arc is running steadily it consists of a bright violet glow confined mainly to the end of the cathode and its immediate neighbourhood, no illumination being visible at the anode. The glow increased in intensity with the current; but if the current was made too large, the anode became red-hot and the discharge tube overheated. The current taken by the arc depends on the pressure of the gas in the discharge tube and on the distance apart of the electrodes, the order of magnitude being about 0·2 amp. for a pressure 50 cm. and distance 1–2 mm. Stronger currents were necessary to run the arc at higher pressures, and the tendency of the anode to overheat was correspondingly greater. The current taken by the arc also increased as the gap between the terminals was decreased. But if too small a gap were used, the behaviour of the arc was unsteady and sputtering was liable to occur. On the other hand, if the gap were increased beyond a certain limit the arc was extinguished.

An efficient arrangement for cooling the electrodes was found to be of the highest importance in the proper running of the arc. This was obtained by leading a current of cold water down each brass tube on which the electrodes were mounted, by means of a narrow concentric pipe, and back again in the space outside the pipe; the arrangement is shown in the enlargement on the left of the figure.

The light from the source was brought to a line focus on the slit by a sphero-cylindrical condensing lens, the final adjustments in position being

made by visual examination of the spectrum. In the actual arrangement used, the source was 35 cm. and the lens 14 cm. from the slit. The photographic plates employed were Imperial Eclipse 10″ × 2″ for the Violet-Green range, and Ilford Special Rapid Panchromatic 10″ × 2″ for the Green-Red range; the best exposures were found to be 2 and 10 hours respectively.

The hydrogen was supplied by the British Oxygen Company, the purity of the gas being guaranteed within 0·1 per cent. A spectroscopic test for the presence of air impurities was applied by taking photographs of the spectrum of the arc when running in air at atmospheric pressure and comparing the plates with those of the hydrogen spectrum on the Comparator of Dr Jack.* No lines attributable to the presence of air in the hydrogen could be detected in the regions for which the records are given.

Procedure.

When the arc had been struck and finally adjusted to give the brightest source possible, its position with regard to the spectrograph was arranged to give the best results at the camera end. In practice it was found that more illumination was received at the camera if, instead of having the line of the electrodes and arc "square on" to the direction of the slit and lens, it was placed (still horizontal) at an appreciable angle to this direction. The effect of this was to increase the solid angle subtended by the end of the cathode, on which most of the glow is concentrated, at the slit, and consequently to produce greater illumination. The best position was found by trial and the gain in intensity was quite considerable.

During the exposure of a photographic plate it was important to watch two things carefully: (1) the running of the arc, (2) the room temperature. As regards the first, the chief troubles are sputtering and glowing; the arc would also sometimes suddenly fail and require to be restruck. Variation of room temperature was kept under supervision by running a clockwork thermograph while an experiment was in progress. There was generally no appreciable change in pressure.

In the present research all that was required was an identification of the lines and not an accurate determination of their wave-length; therefore it was sufficient to have a single comparison spectrum. For this purpose a photograph of the iron arc was taken on the same plate. After the more prominent lines had been identified, it was possible to complete the work without further reference to the comparison spectrum.

* *Proc. Roy. Soc. Edin.*, 1929-30, Vol. L, Part II, No.16, pp. 200–203.

The estimates of the intensities of the lines in the case of the spectrum at low pressures were made from photographic records taken on the same spectrograph and with similar plates by Mr A. S. Roy. The pressure in the discharge tube was less than 0·001 mm. of mercury, and the exposures were of two hours' duration.

IDENTIFICATION OF THE LINES.

The plates were examined by means of a Hilger L 18 measuring micrometer. This instrument is capable of 15 cm. travel by screw motion, and settings of its position can be made by double verniers with an accuracy 0·001 mm.

The wave-lengths were calculated from the formula of Hartmann and identified with those of hydrogen lines given in the tables of Gale, Monk, and Lee.* The calculated wave-lengths seldom differed from those of the lines with which they were identified by more than 0·1 A.U. in the violet range, or 0·2 A.U. in the red. This margin was sufficient in most cases to enable the identification to be made with precision.

It was sometimes found that a later plate was more successful in bringing out the lines than an earlier one which had already been measured. In such cases the Comparator of Dr Jack † provided an easy means of determining exactly how two plates differed in quality and of avoiding unnecessary labour in calculation.

Photometric records of three of the spectra in the violet range were taken from the microphotometer of the Observatory in Edinburgh. These records show graphically the variation in intensity along the whole range of the spectrum on a linear scale which is about twenty times that of the photographic plate, and were found to be of value in the examination of the spectrum—particularly in some cases where it was not possible to determine by eye whether or not a line was composite.

The eye estimates of the intensities of the lines on a scale of ten units were recorded for the two ranges, the resulting tables referring to some seven hundred lines in the region of wave-length 6540 A.U. to 4290 A.U. As regards the consistency of the measurements it was found that two independent readings of the same plate were, with few exceptions, consistent to within one unit, while in the majority of cases the agreement was exact.

In the tables the most intense lines are marked 10 and the weakest for which the edges are visible 0. Very faint lines with indistinct edges are marked 00. The letters "d" or "b" refer to lines which appear diffuse or

* *Loc. cit.* † *Loc. cit.*

unusually-broad; in such cases identification is troublesome, as the line is composite. Where the identification was doubtful on account of the limited dispersion of the spectrograph, the corresponding lines given by Gale, Monk, and Lee are bracketed. For these the intensity recorded might be due to one or more of the lines mentioned. In Table I are given those lines

TABLE I.

15536·11	2	1	3	20302·96	3	0	5	21740·19	3	3	8
15606·61	0	..	2	20365·97	5	4	8	21771·84	2	2	4
15706·57	1	1	3	20396·79	0	0	3	21820·64	4	3	10
15877·13	0	0	4	20417·18	2	2	5	21862·76	8	7	10
15963·63	2	00	2	20620·86	1	00	3	21905·79	3	3	8
16004·39	0	..	3	20648·91	0	1	2	21967·13	5	4	8
16068·21	2	1	7	20686·21	5	3	9	21972·38	3	2	6
16192·25	6	4*b*	8	20721·47	4	2	6	21985·24	1	1	5
16230·41	2	2	6	20777·30	1	0	6	22002·39	5	3	7
16279·56	0	0	2	20780·79	00*a*	0	3	22014·85	2	2	4
16338·02	1	0	7	20789·00	0	0	3	22028·48	2	1	8
16430·10	0	0	5	20795·75	2	2	8	22059·41	1	1	5
16507·71	2	00	6	20859·96	1	00	4	22117·91	3	1	7
16710·63	6	6	8	20888·53	2	2	10	22122·01	3	1	5
16744·92	5	6	8	20903·99	1*a*	1	4	22124·27	1	1	3
16753·44	4	3	8	20960·64	1	00	3	22145·57	3	2	8
16897·19	5 }	8	7	20967·65	0	0	3	22188·25	6	6	9
16898·48	5 }			21009·39	2	0	5	22206·34	6	5	8
16930·14	0	0	2	21039·79	2	1	8	22215·76	1	00	5
16940·99	1	1	4	21081·77	3	4	5	22269·29	1	0	3
16961·69	0	1	3	21112·28	0*h*	0	2*b*	22297·72	1	00	3
17033·21	4	6	8	21132·38	2	1	6	22304·85	1	2	7
17055·50	1	1	4	21140·73	1	1	6	22311·57	00	1	2
17084·56	2	1	5	21163·32	1	1	4	22395·79	1	1	6
17144·93	4	1	3	21220·69	3	2	7	22423·83	1	3	9
17179·50	3	0	8	21300·46	0	00	2	22468·14	0	00	2
17192·31	6	3	6	21333·56	3	3	5	22543·76	1	1	5
17222·72	2	00	5	21365·71	5	4	7	22547·31	1	1	4
17233·19	2*h*	1	4	21369·03	0*a*	00	2	22684·08	0	0	2
17252·65	2	1	5	21401·31	7	4	9	22712·42	0*a* }	0	7
17260·69	4	4	6	21452·78	1	0	6	22713·65	0 }		
17296·19	2*h*	2	7	21463·10	2	1	4	22730·82	1	2	7
17335·25	2 }	3	8	21480·64	3	1	8	22748·03	00	00	3
17337·39	3 }			21497·62	1	1	3	22763·71	1	1	3
17364·30	5	1	8	21505·92	1*h*	0	4	22801·45	1	1	7
17527·44	4	4	9	21520·92	3	4	8	22894·44	1	1	4
17529·00	5	4	9	21530·64	0	0	3	22903·51	0	00	3
17545·14	1	1	4	21533·33	1	0	3	22979·81	1*a*	00	3
17589·17	3	5	8	21543·49	1	0	3	23086·00	1	1	3
17591·22	4	5	8	21585·45	8	7	10	23164·38	0	1	2
18035·41	4	2	6	21614·14	5	5	9	23186·21	1	1	4
18411·60	0	00	2	21645·54	2 }	5	10*b*	23217·88	1	1	6
19116·21	0	0	5	21646·92	5 }			23227·87	1	0	3
19122·16	00	00	4	21698·15	4	4	9	23272·91	1*h*	2	4
20176·00	1	1	6	21707·75	2	1	6				

which show a distinct increase in intensity at the higher pressures, while Table II refers to those which are diminished in intensity. In the case of other lines which appear in Gale, Monk, and Lee's tables, but which are not tabulated here, it may be assumed that no evidence was found of any

marked change in intensity. In each table the first column gives the wave number (*in vacuo*) and the second the intensity, as recorded by Gale, Monk,

TABLE II.

15552·37	10	8	3
15621·96	10	7	2
15800·76	10	8	1
15870·10	10	8	1
15905·53	10	7	2
15941·22	5	4	1
16025·36	10	7	4
16046·28	7	5	2
16060·31	10	10	5
16126·19	10	7	3
16168·95	10	9	2
16203·93	7	3	1
16225·06	8	4	1
16330·59	10	10	6
16399·79	10	8	1
16440·72	10	10	6
16445·80	3	3	0
16488·15	8	6	4
16584·72	10	9	6
16596·37	10	7	1
16603·20	9	7	4
16611·43	10	10	6
16654·24	9	7	2
16764·12	4	3	1
16802·37	10	8	5
16854·86	10	10	7
16873·46	9	9	2
16884·94	7	4	1
17006·43	10	8	2
17025·41	6	7	2
17091·29	8	5	3
17138·90	4*h*	4	1
17169·22	10	8	4
17218·50	8	6	5
17676·23	9	7	1
18053·90	10	8	4
18158·55	10	5	3
18239·51	10	7	3
18509·78	8	3	00

18554·05	10	7	3
18631·03	7	5	00
18665·78	7	2	00
18799·23	5	2	1
18851·17	4	7	1
18851·66	9		
18984·32	10	5	2
19238·85	8	3	1
19319·42	6	2	0
19425·89	5	3	0
19660·83	9	7	2
19677·66	7	4	0
19725·70	7	4	1
19742·22	6	4	2
19776·54	9	8	1
19804·30	5	2	00
19836·46	9	5	1
19873·74	9	6	0
19942·45	10	9	6
19949·79	8	3	2
19960·91	9	6	1
20072·80	6	4	0
20118·28	5	4	1
20127·64	6	6	2
20260·90	10	9	7
20269·03	7	6	3
20438·89	4	5	1
20459·24	2	3	0
20493·95	3	1	0
20503·04	6	6	4
20662·91	6	6	4
20728·46	8	6	4
21076·13	5	4	3
21086·78	5	4	3
21207·81	6	5	3
21306·76	5	5	2
21330·69	6	6	4
21340·34	4	3	2
21446·79	8	6	3

21746·27	4	2	0
21786·69	3	3	1
21806·54	5	3	1
21827·99	10	9	7
21847·63	8	7	6
21913·02	4	4	2
21936·14	5	4	1
21937·52	4		
21951·80	10	8	3
21981·71	5	2	1
22063·07	3	2	0
22139·45	3	2	1*d*
22225·34	10	7	4
22247·21	5	5	1
22263·25	9	9	5
22284·92	8	6	2*d*
22286·17	2		
22343·81	6	6	0
22355·31	3	2	0
22379 40	8	7	1
22410·40	10	9	4
22451·67	3	4	2
22466·06	6	3	3
22476·06	6	7	4
22477·97	9	7	4
22489·64	7	6	4
22494·87	3	3	0
22501·76	2	1	0
22643·75	3	4	1
22647·73	4	4	1
22657·81	8	9	4
22717·07	4	3	2
22827·77	4	5	1
22958·10	3	3	1
23099·28	5	5	1
23202·73	2	2	00
23215·41	3	3	1

and Lee. In the third and fourth columns the numbers are the author's estimates of the intensities of the lines in the spectrum at low and high pressure respectively.

GENERAL REMARKS.

The effects of pressure of the order of one atmosphere on the intensity of the lines appear to be selective. Compared with the spectrum of a "vacuum" tube, the lines fall into three groups: (1) those which are distinctly enhanced under the pressure conditions; (2) those which show little or no alteration; (3) those which show marked diminution in intensity.

Photographs were taken with pressures ranging from 30 cm. of

mercury to atmospheric, but the conclusion arrived at from a comparison of these was that variation of the pressure within this range had little effect on the relative intensity of the lines: the effect of increasing the pressure appeared merely to be to scale down the intensities—the stronger lines appeared fainter, and the fainter lines failed to appear at the higher pressure.

In the tables, therefore, the numbers given are for the best plates, viz. for a pressure of 40 cm. of mercury for the Red-Green range and 50 cm. of mercury for the Green-Violet.

The observations have been used to isolate certain groups of related lines from the spectrum; their theoretical significance is still under investigation.

The author's grateful acknowledgments are due to Professor H. S. Allen for advice and criticism during the progress of the work; to Professor Sampson for kindly granting permission to use the microphotometer in the Royal Observatory, and to his assistant, Dr Baker, for much valuable help; also to Mr A. S. Roy for the use of some of his photographic plates.

BIBLIOGRAPHY.

1. Leo Arons, 1900, "On electric arcs between metallic electrodes in nitrogen and hydrogen," *Annal. der Phys. und Chem.*, **1**, 700.

2. T. R. Merton and S. Barratt, 1922, "On the spectrum of hydrogen," Bakerian Lecture of the Royal Society.

3. Kiuti, 1923, *Proc. Phys. Math. Soc. Japan*, **5**, 9.

4. I. Sandeman, 1925, 1926, "On the secondary spectrum of hydrogen at higher pressures," *Proc. Roy. Soc.*, **108**, 607–16; **110**, 326–41.

5. Y. Hukumoto, 1930, "On some relationships between the continuous and the many-lined spectrum of hydrogen," *Science Reports of Tohoku University*, 1st Series, **19**, 773–92.

A number of papers by D. Chalonge in *Comptes Rendus* (1928–1930) deal with the effect of pressure on the continuous spectrum of hydrogen.

(*Issued separately July* 8, 1932.)

XVII.—The Adrenal Gland of *Xenopus laevis*. By H. Zwarenstein and I. Schrire, from the Department of Physiology, University of Cape Town. *Communicated by* Sir E. A. Sharpey-Schafer, F.R.S., P.R.S.E. (With Three Text-figures.)

(MS. received March 22, 1932. Read May 2, 1932.)

The adrenal gland of Amphibia represents a transition stage between the adrenal system of Cyclostomes and Fishes in which the two components, cortical and chromaphil cells, remain separate and the adrenals of Amniota in which the two components are intimately associated with one another and form a definite and discrete organ. In Amphibia the cortical and chromaphil portions are partially amalgamated.

In the Anura the gland is present as a distinct golden-yellow body on the ventral surface of each kidney in intimate association with the venæ renales revehentes. It consists histologically of interlacing columns of cortical cells and of chromaphil cells which are irregularly distributed around and within the cell-columns.

In Urodela the adrenals occur as a series of yellow streaks on the ventral surface of each kidney, also in close association with the renal veins. The microscopic structure is similar to that of the Anuran adrenal. In addition, however, isolated masses of chromaphil cells are also present.

The following is an account of the adrenal system of the South African clawed toad. *Xenopus lævis*:—

Morphology.—By superficial inspection of the brightly illuminated ventral surface of the kidney under a low-power, binocular, dissecting microscope, it can be seen that the adrenal occurs as a series of yellow strips and islets closely packed round the renal veins and their branches. Isolated islets occur on the walls of the veins, and larger masses of adrenal tissue can be seen irregularly distributed in the wall of the inferior vena cava (fig. 1).

The adrenal extends from the superior to the inferior poles of the kidney, but laterally and medially narrow bands of kidney surface are left uncovered. The lateral uncovered area is broader than the medial, and coursing over it a number of superficial terminal branches of the renal veins can be seen. Occasionally an isolated yellow islet occurs on this lateral area. The medial area is covered by the lateral edges of the inferior vena cava, and at intervals by the large junctional renal veins. With strong illumination, yellow islets can also be seen at deeper levels

and bordering the large venous sinuses from which the renal veins take their origin (fig. 2).

Histology.—Both kidneys with the inter-renal blood vessels, etc., were dissected out and placed in 3 per cent. potassium bichromate. After half an hour's fixation, 10 per cent. formol was added and the tissues left in the mixture for three to four days. After washing in running water over-

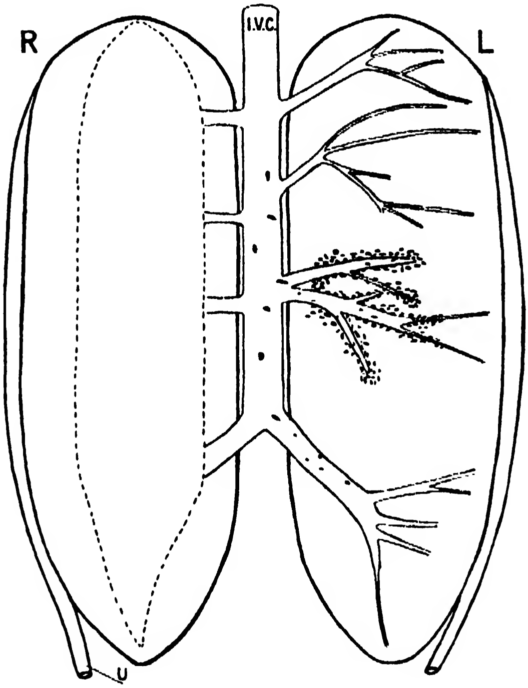


Fig. 1. [Semi-diagrammatic.]

Right Kidney—Area of distribution of yellow strips and islets of adrenal tissue indicated by dotted line.

Relation of strips and islets to venæ renales revehontes of left kidney and to inferior vena cava (I.V.C.). U, Ureter.

night, the tissues were embedded in paraffin. Serial transverse sections, 15 μ thick, were cut and stained in hæmatoxylin and dilute eosin.

Study of the serial sections and the construction of a model by means of R. W. Haines' (private communication) celluloid method showed the following distribution of cortical and chromaphil cells (fig. 3):—

Venæ renales revehentes issue from the hilum of the kidney and pass towards the mid-line to form the inter-renal portion of the inferior vena cava. The veins commence in large sinusoidal blood spaces which penetrate some distance into the substance of the kidney. These spaces

are surrounded by columns and groups of cortical cells forming the walls of the spaces, and in many cases protruding into the lumen. Interspersed among the cortical cells are the chromaphil cells. They occur at the periphery of the cortical groups, in some cases forming a complete capsule, and they also occur irregularly distributed within the cortical cell-columns. Isolated groups of chromaphil cells are also present, but they are not numerous. This arrangement of anastomosing columns of cortical cells with chromaphil cells and blood sinuses in the hilum of the kidney constitutes the adrenal gland proper.

In some parts of the kidney the ciliated funnels or nephrostomes bound the gland laterally, in other parts they traverse the lateral portion of the gland. Certain areas are quite free from nephrostomes.

Elongated groups of cortical and chromaphil cells and large isolated masses of chromaphil cells are found in the walls of the renal veins and inferior vena cava. Masses of adrenal tissue extend throughout the thickness of the vena cava and frequently bulge into the lumen.

Accessory chromaphil bodies are numerous in the connective tissue between the abdominal aorta and the vena cava in close association with sympathetic ganglia, small blood-vessels, and lymphatics. They also occur in the tunica adventitia of the aorta and renal arteries.

Fig. 2. Photograph of surface of both kidneys showing distribution of yellow streak.

The adrenal gland proper is thus similar in distribution and structure to that of Urodeles. In this connection it may be pointed out that Rimer * has recently shown that the pituitary of Xenopus "essentially conforms to that of the salamandrine type in the incomplete separation of the pars tuberalis from the pars anterior."

The occurrence of groups of cortical and chromaphil cells in the walls of the blood-vessels in Xenopus is of exceptional interest, as a similar distribution is found in some Cyclostomes and Fishes. In Petromyzon cortical bodies occur in the wall of the posterior cardinal veins, and bands of chromaphil tissue are intimately associated with the large arteries and their branches. In Teleosts and Ganoids cortical and chromaphil cells occur in the walls of the cardinal veins, and in Ganoids chromaphil cells have been described in the walls of the venæ renales revehentes.

* Rimer, G., 1931, *Trans. Roy. Soc. S.A.*, vol. xix, p. 341.

Gunn and collaborators have experimentally demonstrated to the Royal Society of South Africa (1931) the presence of adrenaline in extracts of

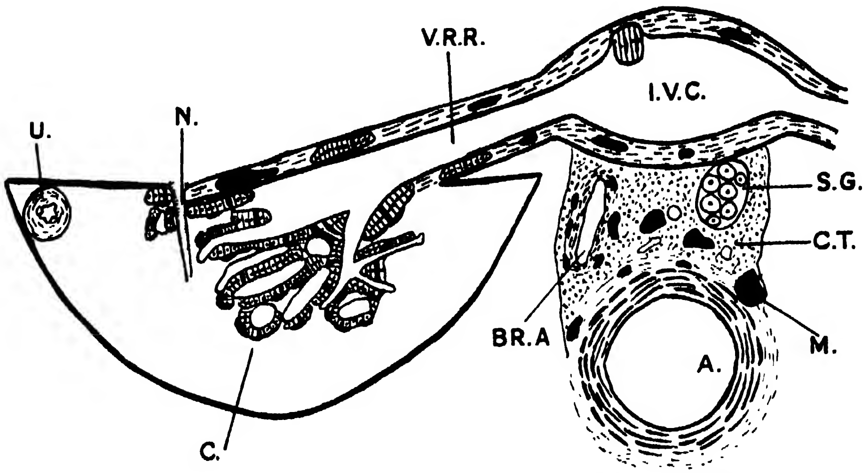


FIG. 3. Transverse section of kidney and of structures in mid-line. [Semi-diagrammatic]

[cortical cells symbol]	Cortical cells.
[chromaphil cells symbol]	Chromaphil cells.

A Abdominal aorta.
BR.A. Branch of aorta.
C. Columns and groups of adrenal tissue with large blood sinuses.
C.T. Connective tissue with blood-vessels, lymphatics, and groups of chromaphil cells.
M. Mass of chromaphil in tunica adventitia of aorta.
I V.C. Inferior vena cava.
N. Nephrostome.
S.G. Sympathetic ganglion.
U. Ureter.
V.R.R. Vena renalis revehens.

the kidneys of Xenopus. They found that extracts made from the dorsal portions of the kidney were inactive, but that extracts prepared from the ventral part, which includes the yellow streak area, gave the usual pharmacological reactions for adrenaline.

SUMMARY.

1. The adrenal gland of Xenopus is essentially similar in distribution and structure to the adrenal of Urodeles.

2. In addition, adrenal tissue occurs in the walls of the renal veins, inferior vena cava, aorta and renal arteries, a condition similar to that found in some Cyclostomes and Fishes.

We wish to thank Mr B. McManus for his assistance in the preparation of sections.

(*Issued separately July* 8, 1932.)

XVIII.—The Geodesics in Einstein's Unified Field Theory.

By **A. Blackwell**, M.Sc. *Communicated by* Professor E. T. Whittaker, F.R.S.

(MS. received January 30, 1932. Read May 2, 1932.)

§ 1. Einstein has shown * that the $g_{\mu\nu}$ giving the metric, $ds^2 = g_{\mu\nu}dx_\mu dx_\nu$, for a unified field theory can be expressed in terms of the two tensors ${}^a h_\mu$, ${}^a h_\nu$ by

$$g_{\mu\nu} = {}^a h_\mu \,.\, {}^a h_\nu \qquad a, \mu, \nu = 1, 2, 3, 4.$$

For the special case of spherical symmetry about a point he (in conjunction with W. Mayer) † obtained the following values of ${}^a h \mu$,

$$\left.\begin{array}{llll} h_s^a = \lambda\delta_{sa}, & a, s = 1, 2, 3, & h_s^4 = 0, & s = 1, 2, 3 \\ h_4^a = \tau x_a, & a = 1, 2, 3, & h_4^4 = \mu & \end{array}\right\} \qquad . \qquad . \qquad (1)$$

where ‡

$$\lambda \equiv \frac{1}{\sqrt[4]{1 - \frac{e^2}{r^4}}}; \qquad \tau \equiv \frac{\lambda}{r^3}e; \qquad \mu \equiv 1 - m\int\sqrt[4]{1 - \frac{e^2}{r^4}} \cdot \frac{dr}{r^2}, \qquad . \qquad . \qquad (2)$$

e, m being constants, identified respectively with the electrical charge and mass of a particle at the centre of symmetry, and r the radius vector drawn from that point.

The object of the present paper is to obtain the path of a particle in the field represented by these h_μ^a. We must first find the corresponding $g_{\mu\nu}$.

§ 2. They are given by

$$g_{\mu\nu} = \sum_a \epsilon_a \,.\, {}^a h_\mu \,.\, {}^a h_\nu, \qquad a = 1, 2, 3, 4,$$

where ${}^a h_\mu$ is equal to the cofactor of h_a^μ in $\| h_a^\mu \|$, divided by the value of $\| h_a^\mu \|$, and ϵ_a is to be taken as -1 for $a = 1, 2, 3$ and $+1$ for $a = 4$, to give

---

* *Preussischen Akademie der Wissenschaften Phy.-Math. Klasse*, 1928, "Neue Möglichkeit für eine einheitliche Feldtheorie," and *Ibid.*, 1929, "Zur einheitliche Feldtheorie."

† *Ibid.*, 1930, "Zwei strenge statische Lösungen der Feldgleichungen," equations (53).

‡ *Ibid.*, equations (50), (37), (44). As a matter of fact, in (44) μ is defined as

$$1 + m\int\sqrt[4]{1 - \frac{e^2}{r^4}} \cdot \frac{dr}{r^2};$$

but, as will be shown later, the sign of m must be changed so that the metric obtained when e is zero shall represent an attractive force.

a pseudo-Riemannian structure of space-time. We have

$$\| h^a_s \| \equiv \begin{vmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ \tau x_1 & \tau x_2 & \tau x_3 & \mu \end{vmatrix} = \lambda^3\mu.$$

Thus we find

$$g_{11} = g_{22} = g_{33} = -1/\lambda^2,$$
$$g_{23} = g_{31} = g_{12} = 0,$$
$$g_{\nu 4} = \frac{e x_\nu}{\lambda\mu r^3}, \qquad \nu = 1, 2, 3,$$
$$g_{44} = \frac{1}{\lambda^4\mu^2}.$$

Thus the metric is

$$ds^2 = -\frac{1}{\lambda^2}(dx_1^2 + dx_2^2 + dx_3^2) + \frac{2e}{\lambda\mu r^3}(x_1 dx_1 + x_2 dx_2 + x_3 dx_3)dx_4 + \frac{dx_4^2}{\lambda^4\mu^2}.$$

Transforming the co-ordinates (x_1, x_2, x_3) into spherical polars (r, θ, ϕ) and writing t instead of x_4, we get

$$ds^2 = -\frac{1}{\lambda^2}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \cdot d\phi^2) + \frac{2e}{\lambda\mu r^2} dr dt + \frac{dt^2}{\lambda^4\mu^2} \quad . \quad . \quad (3)$$

§ 3. It is clear that $\phi = \text{const.}$ must be an integral of the equation of the geodesics.* Taking motion in this plane, we can define a "kinetic energy" by

$$2T = -\frac{u'^2}{u^4\lambda^2} - \frac{\theta'^2}{u^2\lambda^2} - \frac{2e}{\lambda\mu}u't' + \frac{t'^2}{\mu^2\lambda^4} = 1 \quad . \quad . \quad . \quad (4)$$

where

$$u = \frac{1}{r}; \; u' = du/ds; \; \theta' = d\theta/ds; \; t' = dt/ds.$$

The equations to the geodesic are now given by

$$\frac{d}{ds}\left(\frac{\partial T}{\partial \theta'}\right) - \frac{\partial T}{\partial \theta} = 0, \text{ etc.};$$

which give

$$\frac{\theta'}{u^2\lambda^2} = k^* \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and k, K, constants

$$-\frac{eu'}{\lambda\mu} + \frac{t'}{\lambda^4\mu^2} = K \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

---

* In the General Theory of Relativity, Einstein obtained $\theta'/u^2 = k$; e.g. *The Mathematical Theory of Relativity*, by A. S. Eddington (C.U.P.), second edition, p. 86, equation (39·41). Equation (5) reduces to this when $e = 0$.

From (4) and (6) we get

$$\frac{u'^2\lambda^2}{u^4}+\frac{\theta'^2}{u^2\lambda^2}=-1+K^2\lambda^4\mu^2,$$

and dividing by the square of (5),

$$\frac{\lambda^2}{\mu^2}\left(\frac{du}{d\theta}\right)^2+\frac{u^2\lambda^2k^2+1}{k^2\lambda^4\mu^2}=C,\text{ where } C=\left(\frac{K}{k}\right)^2 \quad . \quad . \quad . \quad . \qquad (7)$$

At an apse, $u'=0$. If then we let the suffix 0 denote that the value is taken at an apse, we have from (5) and (6)

$$\frac{\lambda_0^2\mu_0^2}{u_0^2}\left(\frac{d\theta}{dt}\right)_0=\frac{k}{K} \quad\text{or}\quad C=\frac{1}{k^2\mu_0^4\lambda_0^4} \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

§ 4. It should be noted that if we neglect e^2 in the above equations it is effectively the same as neglecting e altogether, for the values of λ, μ are the same in both cases, viz.

$$\lambda=1, \qquad \mu=1+mu \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

This is the case of gravitation only.

Equation (7) now gives

$$\left(\frac{du}{d\theta}\right)^2+u^2+\frac{1}{k^2}=C(1+mu)^2 \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

Differentiating with respect to u, we get

$$\frac{d^2u}{d\theta^2}+u=Cm(1+mu)^* \quad . \quad . \quad . \quad . \quad . \quad . \qquad (11)$$

and from (8) and (9) we see that C is given by

$$C=\frac{1}{h^2(1+mu_0)^2}, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

where h, the areal velocity, is $\frac{1}{u_0^2}\left(\frac{d\theta}{dt}\right)_0$.

§ 5. We may now find the motion of perihelion of a planet, for (10) is the path of a planet. We get

$$\theta=\pm\int\frac{du}{\sqrt{B-(1-m^2C)u^2+2mCu}}+A$$

where B and A are constants. This gives

$$u(1-m^2C)=mC+\sqrt{B(1-m^2C)+m^2C^2}\cdot\cos\{\sqrt{1-m^2C}(\theta-\varpi)\} \quad . \qquad (13)$$

where ϖ is a constant, which may be made zero by suitable choice of the initial point.

At perihelion $du/d\theta=0$, so that

$$\sin\sqrt{1-m^2C}\cdot\theta=0 \quad . \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

---

* Equation (11) is the justification for changing the sign of m in the definition of μ (see note on p. 327); otherwise (11) would have represented a repulsive force.

One solution of this is $\theta=0$, and we expect the next solution to be near $\theta=\pi$, say at $\pi+\epsilon$, where ϵ/π is small. Thus, substituting in (14) for C from (12); writing $\frac{m^2}{h^2}$ for mu_0; and neglecting terms of higher order than $\frac{m^2}{h^2}$, we have

$$\epsilon=\frac{m^2}{2h^2}\pi \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

In his 1916 Theory * Einstein got ϵ to be six times as big, viz., $\frac{3m^2}{h^2}\pi$, the value required to account for the observed facts.

§ 6. We may now obtain the path of a ray of light on passing through a field as represented by (11). Taking $ds=0$ to be the path of a light ray, (4) becomes

$$-\frac{u'^2}{u^4\lambda^2}-\frac{\theta'^2}{u^2\lambda^2}-\frac{2e}{\lambda\mu}u't'+\frac{t'^2}{\mu^2\lambda^4}=0,$$

which, instead of (7), gives

$$\left(\frac{du}{d\theta}\right)^2\frac{\lambda^2}{\mu^2}+\frac{u^2}{\lambda^2\mu^2}=\frac{u_0^2}{\lambda_0^2\mu_0^2},$$

so that the angle θ, between the asymptote and the initial line, is given by

$$\theta=\int_{1/a}^{0}\frac{du}{\left(\frac{u_0^2}{\lambda_0^2\mu_0^2}-\frac{u^2}{\lambda^2\mu^2}\right)^{1/2}\frac{\mu}{\lambda}},$$

a being the shortest distance between the light ray and the centre of attraction. Using the values of λ, μ, λ_0, μ_0 given by (9), expanding, and neglecting m^2, we find

$$\theta=\pi/2+m/a.$$

So that the total deflection of a light ray is $\frac{2m}{a}$, which is half that obtained by Einstein in his 1916 † Theory.

§ 7. And, following Eddington, ‡ we have that the times of vibration of similar but differently situated atoms is $\sqrt{1-\frac{2m}{a}}$, which is the same as Einstein got in 1916.

In conclusion I should like to express my gratitude to Professor Brodetsky, who suggested this problem, for the kind assistance and encouragement he has given me whilst preparing this paper.

* *The Mathematical Theory of Relativity*, p. 88, equation (40.6).
† *Ibid.*, p. 91, § 41. ‡ *Ibid.*, p. 91, § 42.

(*Issued separately July* 8, 1932.)

XIX.—Filial and Fraternal Correlations in Sex-linked Inheritance.

By **Professor Lancelot Hogben**, M.A., D.Sc. (From the Department of Social Biology in the University of London.)

(MS. received May 10, 1932. Read June 20, 1932.)

1. Introduction.

The ancestral and fraternal correlations arising from the principle of Segregation in its original form have been made the subject of a series of memoirs by Professor Karl Pearson (3), Snow (4), and Brownlee (1). The solution of the general case with no restriction as regards dominance or the number of genes was given in an important publication by Fisher (2). The object of this communication is to direct attention to the type of correlation arising when the variance is due to X-borne genes. The problem was suggested by the comparison of fraternal coefficients for dizygotic twins of unlike sex and for boy pairs and girl pairs of dizygotic twins. The examination of this case in its simplest form, when a single sex-linked gene substitution is involved, leads to conclusions which may assist to throw light on the extent to which sex-linked inheritance contributes to the observed variance for a given measurable character in a mixed population. Since selection acts more rapidly upon sex-linked recessive conditions than upon autosomal recessives, the contribution of X-borne genes to variance of a given type is of practical as well as theoretical interest.

2. The Genotypic Ratios in a Stable Population.

The several genotypes involved in a single sex-linked gene substitution are in equilibrium in a population mating at random in the following proportions:—

$$\tfrac{1}{2}p^2\mathrm{RR} : pq\mathrm{RD} : \tfrac{1}{2}q^2\mathrm{DD} : \tfrac{1}{2}p\mathrm{RY} : \tfrac{1}{2}q\mathrm{DY}.$$

All possible types of matings and the ensuing proportions of sons and daughters may be summarised in the table on p. 332.

The filial and fraternal types of correlation may be calculated from the above. In what follows, the different combinations of parent and offspring or of sibs will be classified separately with respect to sex.

Frequency of Mating.	Type of Mating.	Daughters.			Sons.	
		RR.	RD.	DD.	RY.	DY.
p^3	RY × RR	1	0	0	1	0
$2p^2q$	RY × RD	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$
pq^2	RY × DD	0	1	0	0	1
p^2q	DY × RR	0	1	0	1	0
$2pq^2$	DY × RD	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
q^3	DY × DD	0	0	1	0	1

3. Filial Correlation.

(*a*) The first case, that of *father and son*, is unique. As there is no XX genotype the table is fourfold.

	RY	DY
RY	p^2	pq
DY	pq	q^2

This gives:

$$r = 0.$$

(*b*) The relation for father and daughter is the same as for mother and son, thus:

	RR	RD	DD
RY	p^2	pq	0
DY	0	pq	q^2

When RD is intermediate between RR and DD, this gives $\frac{1}{\sqrt{2}}$ or

$$r = 0{\cdot}71.$$

If RD and DD are indistinguishable the fourfold table is:

	R	D
R	p^2	pq
D	0	q

This gives for r

$$\sqrt{\frac{p}{1+p}}.$$

When $p=\frac{1}{2}=q$,

$$r=\frac{1}{\sqrt{3}}=0{\cdot}58.$$

(*c*) For the relation mother-daughter the table is:

	RR	RD	DD
RR	p^3	p^2q	0
RD	p^2q	pq	pq^2
DD	0	pq^2	q^3

In this case the value obtained when the heretozygote is intermediate is

$$r=0{\cdot}50.$$

When dominance is complete the fourfold table is:

	R	D
R	p^3	p^2q
D	p^2q	$q\,(pq+1)$

The value of r is

$$\frac{p}{1+p}.$$

When $p=\frac{1}{2}=q$,

$$r=0{\cdot}33.$$

4. Fraternal Correlation.

(*a*) The simplest case is the relation of brother to brother. The table is:

	R	D
R	$p^2+\frac{1}{2}pq$	$\frac{1}{2}pq$
D	$\frac{1}{2}pq$	$\frac{1}{2}pq+q^2$

In this case

$$r=0{\cdot}50.$$

(*b*) For the relation brother-sister or sister-brother the complete table is as follows:

	RR	RD	DD
RY	$\frac{1}{2}p^2(1+p)$	$\frac{1}{2}pq(1+2p)$	$\frac{1}{2}pq^2$
DY	$\frac{1}{2}p^2q$	$\frac{1}{2}pq(1+2q)$	$\frac{1}{2}q^2(1+q)$

The value of r is $\frac{1}{\sqrt{8}}$ or 0·35 (approx.), when the heterozygote is intermediate.

When RR and RD are indistinguishable the fourfold table is:

	R	D
R	$\frac{1}{2}p^2(1+p)$	$\frac{1}{2}pq(2+p)$
D	$\frac{1}{2}p^2q$	$\frac{1}{2}q(2-p^2)$

The value of r is

$$\frac{1}{2}\sqrt{\frac{p}{1+p}}.$$

When $p=\frac{1}{2}=q$, this gives

$$r=\frac{1}{2\sqrt{3}}=0{\cdot}29.$$

(*c*) There remains the case of sister-sister correlation. The complete table is:

	RR	RD	DD
RR	$\frac{1}{2}p^2(1+p)$	$\frac{1}{2}p^2q$	0
RD	$\frac{1}{2}p^2q$	$3/2\ pq$	$\frac{1}{2}pq^2$
DD	0	$\frac{1}{2}pq^2$	$\frac{1}{2}q^2(1+q)$

When the heterozygote is intermediate this gives:

$$r=0{\cdot}75.$$

When dominance is complete the fourfold table is:

	R	D
R	$\frac{1}{2}p^2(1+p)$	$\frac{1}{2}p^2q$
D	$\frac{1}{2}p^2q$	$pq(1+q)+\frac{1}{2}q(1+q^2)$

The value of r is then:

$$\frac{1+p-2p^2}{2q(1+p)}.$$

When $p=\frac{1}{2}=q$,

$$r=\frac{2}{3}=0{\cdot}67.$$

5. Summary.

The results obtained may be summarised in the following table comparing the filial and fraternal coefficients for autosomal and sex-linked variation. When dominance is complete the values given are referred to equal frequency of the allelomorphic genes, *i.e.* $p=\frac{1}{2}=q$.

Relationship.	Dominance Complete.		Heterozygote Intermediate.	
	Autosomal.	Sex-linked.	Autosomal.	Sex-linked.
Father-son	0·33	0·00	0·50	0·00
Mother-daughter	0·33	0·33	0·50	0·50
Father-daughter	0·33	0·58	0·50	0·71
Mother-son	0·33	0·58	0·50	0·71
Brother-brother	0·42	0·50	0·50	0·50
Brother-sister	0·42	0·29	0·50	0·35
Sister-sister	0·42	0·67	0·50	0·75

Briefly, then, the contribution of sex-linked genes to the observed variance will tend to lower the correlation between father and son and between brother and sister. It will raise the correlation between father and daughter or between sister and sister conspicuously. These considerations emphasise the interest attaching to a separate investigation of fraternal correlation for sibs of all three sex classes in connection with studies on twins. If the correlation for dizygotic twins of like sex is higher than for dizygotic twins of unlike sex, the difference might be due to the greater similarity of environment for two individuals of the same sex, or it might be due to the contribution of sex-linked genes to the observed variance. In the latter case the correlation for boy pairs should be significantly lower than the correlation for girl pairs. The wide discrepancies in recorded data for fraternal correlations with respect to intelligence quotients might be re-examined from this standpoint. If an appreciable proportion of the total variance is due to sex-linked genes, it is of more importance that a boy should have a clever mother than a clever father.

REFERENCES.

(1) Brownlee, 1910. "The Significance of the Correlation Coefficient when applied to Mendelian Distributions," *Proc. Roy. Soc. Edin.*, vol. xxx, p. 473.

(2) Fisher, 1918. "The Correlation between Relatives on the Supposition of Mendelian Inheritance," *Trans. Roy. Soc. Edin.*, vol. lii, p. 399.

(3) Pearson, 1904. "Mathematical Contributions to the Theory of Evolution. Part XII," *Phil. Trans. Roy. Soc.*, A, vol. cciii, p. 53.

(4) Snow, 1910. "On the Determination of the Chief Correlations between Collaterals in the Case of a simple Mendelian Population Mating at Random," *Proc. Roy. Soc.*, B, vol. lxxxiii, p. 37.

(*Issued separately September* 5, 1932.)

XX.—The Diffusion Coefficients of Bromine-Hydrogen, Bromine-Nitrogen, Bromine-Oxygen, and Bromine-Carbon Dioxide.

By **John E. Mackenzie**, D.Sc., and **Harry W. Melville**, Ph.D. (Chemistry Department, University of Edinburgh).

(MS. received June 23, 1932. Read July 4, 1932.)

Introduction.

The early experiments on gaseous diffusion carried out by Joseph Priestley,* by John Dalton,† and by Berthollet ‡ led to the conclusion that a lighter gas diffuses more rapidly than a heavier one. Faraday § concluded from his experiments that "the actual relative mobilities of the gases are inversely as their specific gravities." Graham's classical work || established quantitative relationships, of which perhaps the most important was: "The diffusion or spontaneous intermixture of two gases in contact is effected by an interchange in position of indefinitely minute volumes of the gases, which volumes are not necessarily of equal magnitude, being, in the case of each gas, inversely proportional to the square root of the density of that gas."

Measurements of diffusion as thus defined, that is diffusion of two gases in contact—without any septum between them—appear to be comparatively few. Graham's experiments in 1863 (*vide ante*) led to the more accurate work of Loschmidt,¶ which may be briefly described as follows: A glass tube was divided into two equal parts and these joined together by glass plates and a movable steel plate so arranged (*a*) that the circular holes cut in the glass plates and steel plate coincided with the bore of each half-tube, thus allowing uninterrupted passage of gas from one end to the other end of the tube; or (*b*) by pushing in the steel plate the upper half was completely cut off from the lower half of the tube. With the steel plate in position (*b*) each half was filled with a gas, the heavier gas being generally put in the lower and the lighter gas in the upper half of the vertical tube.

* *Experiments and Observations on different Kinds of Air*, 1777, iii, 301.

† *Phil. Mag.*, 1806, xxiv, 8.

‡ *Mém. d'Arcueil*, 1809, ii, 463.

§ *Quart. Journal of Sci.*, 1817, iii, 354 ; 1819, vii, 106.

|| *Ibid.*, 1829, ii, 74 ; *Phil. Mag.*, 1833, ii, 175, 269, 351 ; *Brit. Assoc. Report*, 1845, 28 ; *Phil. Trans.*, 1846, iv, 573 ; 1849, ii, 349 ; 1863, 153, 385.

¶ *Wien. Sitz.*, 1870, lxi, 367 ; lxii, 468.

The steel plate was then moved to position (*a*), when diffusion commenced, and after a definite interval of time, say an hour, the steel plate was again brought to position (*b*) and the gas mixture in each half of the tube analysed. In this manner Loschmidt obtained data from which were calculated the diffusion coefficients of CO_2 : Air ; $CO_2 : H_2$; $H_2 : O_2$; $CO_2 : O_2$; $CO_2 : CO$; $CO_2 : CH_4$; $CO_2 : N_2O$; $SO_2 : H_2$; $CO : O_2$; $CO : H_2$.

Stefan * advanced the theoretical treatment of the subject, using the experimental results obtained by Loschmidt (*vide ante*), and his collaborators Wretschko † and Benigar,‡ and also calculated the diffusion coefficients of $(C_2H_5)_2O$: air and CS_2 : air from the rate of evaporation of these liquids contained in glass tubes of 2 to 6 mm. diameter. Winkelmann § extended this method to other vapours and gases, and later Stefan ‖ summarised the results thus obtained.

Schmidt,¶ Jackmann,** Deutsch,†† and Lonius ‡‡ used a similar method, but replaced the glass tube by a steel tube provided with a steel stopcock, with opening of same bore as that of the tube, and in a second apparatus an additional stopcock divided the top half of the tube into an upper and a lower quarter. The data obtained from experiments with the second apparatus showed that the diffusion constant varied slightly with change in the relative concentration of the two gases.

The method described below presents an attempt to develop a method of measuring diffusion coefficients of vapours into gases. It allows of continuous observation of the process of diffusion and the making of a large number of measurements in one experiment. For this purpose the vapour of bromine is admirably adapted and its diffusion into a colourless gas is readily followed visually. Balard §§ in describing the newly discovered element said: "Il se volatilise aisément, et cette grande volatilité contraste beaucoup avec sa pesanteur spécifique: il suffit de mettre une goutte de brôme dans un vase quelconque, pour que sa capacité soit à l'instant remplie par une vapeur rutilante très foncée. . . ."

* *Wien. Sitz.*, 1871, lxiii, 63 ; 1872, lxv, 323 ; 1873, lxviii, 385.

† *Ibid.*, 1870, lxii, 575.

‡ *Ibid.*, 1870, lxii, 687.

§ *Ann. d. Physik*, 1884, xxiii, 203 ; 1885, xxvi, 105.

‖ *Ibid.*, 1890, xli, 725.

¶ *Ann. d. Physik*, 1904, xiv, 801.

** *Inaug. Diss. Halle*, 1906.

†† *Ibid.*, 1907.

‡‡ *Ann. d. Physik*, 1909, xxix, 664.

§§ *Ann. Chim. Phys.*, 1826, (2), 32, 337.

EXPERIMENTAL.

The apparatus (fig. 1) is a glass tube A, 1500 mm. in length and 29·8 mm. in diameter (internal). A tap funnel B is sealed to the lower end of A and the upper end is closed by a rubber stopper C provided with a glass tap D. A is filled with hydrogen or other gas, and bromine is then poured into B. The barometric pressure and room temperature having been measured, bromine is run from B into A, so that the surface of the liquid bromine is about 25 mm. in diameter, the time is noted and tap D opened momentarily to equalise the pressure. A white filter-paper screen behind the tube showed up the reddish-brown vapour of bromine. At short intervals of time the height above the surface of the liquid bromine at which the colour of the bromine vapour just becomes perceptible is measured.

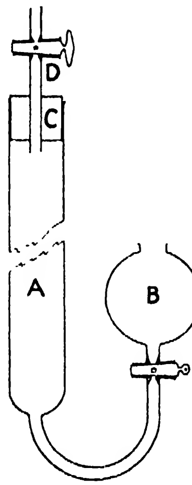


FIG. 1.

The observations were made in a room with a very large window having an uninterrupted northern exposure. Experiments made on different days and by different observers (having normal colour vision) gave closely concordant data, so that it may be concluded that slight variations in the brightness of the northern sky did not appreciably affect the readings of the heights of the bromine-vapour column. The hydrogen, nitrogen, oxygen, and carbon dioxide were supplied by the British Oxygen Company, and before introducing into the tube A were bubbled through concentrated sulphuric acid.

In this manner measurements were made with pairs of gases and bromine vapour, using two diffusion tubes side by side in order that relative as well as absolute measurements might be made. A large number of experiments were made, and the data given in Table I and plotted along with others, in fig. 2, are typical.

In order to calculate the absolute value of the diffusion coefficient (D), it is necessary to know the concentration of bromine vapour (n), which can just be seen in the apparatus used for these experiments. This concentration was measured as follows. A 12-litre flask with a neck of the same diameter as that of the diffusion tube was filled with hydrogen, and then bromine vapour was introduced until, after thorough mixing, it was just visible in the neck. The use of the flask instead of the diffusion tube rendered the adjustment of the bromine concentration much easier owing to the gradation of intensity of the colour of the bromine vapour from the middle to the neck

of the flask. In addition the use of a large flask permitted of a more accurate determination of the concentration of the bromine vapour. The flask was connected on one side to a gas wash-bottle containing sulphuric acid and

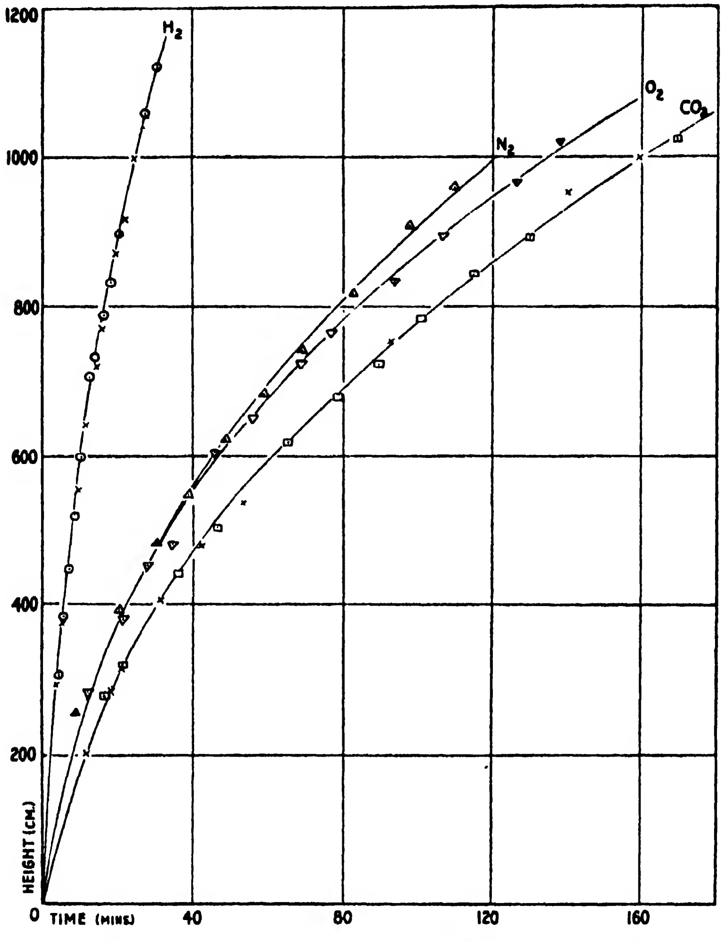


FIG. 2. × denotes readings made on 23.2.32 at 15° and 767 mm. for $Br_2 : H_2$, and on 2.24.32 at 15° and 764 mm. for $Br_2 : CO_2$.

on the other to two sets of Geissler bulbs in series containing potassium iodide solution and to a large aspirator. Air was then drawn through the apparatus until all the bromine had been transferred from the flask to the Geissler bulbs. The amount of iodine liberated was estimated by standard thiosulphate solution. Three estimations gave the following results: 2·66, 2·57, 2·79 grm. of Br_2 per 10^6 c.c., which is equivalent to a pressure of Br_2 of 0·275 mm.

TABLE I.

Bromine-Hydrogen.		Bromine-Nitrogen.		Bromine-Oxygen.		Bromine-Carbon Dioxide.	
Time. (min.).	Height at which Bromine Vapour is just visible. (mm.)	Time. (min.)	Height at which Bromine Vapour is just visible. (mm.)	Time. (min.)	Height at which Bromine Vapour is just visible. (mm.)	Time. (min.)	Height at which Bromine Vapour is just visible. (mm.)
0	..	0	..	0	..	0	..
4	306	8	254	11	285	17	280
5	386	20	389	21	385	22	320
6	446	30	479	28	455	32	375
8	518	39	544	35	485	44	485
10	601	49	619	45	605	55	565
13	709	59	679	56	655	66	620
14	736	71	739	68	725	79	675
17	791	83	814	77	765	90	720
19	836	98	909	93	835	101	780
22	906	110	959	106	905	115	845
27	1058	121	1024	119	955	130	890
31	1118	147	1089	124	965	143	925
36	1258	..	..	148	1025	170	1025
..	..	..	..	163	1065	..	..
..	..	..	..	182	1135	..	..
Pressure . 774 mm. Temp. . 286° K. Date . 18.2.32		Pressure . 772 mm. Temp. . 286° K. Date . 22.2.32		Pressure 766·5 mm. Temp. . 286° K. Date . 1.3.32		Pressure . 768 mm. Temp. . 286° K. Date . 25.2.32	

CALCULATION OF DIFFUSION COEFFICIENT.

The diffusion coefficient of a single gas may be defined by the general equation

$$\frac{dn}{dt}=\mathrm{D}\left(\frac{d^2n}{dx^2}+\frac{d^2n}{dy^2}+\frac{d^2n}{dz^2}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where n is the number of molecules per c.c., x, y, and z are co-ordinates of a given point in the gas, t being the time and D the diffusion coefficient. If, for simplicity, diffusion in a long cylindrical tube is considered, then equation (1) may be simplified to

$$\frac{dn}{dt}=\mathrm{D}\frac{d^2n}{dx^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

x being measured along the axis of the tube, this equation being the expression of Fick's diffusion law. In order to integrate (2) the limiting

conditions must be known. If it is assumed that the concentration is kept constant at one end of the tube, i.e. $n=n_0$ for $x=0$, then (2) becomes, on integration,*

$$n=n_0e^{-x^2/2Dt}. \qquad (3)$$

Hence if n is measured at x after a given time interval t, D may be calculated from (3).

Before it is possible to calculate D from the above data an important correction must be applied. Since the tap at the top of the diffusion tube is kept shut, the pressure will gradually rise as the bromine evaporates. If $\bar{p}$ is the mean pressure of the bromine, x_0 the length, and A the area of cross-section of the diffusion tube,

$$\bar{p}Ax_0=A\int_0^{x_0}p\,dx$$

or

$$\bar{p}=\frac{1}{x_0}\int_0^{x_0}p\,dx=\frac{1}{x_0}\int_0^{x_0}p_0e^{-x^2/2Dt}dx.$$

The integral is most conveniently evaluated graphically. Now $p=p_0e^{-kx^2}$, and let $k=1/2Dt$. Since p and p_0 are known, then the value of k for any value of x may be computed and therefore the curve of p against x plotted. This was done for three values of x, viz. 70, 87·5, and 110 cm., the area of the curves determined by a planimeter, and the mean value of the pressure of the bromine vapour obtained from the above equation.

The following results were obtained:

x	70	87·5	110 cm.
p	20·2	24·8	32·5 mm.

The values of t for $x=70$, 87·5, and 110 cm. were obtained from fig. 2. Using equation (3), D was calculated for the pressure prevailing in the diffusion tube, and then by multiplication by the factor—total pressure/760—D_{760} was evaluated. The results of the calculation are shown in Table II.

D calc. may be obtained in various ways by using the different equations which have been advanced from time to time, and the values thus got are in fairly close agreement. In a recent comprehensive survey of diffusion coefficients, Arnold † concludes that the diffusion equation given by Jeans ‡ agrees best with the experimental data. The diffusion coefficient is given by the equation

$$D=\frac{K}{\sigma_{12}^2}\sqrt{\frac{1}{M_1}+\frac{1}{M_2}}, \qquad (4)$$

* *Cf.* Eucken, *Lehrbuch der chemischen Physik*, p. 522.
† *Industrial and Engineering Chem.*, 1930, xxii, 1091.
‡ *Dynamical Theory of Gases*, 4th ed., chap. xiii.

where K is a constant provided the temperature is a constant, σ_{12} the sum of the radii of the molecules, and M_1 and M_2 the molecular weights of the gases 1 and 2.

TABLE II.

Gas Mixture.	x (cm.).	t (min.).	p (mm.).	Total Press.	D cm.^{2} sec.$^{-1}$	D_{760}.	D calc.
Br_2/H_2	70·0	13·0	20·0	794·0	0·513	0·538	..
	87·5	20·0	25·0	799·0	0·521	0·550	0·598
	110·0	30·0	32·5	806·5	0·546	0·580	..
Br_2/N_2	70·0	61·0	20·0	792·0	0·110	0·115	..
	87·5	92·0	25·0	799·0	0·113	0·119	0·132
	110·0	146·0	32·5	806·0	0·113	0·120	..
Br_2/O_2	70·0	63·0	20·0	786·0	0·104	0·109	..
	87·5	101·0	25·0	791·0	0·103	0·108	0·133
	110·0	178·0	32·5	806·0	0·093	0·099	..
Br_2/CO_2	70·0	84·0	20·0	788·0	0·079	0·083	..
	87·5	126·0	25·0	793·0	0·083	0·087	0·107
	110·0	..	..	..	..	..	..

For $H_2 : O_2$ Arnold concludes from theoretical considerations that the most probable value of D_{12} is 0·70. Using this value, the constant K of equation (4) was determined by substituting the appropriate values for σ and M_1 and M_2, and found to be $7{\cdot}02 \times 10^{-16}$. In Table II, D calc. was got by using this *theoretical* value of K and the following values of σ—Br_2, $1{\cdot}71 \times 10^{-8}$ cm.; H_2, $1{\cdot}15 \times 10^{-8}$ cm.; N_2, $1{\cdot}55 \times 10^{-8}$ cm.; O_2, $1{\cdot}45 \times 10^{-8}$ cm.; CO_2, $1{\cdot}60 \times 10^{-8}$ cm.

It is to be noted that the observed diffusion coefficients are consistently lower than the calculated values. A similar difference between observed and calculated diffusion coefficients is found in the data given by Arnold * for mixtures of gases, and of vapours and hydrogen, but not of vapours with air or carbon dioxide.

With the one exception of $Br_2 : O_2$ at $t = 178$ min. (which may be due to experimental error), D_{760} increases with time of diffusion. Hence the diffusion coefficient is dependent to some slight extent on the composition of the vapour-gas mixture, as has been previously found to be the case with most gas mixtures (*see* p. 338). It is probable that the experimental values of D would approach closely to the calculated values if the diffusion were allowed to proceed further until an approximately equimolecular mixture was obtained.

* *Industrial and Engineering Chem.*, 1930, xxii, 1091.

Summary.

A brief history of the measurement of diffusion of gases (excluding "effusion," "transpiration," and other processes from which data for the calculation of the diffusion coefficient may be obtained) is given.

A visual method of following the diffusion of a strongly coloured vapour (bromine) into colourless gases is described.

The diffusion coefficients calculated from the observed data are in good accordance with those obtained theoretically.

The authors desire to thank Miss Netta Macnaughton for valuable assistance in the experimental work.

(*Issued separately September 5, 1932.*)

XXI.—Relative Co-ordinates. By A. G. Walker, B.A. (University of Edinburgh). *Communicated by* H. S. Ruse, B.A.

(MS. received May 17, 1932. Read July 4, 1932.)

Introduction.

If C is a given curve in a Riemannian space V_n, a system of co-ordinates $(z^1, z^2, \ldots, z^n)$ can be set up at each point P of C, thus generalising moving axes along a twisted curve. If in these co-ordinates a point Q is defined relative to the point P, then Q traces some curve as P moves along C; any curve C′ can be defined in this way by setting up a (1, 1) correspondence between points of C and C′. We shall take the relative co-ordinates at P to be normal co-ordinates with origin at P, the parametric directions at P being the directions of an orthogonal ennuple defined at points of C. In general, this work is too heavy except for the consideration of points within a certain distance from the curve C, and we shall therefore consider only points the cube of whose distance from C may be neglected. This is sufficient for the application to such problems as the motion of a small rigid body in space-time.

§ 1. Relative Co-ordinates.

Let V_n be a Riemannian space, co-ordinates $(x'^1, x'^2, \ldots, x'^n)$, and let C be a curve in V_n, defined by the equations

$$x'^i = x^i(s), \qquad (i = 1, 2, \ldots, n) \quad . \quad . \quad . \quad . \quad (1.1)$$

where s is the arc, or a parameter if C is null. Let $\lambda_{\sigma|}{}^i$ be the components of the vectors of an orthogonal ennuple* defined at points of C. The ennuple chosen will depend in general on the particular problem under consideration. The vectors of the ennuple may be given explicitly, or they may be solutions of a set of differential equations. An example of the former is obtained when the ennuple is formed by the tangent and principal normals of C, and an example of the latter when the ennuple is defined by Levi-Civita parallel transport from a given ennuple at some point of C. In both cases the vectors of the ennuple are solutions of displacement equations of the form

$$dV^i + C^i_{jk} V^j dx^k = 0, \qquad C^i_{jk} = \Gamma^i_{jk} + A^i_{jk} \quad . \quad . \quad . \quad (1.2)$$

* The notation is that of Eisenhart, *Riemannian Geometry* (1926). In $\lambda_{\sigma|}{}^i$, σ indicates the vector of the ennuple and i the component.

where Γ^i_{jk} are the Christoffel symbols of the second kind, and A^i_{jk} are the components of a tensor. For if the vectors are given explicitly, and invariants $\gamma_{\theta\sigma}$ are defined by

$$\gamma_{\theta\sigma} = -\lambda_{\theta|i}\left(\frac{d\lambda_{\sigma|}{}^i}{ds} + \Gamma^i_{jk}\lambda_{\sigma|}{}^j\frac{dx^k}{ds}\right) \quad . \quad . \quad . \quad . \quad . \quad (1.3)$$

then, multiplying by $e_\theta\lambda_{\theta|}{}^l$, and summing for θ, we have

$$\begin{aligned}\frac{d\lambda_{\sigma|}{}^l}{ds} + \Gamma^l_{jk}\lambda_{\sigma|}{}^j\frac{dx^k}{ds} &= -\sum_\theta e_\theta\gamma_{\theta\sigma}\lambda_{\theta|}{}^l \\ &= -\sum_{\theta,\,\phi} e_\theta e_\phi\gamma_{\theta\phi}\lambda_{\theta|}{}^l\lambda_{\phi|j}\lambda_{\sigma|}{}^j.\end{aligned}$$

Hence, the vectors $\lambda_{\sigma|}{}^i$ are solutions of (1.2), where

$$A^i_{jk}\dot{x}^k = \sum e_\theta e_\phi\gamma_{\theta\phi}\lambda_{\theta|}{}^i\lambda_{\phi|j}, \qquad \dot{x}^k = dx^k/ds.$$

From this equation we see that if the vectors are given as solutions of the equations (1.2), the invariants $\gamma_{\theta\phi}$ are given by

$$\gamma_{\theta\phi} = A^i_{jk}\dot{x}^k\lambda_{\theta|i}\lambda_{\phi|}{}^j. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.4)$$

By differentiating the equations of orthogonality, $g_{ij}\lambda_{\theta|}{}^i\lambda_{\phi|}{}^j = e_\phi\delta^\theta_\phi$, it can be seen at once from (1.3) that the invariants satisfy

$$\gamma_{\theta\phi} + \gamma_{\phi\theta} = 0, \qquad (\theta, \phi = 1, 2, \ldots, n). \quad . \quad . \quad . \quad (1.5)$$

It may here be of interest to discuss the forms (1.2) must take in order that the vectors should satisfy certain conditions. It is always necessary that the magnitude of a vector and the angle between two vectors should be unaltered by the displacement, *i.e.*, if u^i, v^i are any two solutions, the expression $g_{ij}u^iv^j$ must be constant. Differentiating this expression along the curve, and substituting from (1.2), the required conditions become

$$(g_{ij}A^i_{kl} + g_{ik}A^i_{jl})\dot{x}^l = 0.$$

Thus, if this is all that is required, the most simple solution is $A^i_{jk} = 0$, giving the ordinary parallel transport.

If it is also required that the vectors should make constant angles with the curve C, then $\dot{x}^i$ must be a solution of the equations, *i.e.* substituting in (1.2),

$$A^i_{jk}\dot{x}^j\dot{x}^k = -\eta^i = -(\ddot{x}^i + \Gamma^i_{jk}\dot{x}^j\dot{x}^k),$$

η^i being the curvature vector of the curve. The most simple solution of these two sets of equations is evidently

$$A^i_{jk}\dot{x}^k = eg_{jk}(\dot{x}^i\eta^k - \dot{x}^k\eta^i), \qquad e = g_{ij}\dot{x}^i\dot{x}^j. \quad . \quad . \quad . \quad (1.6)$$

Let a point P of C be given by $s = s_0$ in (1.1), and let the subscript 0

indicate the value at this point. Then with P as origin, a system of Riemannian co-ordinates $(y^1, y^2, \ldots, y^n)$ is defined by the equations

$$x'^i = x_0{}^i + y^i - \frac{1}{2!}(\Gamma^i_{\alpha\beta})_0 y^\alpha y^\beta - \frac{1}{3!}(\Gamma^i_{\alpha\beta\gamma})_0 y^\alpha y^\beta y^\gamma - \ldots, \qquad (1.7)$$

where the coefficients are certain well-known functions of the Christoffel symbols and their derivatives. Any other system of Riemannian co-ordinates is now given by a linear transformation of the y's with constant coefficients, and a system of normal co-ordinates is given by such a transformation so that the parametric directions at P are mutually orthogonal. We now wish to find the transformation of the y's giving normal co-ordinates with directions $(\lambda_{\sigma|}{}^i)_0$ at P.

If $(\lambda'_{\sigma|}{}^\alpha)_0$ are the components in the y's of the vectors $(\lambda_{\sigma|}{}^i)_0$, the required normal co-ordinates $(z^1, z^2, \ldots, z^n)$ are evidently given by

$$y^\alpha = (\lambda'_{\sigma|}{}^\alpha)_0 z^\sigma.$$

We have $(\lambda'_{\sigma|}{}^\alpha)_0 = (\lambda_{\sigma|}{}^i \partial y^\alpha/\partial x'^i)_0$, and from (1.7), $(\partial y^\alpha/\partial x'^i)_0 = \delta^\alpha_i$. Hence $(\lambda'_{\sigma|}{}^\alpha)_0 = (\lambda_{\sigma|}{}^\alpha)_0$, and the normal co-ordinates are given by

$$y^\alpha = (\lambda_{\sigma|}{}^\alpha)_0 z^\sigma. \qquad (1.8)$$

Substituting in (1.7), and dropping the subscript, we have at the point P(s) of C a transformation of the form

$$x'^i = x^i + F^i(s, z), \qquad (1.9)$$

this being given by (1.7), which can be written in the form

$$x'^i = x^i + f^i(x, y); \qquad (1.10)$$

and by

$$y^\alpha = \lambda_{\sigma|}{}^\alpha z^\sigma, \qquad (1.11)$$

the x's and λ's being known functions of s. The relative co-ordinates are now completely defined at each point of C.

§ 2. Fundamental Formulæ.

Let Q be the point (z) in the co-ordinates at P(s), and let Q′ be the point $(z+dz)$ in the co-ordinates at P($s+ds$), and $(z+\delta z)$ in the co-ordinates at P. We proceed to find the relation between δz and dz where dz is small. If Q′ is the point $(\xi^1, \xi^2, \ldots, \xi^n)$ in the co-ordinates of V_n, we have from (1.9),

$$\xi^i = x^i(s) + F^i(s, z + \delta z)$$

and

$$\xi^i = x^i(s + ds) + F^i(s + ds, z + dz).$$

Expanding to the first order and equating, we get

$$\frac{\partial F^i}{\partial z^\sigma}\frac{Dz^\sigma}{ds} = \dot{x}^i + \frac{\partial F^i}{\partial s}, \qquad (2.1)$$

where

$$Dz^\sigma = \delta z^\sigma - dz^\sigma.$$

From (1.10) and (1.11) we have, writing $f^i_a = \partial f^i / \partial y$

$$\frac{\partial F^i}{\partial z^\sigma} = f^i_a \lambda_{\sigma|}{}^a, \quad . \quad . \quad . \quad . \quad . \quad (2.2)$$

and

$$\frac{\partial F^i}{\partial s} = \frac{\partial f^i}{\partial x^k}\dot{x}^k + f^i_a \frac{d}{ds}(\lambda_{\sigma|}{}^a) z^\sigma ;$$

i.e. from (1.2),

$$\frac{\partial F^i}{\partial s} = \frac{\partial f^i}{\partial x^k}\dot{x}^k - f^i_a C^a_{jk}\dot{x}^k y^j. \quad . \quad . \quad . \quad . \quad . \quad (2.3)$$

If $\bar{f}^a_i$ is the minor of f^i_a in $|f^\beta_\mu|$, divided by the determinant, then substituting (2.2) and (2.3) in (2.1) and multiplying by $\bar{f}^l_i$, we get

$$\lambda_{\sigma|}{}^l \frac{Dz^\sigma}{ds} = \bar{f}^l_i\left(\dot{x}^i + \frac{\partial f^i}{\partial x^k}\dot{x}^k\right) - C^l_{jk} y^j \dot{x}^k. \quad . \quad . \quad . \quad . \quad . \quad (2.4)$$

In expanding this expression, we shall neglect terms of order greater than the second in the z's (or y's). The definition of relative co-ordinates being independent of the particular system of original co-ordinates of V_n, the coefficients in the expansion are invariants with respect to these co-ordinates. We may therefore assume that the x co-ordinates are geodesic at P, and if we express the result in invariant form, the expression so obtained will be true whatever the original co-ordinates may have been.

We have

$$f^i(x, y) = y^i - \frac{1}{2!}\Gamma^i_{\alpha\beta} y^\alpha y^\beta - \frac{1}{3!}\Gamma^i_{\alpha\beta\gamma} y^\alpha y^\beta y^\gamma, \quad . \quad . \quad . \quad (2.5)$$

and hence at P,

$$f^i_a = \delta^i_a - \tfrac{1}{2}\Gamma^i_{a\beta\gamma} y^\beta y^\gamma, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.6)$$

for the Christoffel symbols now vanish at P. From $f^i_a \bar{f}^a_j = \delta^i_j$ we have

$$\bar{f}^a_i = \delta^a_i + \tfrac{1}{2}\Gamma^a_{i\beta\gamma} y^\beta y^\gamma. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.7)$$

Also, from (2.5),

$$\frac{\partial f^i}{\partial x^k} = -\tfrac{1}{2}\Gamma^{\ \ i}_{\alpha\beta.k} y^\alpha y^\beta, \qquad \Gamma^{\ \ i}_{\alpha\beta.k} = \frac{\partial}{\partial x^k}\Gamma^i_{\alpha\beta},$$

and hence

$$\bar{f}^l_i\left(\dot{x}^i + \frac{\partial f^i}{\partial x^k}\dot{x}^k\right) = \dot{x}^l + \tfrac{1}{2}(\Gamma^l_{k\beta\gamma} - \Gamma^{\ \ l}_{\beta\gamma.k}) y^\beta y^\gamma \dot{x}^k.$$

From the expression * for $\Gamma^i_{k\beta\gamma}$, we have, at P,

$$\begin{aligned}\Gamma^l_{k\beta\gamma} - \Gamma^{\ \ l}_{\beta\gamma.k} &= \tfrac{1}{3}(\Gamma^{\ \ l}_{k\beta.\gamma} + \Gamma^{\ \ l}_{\beta\gamma.k} + \Gamma^{\ \ l}_{\gamma k.\beta}) - \Gamma^{\ \ l}_{\beta\gamma.k} \\ &= \tfrac{1}{3}[(\Gamma^{\ \ l}_{\beta k.\gamma} - \Gamma^{\ \ l}_{\beta\gamma.k}) + (\Gamma^{\ \ l}_{\gamma k.\beta} - \Gamma^{\ \ l}_{\gamma\beta.k})] \\ &= \tfrac{1}{3}(R^l_{\beta\gamma k} + R^l_{\gamma\beta k}). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.8)\end{aligned}$$

---

* Eisenhart, *op. cit.*, p. 52.

Substituting from (1.3) and (2.8) in (2.4),

$$\lambda_{\sigma|}{}^{l}\frac{Dz^{\sigma}}{ds}=\dot{x}^{l}-A^{l}_{jk}\dot{x}^{k}y^{j}+\tfrac{1}{3}R_{\beta\gamma k}{}^{l}\dot{x}^{k}y^{\beta}y^{\gamma}, \qquad . \quad . \quad . \quad (2.9)$$

this being true whatever the original co-ordinates may be. From (2.9) we get at once

$$e_{\sigma}\frac{Dz^{\sigma}}{ds}=\dot{x}^{i}\lambda_{\sigma|i}-A^{i}_{jk}\dot{x}^{k}\lambda_{\sigma|i}y^{j}+\tfrac{1}{3}R_{i\beta\gamma k}\dot{x}^{k}\lambda_{\sigma|}{}^{i}y^{\beta}y^{\gamma},$$

a repeated index not indicating summation if it is attached to $e=\pm 1$. From (1.4) and (1.11) we now have, writing $u^{\sigma}=e_{\sigma}\lambda_{\sigma|i}\dot{x}^{i}$,

$$\frac{\delta z^{\sigma}}{ds}=\frac{dz^{\sigma}}{ds}+u^{\sigma}-e_{\sigma}\gamma_{\sigma\nu}z^{\nu}+\tfrac{1}{3}e_{\sigma}\gamma_{\sigma\mu\nu h}u^{h}z^{\mu}z^{\nu}. \qquad (\sigma=1,\,2,\,.\,.\,.,\,n), \quad . \quad (2.10)$$

where $\gamma_{\sigma\mu\nu h}$ is the well-known invariant $R_{lijk}\lambda_{\sigma|}{}^{l}\lambda_{\mu|}{}^{i}\lambda_{\nu|}{}^{j}\lambda_{h|}{}^{k}$.

If C is a geodesic, we can use Levi-Civita parallel transport along C, taking $\lambda_{1|}{}^{i}=\dot{x}^{i}$, in which case we have $\gamma_{\sigma\nu}=0$, $\mu^{\sigma}=\delta^{1}_{\sigma}$, and hence

$$\frac{\delta z^{\sigma}}{ds}=\frac{dz^{\sigma}}{ds}+\delta^{1}_{\sigma}+\tfrac{1}{3}e_{\sigma}\gamma_{\sigma\mu\nu 1}z^{\mu}z^{\nu}. \qquad . \quad . \quad . \quad . \quad (2.11)$$

If C is not a geodesic, an interesting set of relative co-ordinates is found by taking the vectors of reference to be in the principal directions of the curve. In this case the Serret-Frenet formulæ give

$$\gamma_{\sigma\nu}=-\kappa_{\sigma-1}\delta^{\nu}_{\sigma-1}+\kappa_{\sigma}\delta^{\nu}_{\sigma+1},$$

and hence

$$\frac{\delta z^{\sigma}}{ds}=\frac{dz^{\sigma}}{ds}+\delta^{1}_{\sigma}+e_{\sigma}\kappa_{\sigma-1}z^{\sigma-1}-e_{\sigma}\kappa_{\sigma}z^{\sigma+1}+\tfrac{1}{3}e_{\sigma}\gamma_{\sigma\mu\nu 1}z^{\mu}z^{\nu}, \qquad . \quad . \quad (2.12)$$

where the κ's are the principal curvatures of C. If V_n is a flat space, we have without any approximation

$$\frac{\delta z^{\sigma}}{ds}=\frac{dz^{\sigma}}{ds}+\delta^{1}_{\sigma}+e_{\sigma}\kappa_{\sigma-1}z^{\sigma-1}-e_{\sigma}\kappa_{\sigma}z^{\sigma+1}. \qquad . \quad . \quad . \quad (2.13)$$

This is an evident generalisation of the moving axes formulæ for a twisted curve, where $n=3$, and κ_1 is the curvature, κ_2 the torsion.

§ 3. The Application to Neighbouring Curves.

For the further consideration of relative co-ordinates we require the metric of V_n referred to normal co-ordinates with a given point P as origin. Let $a_{\alpha\beta}$ be the fundamental tensor for the y co-ordinates defined by (1.7), and $b_{\sigma\nu}$ the tensor for the z co-ordinates defined by (1.7) and (1.8). Then from the law of transformation, we have

$$a_{\alpha\beta}=g_{ij}\frac{\partial x'^{i}}{\partial y^{\alpha}}\frac{\partial x'^{j}}{\partial y^{\beta}}, \qquad b_{\sigma\nu}=a_{\alpha\beta}\frac{\partial y^{\alpha}}{\partial z^{\sigma}}\frac{\partial y^{\beta}}{\partial z^{\nu}}. \qquad . \quad . \quad . \quad . \quad (3.1)$$

By a method similar to that used in expanding (2.4) it can easily be shown that we get

$$a_{\alpha\beta} = (g_{\alpha\beta})_0 + \tfrac{1}{3}(R_{\alpha\gamma\delta\beta})_0 y^\gamma y^\delta \quad . \quad . \quad . \quad . \quad . \quad (3.2)$$

to the required order of approximation, and hence from (3.1) and (1.8),

$$b_{\sigma\nu} = e_\sigma \delta^\nu_\sigma + \tfrac{1}{3}(\gamma_{\sigma rs\nu})_0 z^r z^s. \quad . \quad . \quad . \quad . \quad . \quad (3.3)$$

If the co-ordinates of a point Q relative to the point P(s) of C are given as functions of s, then as P moves along C, Q traces a curve C′. The tangent vector of this curve at Q in the co-ordinates at P is $\delta z^\sigma/d\bar{s}$, where $\bar{s}$ is the arc, and is given by

$$\pm\left(\frac{d\bar{s}}{ds}\right)^2 = b_{\sigma\nu}\frac{\delta z^\sigma}{ds}\frac{\delta z^\nu}{ds}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.4)$$

Substituting from (2.10) and (3.3) we get

$$\begin{aligned}\pm\left(\frac{d\bar{s}}{ds}\right)^2 = e + 2\gamma_{\sigma h}u^h z^\sigma + 2\sum_\sigma e_\sigma u^\sigma \dot{z}^\sigma + \sum_\sigma e_\sigma(\dot{z}^\sigma)^2 + 2\gamma_{\sigma\nu}z^\sigma\dot{z}^\nu \\ + \sum_\sigma e_\sigma\gamma_{\sigma\mu}\gamma_{\sigma\nu}z^\mu z^\nu + \gamma_{h\mu\nu k}u^h u^k z^\mu z^\nu,\end{aligned} \qquad (3.5)$$

where $e = \sum\limits_\sigma e_\sigma(u^\sigma)^2 = \pm 1$, 0 according as C is not, or is null. If C is not null, the sign on the left is evidently that of e.

If C is not null, and C′ is a curve near C, we can take C′ to be the locus of a point Q where the geodesic QP is orthogonal to C at P. We now take $\lambda_{1|}{}^i = \dot{x}^i$, and C′ is given by $z^1 = 0$, and $z^2, z^3, \ldots, z^n$ as functions of s. If C is not a geodesic, the simplest displacement of vectors along C which transports the tangent at one point to the tangent at any other point was found in § 1. This is defined by

$$\Lambda^i_{jk}\dot{x}^k = e g_{jk}(\dot{x}^i\eta^k - \dot{x}^k\eta^i); \qquad \eta^i = \dot{x}^i{}_{,k}\dot{x}^k, \quad . \quad . \quad . \quad . \quad (3.6)$$

from which we get

$$\gamma_{\sigma\nu} = \delta^1_\sigma v_\nu - \delta^1_\nu v_\sigma; \qquad v_\sigma = \eta^i\lambda_{\sigma|i}. \quad . \quad . \quad . \quad . \quad . \quad (3.7)$$

Substituting in (2.10) and (3.5), we have

$$\frac{\delta z^1}{ds} = 1 - e v_r z^r + \tfrac{1}{3}e\gamma_{1rs1}z^r z^s, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.8)$$

$$\frac{\delta z^r}{ds} = \dot{z}^r + \tfrac{1}{3}e_r\gamma_{rst1}z^s z^t,$$

$$\left(\frac{d\bar{s}}{ds}\right)^2 = 1 - 2e v_r z^r + e\sum_r e_r(\dot{z}^r)^2 + (v_r z^r)^2 + e\gamma_{1rs1}z^r z^s$$

i.e.

$$\frac{d\bar{s}}{ds} = 1 - e v_r z^r + \tfrac{1}{2}e\sum_r e_r(\dot{z}^r)^2 + \tfrac{1}{2}e\Gamma_{rs}z^r z^s \quad . \quad . \quad . \quad . \quad . \quad (3.9)$$

where $\Gamma_{rs} = \Gamma_{sr} = \gamma_{1rs1} = \mathrm{R}_{hijk}\dot{x}^h\dot{x}^k\lambda_{r|}{}^i\lambda_{s|}{}^j$, and r, s, t take the values $2, 3, \ldots, n$. These equations hold if C is a geodesic, for in this case we have $v_r = 0$, $(r = 1, 2, \ldots, n)$, and the above transport reduces to Levi-Civita parallel transport.

§ 4. Geodesics in Relative Co-ordinates.

To find the equations of the geodesics near C (not null) we use the variational definition

$$\delta \int \mathrm{T}\, ds = 0, \qquad (4.1)$$

where $\mathrm{T} = d\bar{s}/ds$, and $z^2, z^3, \ldots, z^n$ are varied as functions of s. This equation gives the differential equations

$$\frac{d}{ds}\left(\frac{\partial \mathrm{T}}{\partial \dot{z}^r}\right) - \frac{\partial \mathrm{T}}{\partial z^r} = 0, \qquad (4.2)$$

and substituting from (3.9), we get

$$e_r\ddot{z}^r - \Gamma_{rs}z^s = v_r, \qquad (r = 2, 3, \ldots, n). \qquad (4.3)$$

These equations are only true when $(z)^2$ may be neglected. They show, as expected, that v_r must be of the order of smallness of z, *i.e.* C cannot lie near a geodesic unless the first curvature of C is small.

If C is a geodesic, we have $v_r = 0$, and the geodesics near C are given by

$$\ddot{z}^r - e_r\Gamma_{rs}z^s = 0 \qquad (r = 2, 3, \ldots, n). \qquad (4.4)$$

We can at once find the general solution of equations (4.4) when V_n is a space of constant curvature. In this case we have

$$\mathrm{R}_{hijk} = \mathrm{K}(g_{hj}g_{ik} - g_{hk}g_{ij}),$$

where K is constant, and hence

$$\Gamma_{rs} = \mathrm{K}\lambda_{r|i}\lambda_{s|j}(\dot{x}^i\dot{x}^j - eg^{ij}) = -ee_r\mathrm{K}\delta^s_r. \qquad (4.5)$$

Equations (4.4) now become

$$\ddot{z}^r + e\mathrm{K}z^r = 0$$

and the solution is

$$z^r = \mathrm{A}^r \cos(\sqrt{e\mathrm{K}}\, s + a^r), \qquad \mathrm{A}^r \cosh(\sqrt{-e\mathrm{K}}\, s + a^r), \qquad (4.6)$$

according as $e\mathrm{K}$ is $+ve$ or $-ve$, where A^r, a^r are arbitrary constants.

§ 5. Rigid Motion in a Riemannian Space.

Let L_p, L_q, . . . be the paths, or world-lines, of a system of neighbouring particles. Then we may say that the system of particles moves as a rigid body if the orthogonal distance between any two world-lines is

constant along these lines. We shall suppose that the lines are sufficiently near together so that the cube of the greatest distance between them may be neglected, and we shall refer to one line C and use relative co-ordinates with the transport (1.6) in referring to the other lines.

In the co-ordinates at a point P of C it is evident from (3.3) that the geodesics are straight lines to the required order of smallness. Consider any two lines L, L′ of the system; let L meet the V_{n-1} orthogonal to C at P at the point $Q(z^2, z^3, \ldots, z^n)$, and let L′ meet this space at the point $Q'(z'^2, z'^3, \ldots, z'^n)$. Let $Q\bar{Q}$ be the geodesic orthogonal to L at Q, meeting L′ at the point $\bar{Q}$. Then it can easily be verified that $Q\bar{Q} = QQ'$ to the required order of approximation, *i.e.* we have

$$\pm (Q\bar{Q})^2 = \sum_{r=2}^{n} e_r (z'^r - z^r)^2. \qquad (5.1)$$

Hence, for rigid motion, this expression must be independent of s, and so for each pair of world-lines. Thus if L_p is the locus of $Q_p(z_p^2, z_p^3, \ldots, z_p^n)$, the conditions are

$$\frac{d}{ds}\sum_{r=2}^{n} e_r (z_p^r - z_q^r)^2 = 0 \qquad (5.2)$$

for all values of p, q. We have taken C to be the world-line of one of the particles, and hence, from (5.2), the conditions are

$$\frac{d}{ds}\sum_r e_r (z_p^r)^2 = 0, \qquad \frac{d}{ds}\sum_r e_r z_p^r z_q^r = 0; \qquad (5.3)$$

i.e. the geodesic distance PQ_p, and the angle at P between the geodesics PQ_p, PQ_q are constant along C. Hence, we can write

$$z_p^r = \xi_s^r v_p^s \qquad (5.4)$$

where the v's are constants, and ξ_s^r, $(r, s = 2, 3, \ldots, n)$ are the coefficients of an orthogonal transformation in V_{n-1} and are dependent only on s. *Equations* (5.4) *are the necessary and sufficient conditions that the lines* (z_p) *should be the world-lines of the particles of a small rigid body, referred to relative co-ordinates along one of the world-lines.*

Thomsen* has discussed the equations of motion of a rigid body in space-time by considering the variation of the energy integral. For this we require the energy of the particles, which is at once given by (3.9) and

* Thomsen, *Math. Zeitschrift*, 29, 1929, 96.

(5.4). For the particle $Q(z)$ the energy is $md\bar{s}/dt$, and we can write $d\bar{s}/dt$ in the form

$$\frac{d\bar{s}}{dt} = \frac{ds}{dt}(1 - eT_r\xi'_s v^s) + \tfrac{1}{2}e\frac{dt}{ds}(\Sigma e_r \xi'^r_s \xi'^r_t + R_{hijk}x'^h x'^k \lambda_{u|}{}^i \lambda_{v|}{}^j \xi''_s \xi''_t)v^s v^t, \quad . \quad (5.5)$$

where a dash denotes differentiation with respect to t, and T_r is written for $\eta^i\lambda_{r|i}$, $\eta^i = \dot{x}^i{}_{,k}\dot{x}^k$. In space-time we have $n=4$, $e=1$, $e_r = -1$ $(r=2, 3, 4)$, and we have Thomsen's form, with a slight change of notation.

(Issued separately October 12, 1932.)

XXII.—Tables of the Elliptic-cylinder Functions.

By E. L. Ince, M.A. (Cantab.), D.Sc. (Edin.).

(MS. received April 30, 1930. Revised MS. and Tables received November 16, 1931. Read June 2, 1930.)

§ 1. Introduction.

Although the physical problem which originally led to the *Mathieu equation* *

$$\frac{d^2y}{dx^2}+(a-2\theta\cos 2x)y=0$$

is now well over sixty years old, and although other problems studied in more recent years, such as the scattering of electric waves by an elliptic cylinder, depend essentially on the same equation, no thorough investigation of detail has hitherto been possible owing to the lack of tables of the functions defined by the equation. In fact, until 1924 all attempts to reduce the functions to a form suitable for calculation had failed. In that year a successful method of attack was discovered by the present writer, † who then undertook the computation of the tables which are here published.

These tables will enable the solutions of the classical problems of the ellipse and the elliptic cylinder in mathematical physics to be studied more minutely; but it is believed that they will be of greater value in furthering the attack on more modern problems, such as the theory of oscillating electrical circuits, which depend upon the periodic solutions of a Mathieu equation.

§ 2. Definitions.

Any solution of the Mathieu equation will be known as a *Mathieu function.* The conditions of the classical problems demand solutions which are periodic, with period π or 2π. Owing to their physical significance in the scattering of waves by an elliptic cylinder these solutions will be known as *elliptic-cylinder functions.* Such a function can only exist when a has, for any given θ, one or other of a discrete set of values. These *characteristic numbers* are continuous functions of θ and reduce to the set, $0, 1, 4, \ldots, n^2, \ldots$ for $\theta=0$.

* Émile Mathieu, "Mémoire sur le mouvement vibratoire d'une membrane de forme elliptique," *Journal de Mathématiques* (Liouville) (2), xiii, 1868, 137–203. See also Whittaker and Watson, *Modern Analysis*, chap. xix, and the Monograph by P. Humbert, "Fonctions de Lamé et Fonctions de Mathieu" (*Mémorial des Sciences mathématiques*, x, Paris, 1926), in which will be found a bibliography, practically complete up to 1924.

† Ince, "Researches into the Characteristic Numbers of the Mathieu Equation," *Proceedings of the Royal Society of Edinburgh*, xlvi, 1925–26, 20–29, 316–322; xlvii, 1926–27, 294–301.

If $y=u(x)$ is an elliptic-cylinder function, then $y=u(ix)$ will be a solution of the allied equation

$$\frac{d^2y}{dx^2}-(a-2\theta\cosh 2x)y=0$$

for the same a, and may be termed a *hyperbolic-cylinder function.*

In the classical problems these functions are almost always associated * as a product of the form $u(\xi)u(i\eta)$; but many of the more recent problems involve the elliptic-cylinder functions alone.

§ 3. Specification of the Elliptic-cylinder Functions.

It is known from purely theoretical considerations that when a periodic solution of period π or 2π exists, † it belongs to one or other of four main types:

(i) $F(\cos 2x)$; (ii) $\sin x\ F(\cos 2x)$; (iii) $\cos x\ F(\cos 2x)$; $\sin x\ \cos x\ F(\cos 2x)$,

where $F(\cos 2x)$ represents, in each case, an integral function of $\cos 2x$. Each type has an infinite number of members, each associated with a characteristic value of a. The members of each set are distinguished from one another by the number of zeros they possess in the interval $(0, \frac{1}{2}\pi)$.

It is usual to denote particular functions as follows:—

$$\text{(i)}\quad ce_{2n}(x,\theta)=\sum_{r=0}^{\infty}A_{2r}(\theta)\cos 2rx,$$

$$\text{(ii)}\ se_{2n+1}(x,\theta)=\sum_{r=0}^{\infty}B_{2r+1}(\theta)\sin(2r+1)x,$$

$$\text{(iii)}\ ce_{2n+1}(x,\theta)=\sum_{r=0}^{\infty}A_{2r+1}(\theta)\cos(2r+1)x,$$

$$\text{(iv)}\ se_{2n+2}(x,\theta)=\sum_{r=1}^{\infty}B_{2r}(\theta)\sin 2rx,$$

where, in every case, n (a positive integer or zero) indicates the number of zeros which each particular function possesses ‡ in the open interval $0<x<\frac{1}{2}\pi$. The corresponding characteristic values of a will be denoted by

$$a_{2n}(\theta),\quad b_{2n+1}(\theta),\quad a_{2n+1}(\theta),\quad b_{2n+2}(\theta)$$

respectively.

The functions are definite apart from a constant multiplier. To complete the specification (which should be such that the typical functions

* An old problem in which $u(\xi)$ is not involved with $u(i\eta)$ is that of a stretched string one of whose ends is subjected to a periodic vibration (Lord Rayleigh, "On Maintained Vibrations," *Philosophical Magazine*, xv, 1883, 229–235; *Scientific Papers*, ii, 188–193).

† Except when $\theta=0$ the Mathieu equation cannot have more than one solution of period π or 2π (*cf.* Ince, *Proceedings of the Cambridge Philosophical Society*, xxi, 1922, 117–120).

‡ The number of zeros of any particular elliptic-cylinder function is independent of θ (*cf.* Ince, *Journal of the London Mathematical Society*, 2, 1925, 47).

reduce respectively to $\cos 2nx$, $\sin(2n+1)x$, $\cos(2n+1)x$, and $\sin 2x$ when $\theta=0$) two main courses are open, namely:

(a) to fix the value of the function, or of its differential coefficient, at an end point of the interval $(0, \pi)$; or

(b) to subject each function to the condition

$$\int_0^{2\pi} y^2 dx = \pi.$$

In the following tables the latter definition* has been adopted, coupled with the condition that, for $x=0$,

$$ce_m(x, \theta) > 0 \qquad (m=0, 1, 2, \ldots);$$
$$\frac{d}{dx} se_m(x, \theta) > 0 \qquad (m=1, 2, \ldots).$$

If θ is changed to $-\theta$ and x is replaced by $\frac{1}{2}\pi - x$, the Mathieu equation remains unaltered. Consequently, if the elliptic-cylinder functions are tabulated for positive values of θ, the values for the corresponding negative values of θ may be deduced by the relations

$$ce_{2n}(x, -\theta) = (-1)^n ce_{2n}(\tfrac{1}{2}\pi - x, \theta); \qquad se_{2n+2}(x, -\theta) = (-1)^n se_{2n+2}(\tfrac{1}{2}\pi - x, \theta);$$
$$ce_{2n+1}(x, -\theta) = (-1)^n se_{2n+1}(\tfrac{1}{2}\pi - x, \theta); \quad se_{2n+1}(x, -\theta) = (-1)^n ce_{2n+1}(\tfrac{1}{2}\pi - x, \theta).$$

The respective characteristic numbers are:

$$a_{2n}(-\theta) = a_{2n}(\theta); \qquad b_{2n+2}(-\theta) = b_{2n+2}(\theta);$$
$$a_{2n+1}(-\theta) = b_{2n+1}(\theta); \quad b_{2n+1}(-\theta) = a_{2n+1}(\theta).$$

§ 4. THE CHARACTERISTIC NUMBER $a_{2n}(\theta)$ AND THE ELLIPTIC-CYLINDER FUNCTION $ce_{2n}(x, \theta)$.

The original method of solution adopted by Mathieu was to develop the characteristic number and the corresponding elliptic-cylinder function simultaneously in ascending powers of θ. The method may possibly serve for values of θ so small that three or four terms are obviously sufficient to give the required number of significant figures. In actual practice, values of θ much larger than these are required, and then not only would the labour of calculating the higher terms be prohibitive, but the convergence of the developments becomes questionable.

If the characteristic numbers can be computed for given values of θ, several methods of calculating the functions are available. The method of attack which proved to be the most fruitful is the direct one, which will now be outlined.

* This definition was suggested by Goldstein (*Transactions of the Cambridge Philosophical Society*, xxiii, 1927, 303), who, however, defined $ce_0(x, \theta)$ exceptionally, so that it should reduce to unity when $\theta=0$. Unfortunately this destroys the asymptotic equality between $ce_0(x, \theta)$ and $se_1(x, \theta)$ as $\theta \to +\infty$. According to the present definition, $ce_0(x, 0)$ has the irrational value $1/\sqrt{2}$, but that is a very slight disadvantage.

Let the equation be supposed to admit of an even periodic solution of period π—that is, an elliptic-cylinder function $ce_{2n}(x, \theta)$. Then the recurrence-relations between the coefficients in the series

$$y=\sum_{r=0}^{\infty} A_{2r}\cos 2x,$$

will be *

$$aA_0-\theta A_2=0,$$
$$(a-4)A_2-\theta(2A_0+A_4)=0,$$
$$(a-4r^2)A_{2r}-\theta(A_{2r-2}+A_{2r+2})=0 \qquad (r \geqslant 2).$$

The last relation may be written

$$\frac{A_{2r}}{A_{2r-2}}=\frac{-\theta/4r^2}{1-\dfrac{a}{4r^2}+\dfrac{\theta}{4r^2}\cdot\dfrac{A_{2r+2}}{A_{2r}}},$$

and thus leads directly to the expression of A_{2r}/A_{2r-2} in terms of an infinite continued fraction involving r, θ, and a.

Let the symbol $\overset{\infty}{\underset{r}{\Phi}}\dfrac{a_\nu}{\beta_\nu}$ denote the continued fraction †

$$\frac{1}{\beta_r+}\,\frac{a_{r+1}}{\beta_{r+1}+}\,\frac{a_{r+2}}{\beta_{r+2}+}\;\ldots,$$

then

$$\frac{A_{2r}}{A_{2r-2}}=-\frac{\theta}{4r^2}\overset{\infty}{\underset{r}{\Phi}}\frac{-\dfrac{\theta^2}{16\nu^2(\nu-1)^2}}{1-\dfrac{a}{4\nu^2}} \qquad (r>2),$$

$$\frac{A_2}{A_0}=-\frac{\theta}{2}\overset{\infty}{\underset{1}{\Phi}}\frac{-\dfrac{\theta^2}{16\nu^2(\nu-1)^2}}{1-\dfrac{a}{4\nu^2}}.$$

These continued fractions are of a well-known type, and converge for all values of a and θ. Moreover, for large r

$$\frac{A_{2r}}{A_{2r-2}}\sim-\frac{\theta}{4r^2},$$

so that the series for y is rapidly convergent. This series will satisfy the Mathieu equation if the above expression for A_2/A_0 is equal to that deduced from the first recurrence-relation; that is, if

$$\frac{a}{\theta}=-\frac{\theta}{2}\overset{\infty}{\underset{1}{\Phi}}\frac{-\dfrac{\theta^2}{16\nu^2(\nu-1)^2}}{1-\dfrac{a}{4\nu^2}}$$

* Note that A_0 cannot vanish, except for $\theta=0$, unless the function vanishes identically. On the other hand, any other coefficient may vanish. Thus A_2 vanishes when $a=0$, A_4 vanishes when $a^2-4a-2\theta^2=0$, and so on. It is therefore impracticable to define $ce_{2n}(x, \theta)$ by stipulating (as Mathieu did) that the coefficient of $\cos 2nx$ in the expansion shall be unity.

† The analogy with Σ and Π for series and products will be evident. Note the convention that the leading numerator is unity.

or

$$a = \frac{-\theta^2/2}{1-\frac{a}{4}-}\frac{\theta^2/64}{1-\frac{a}{16}-}\frac{\theta^2/576}{1-\frac{a}{36}-}\dots.$$

This is the relation which determines the characteristic numbers $a_{2n}(\theta)$. Starting from $a_{2n}(0)=4n^2$ and increasing θ by gradual amounts, it is not difficult to proceed with the tabulation of any particular a_{2n}. The method of procedure was as follows:—

Tables of the constants $\theta^2/2$, $\theta^2/64$, $\theta^2/576$, . . . were drawn up, and then (by extrapolation from previously calculated values of a, for example) an initial value of a as accurate as possible, to correspond to the value of θ in question, is taken. Let it be $4a$. Then calculation proceeds according to the following scheme, from right to left:

$$\begin{array}{lccccc} & 1-a & 1-a/4 & 1-a/9 & \dots & 1-a/n^2 \\ \text{Diff.} = & \frac{\gamma_1}{\phi_1} & \frac{\gamma_2}{\phi_2} & \frac{\gamma_3}{\phi_3} & \dots & \frac{\gamma_n}{\phi_n}. \end{array}$$

In this scheme

$$\gamma_{r-1} = \frac{\theta^2}{16r^2(r-1)^2} \div \phi_r,$$

$$\phi_r = 1 - a/r^2 - \gamma_r,$$

and the scheme ends, on the right, at the stage where it is obviously sufficient, for the degree of precision aimed at, to take $\phi_{n+1}=1$ or $\gamma_n = \theta^2/16n^2(n+1)^2$.

Finally, the relation $a = -\theta^2/2\phi_1$ leads to a value of a whose closeness to the true value depends upon the correctness of the value originally chosen.

It is easy to see whether the original value of a was in excess or defect. An adjustment in the correct sense is made, and the process repeated. Now that two initial and two final values are known, a third initial value is obtained by linear interpolation; unless the first two initial values were badly chosen, this will be a fair approximation to the true value. The process is repeated (with an increasing number of significant figures if desired) until the initial and final values agree. After the first three or four steps the right-hand columns of the scheme alter little, if at all; so that, given a machine of adequate capacity, the correct value can be reached very quickly by a practised computer.

The ratios between the coefficients may now be calculated from the formula

$$\frac{A_{2r}}{A_{2r-2}} = -\frac{\theta}{4r^2\phi_r};$$

and, lastly, the coefficients themselves may be determined in accordance with the definition of the function, which in this case implies the relation

$$2A_0^2 + A_1^2 + A_2^2 + A_3^2 + \dots = 1.$$

§ 5. The Three Remaining Cases.

When the equation admits of an odd solution of period 2π, $y = se_{2n+1}(x, \theta)$, the recurrence-relations between the coefficients in the series

$$y = \sum_{r=0}^{\infty} B_{2r+1} \sin(2r+1)x$$

are

$$(a+\theta-1)B_1 - \theta B_3 = 0,$$
$$\{a-(2r+1)^2\}B_{2r+1} - \theta(B_{2r-1} + B_{2r+3}) = 0 \qquad (r \geqslant 1).$$

These relations lead to the continued fraction

$$\frac{B_{2r-1}}{B_{2r+1}} = -\frac{\theta}{(2r+1)^2} \overset{\infty}{\underset{r}{\Phi}} \frac{-\dfrac{\theta^2}{(4\nu^2-1)^2}}{1-\dfrac{a}{(2\nu+1)^2}}$$

whence

$$a = 1 - \theta - \frac{\theta^2/9}{1-\dfrac{a}{9}-} \, \frac{\theta^2/225}{1-\dfrac{a}{25}-} \, \frac{\theta^2/1225}{1-\dfrac{a}{49}-} \cdots$$

According to the definition of the function the coefficients are such that $B_1^2 + B_3^2 + B_5^2 + \ldots = 1$.

The case of an even solution of period 2π, $y = ce_{2n+1}(x, \theta)$, is very similar; the coefficients in the series

$$y = \sum_{r=0}^{\infty} A_{2r+1} \cos(2r+1)x$$

obey the same laws as the coefficients B_{2r+1}, with the single exception,

$$(a-\theta-1)A_1 - \theta A_3 = 0,$$

and the equation for a differs only in the change of the sign of θ.

When the equation admits of an odd solution of period π, $y = se_{2n+2}(x, \theta)$, the recurrence-relations for the coefficients in the series

$$y = \sum_{r=1}^{\infty} B_{2r} \sin 2rx$$

are

$$(a-4)B_2 - \theta B_4 = 0,$$
$$(a-4r^2)B_{2r} - \theta(B_{2r-2} + B_{2r+2}) = 0 \qquad (r \geqslant 2).$$

Thus

$$\frac{B_{2r}}{B_{2r-2}} = -\frac{\theta}{4r^2} \overset{\infty}{\underset{r}{\Phi}} \frac{-\dfrac{\theta^2}{16\nu^2(\nu-1)^2}}{1-\dfrac{a}{4\nu^2}},$$

whence

$$a = 4 - \frac{\theta^2/16}{1-\dfrac{a}{16}-} \, \frac{\theta^2/576}{1-\dfrac{a}{36}-} \, \frac{\theta^2/2304}{1-\dfrac{a}{64}-} \cdots$$

The definition of the function implies $B_2^2 + B_4^2 + B_6^2 + \ldots = 1$.

§ 6. Procedure.

The machines available for use in the computation were:*

1. *Nova Brunsviga Mod. III.* Capacity, 10:10:15. Two multiplier registers, one having tens transmission.
2. *Trinks-Triplex Brunsviga.* Capacity, 20:12:20. Two multiplier registers, one having tens transmission.
3. *Facit.* Capacity, 9:10:15. Tabulator stops, but no transmission in multiplier register.
4. *Sundstrand.* Adding and listing; capacity, nine figures.

The tabulation of any particular Mathieu function involves three processes, each dependent on the preceding: (a) determination of the characteristic number; (b) calculation of the ratios of consecutive Fourier coefficients, and the subsequent determination of the actual coefficients in accordance with the chosen definition of the function; (c) the computation of the function from the Fourier series.

Each characteristic number was calculated to ten decimal places; the Fourier coefficients also to ten places. After a sufficient interval each characteristic number and set of coefficients was re-calculated and extended to twelve places. In the early stages the functions were defined by the convention that the value for $x=\frac{1}{2}\pi$ (or if that value were zero, the value of the differential coefficients) should be ± 1. Although it appeared that this might not be the best definition of the function, it was adhered to until the twelve Mathieu functions of lowest order had been computed to ten or twelve decimal places for $\theta=1, 2, \ldots, 10$, and differenced.†

The alternative definition was found by experience to be decidedly the better one; the necessary conversion factor for each function was calculated and applied, and the new values differenced.

§ 7. Description of the Tables.

Table I contains the characteristic numbers a_0 to a_5 and b_1 to b_6 for the twenty-one values $\theta=0, 1, 2, \ldots, 10, 12, \ldots, 20, 24, \ldots, 40$ to seven

* The first of these machines was purchased with a grant given by the Government Grant Committee of the Royal Society; the remainder are machines belonging to the Department of Pure Mathematics of the Egyptian University. The capacity is indicated by (i) the number of setting levers; (ii) the number of figures in the multiplier register; and (iii) the number of figures in the product register. A Miniature Brunsviga (9:8:13) was sometimes used for incidental work.

† This work occupied the greater part of the period November 1926 to April 1930. The tables calculated in accordance with this original definition are deposited with the Royal Society of Edinburgh. They are of the same range as the tables now published, but show the functions to eight places of decimals, with second and fourth differences.

places of decimals. The asymptotic relationship $a_r \sim b_{r+1}$ for θ large will be noticed.

Tables II to XIII contain the coefficients* of the twelve functions of lowest order, $ce_0(x, \theta)$, $se_1(x, \theta)$, . . . , $ce_5(x, \theta)$, $se_6(x, \theta)$ to seven decimal places, for the above values of θ, according to the definition

$$\int_0^{2\pi} y^2 dx = 0.$$

Tables XIV to XXV give the actual values of the twelve functions $ce_0(x,\theta)$, $se_1(x, \theta)$, . . . , $ce_5(x, \theta)$, $se_6(x, \theta)$ for each of the values $\theta = 1, 2, \ldots, 10$ to five decimal places, each function occupying four pages. The first page gives the values for $\theta = 1, 2, \ldots, 5$ from $x = 0°$ to $x = 45°$, with tabular interval of 1°; the second page from $x = 45°$ to $x = 90°$. The third and fourth pages give the corresponding values for $\theta = 6, 7, \ldots, 10$.

Second differences are printed in order to facilitate interpolation in x by the Laplace-Everett formula. Interpolation in θ for a given x is a different matter, particularly for low values of θ in the functions of low order. Its difficulty could be minimised by shortening the interval between consecutive values of θ, but for the process to become really effective the bulk of the tables would have to be increased manyfold. Nor can the difficulty be got over by interpolating in the Fourier coefficients where, generally speaking, interpolation is more difficult than in the tables themselves. It seems that general methods of interpolation are impracticable, and that there is scope for research in finding a particular method of interpolation dependent upon the characteristic properties of the elliptic-cylinder functions.

The functions were computed for the first ten integral values of θ—that is, for only one half of the number of cases for which the coefficients were worked out, and one quarter of their range. These values are, however, sufficient to reveal the salient features of the functions and their mode of dependence upon θ.

§ 8. Conclusion.

The author desires to express his thanks to the Government Grant Committee of the Royal Society for a grant towards the purchase of the Nova Brunsviga calculating machine on which the bulk of the computation was done, and to the Carnegie Trust for the Universities of Scotland for a grant in aid of the publication of the tables in this paper. He also gratefully acknowledges the help given by Mansy Shehata Effendi, Assistant in the Department of Pure Mathematics of the Egyptian University.

Tables of the zeros and turning-points of the functions form the subject of a separate communication.

* Five-figure tables of the Fourier coefficients of the five functions of lowest order, including some very high values of the parameter, were given by Goldstein, *loc. cit.*, after p. 336.

TABLE I.—CHARACTERISTIC NUMBERS.

θ	a_0	b_1	a_1	b_2	a_2	b_3
0	0·0000000	1·0000000	1·0000000	4·0000000	4·0000000	9·0000000
1	− 0·4551386	− 0·1102488	1·8591081	3·9170248	4·3713010	9·0477393
2	− 1·5139569	− 1·3906765	2·3791999	3·6722327	5·1726651	9·1406277
3	− 2·8343919	− 2·7853797	2·5190391	3·2769220	6·0451969	9·2231328
4	− 4·2805188	− 4·2591829	2·3180082	2·7468810	6·8290748	9·2614461
5	− 5·8000460	− 5·7900806	1·8581875	2·0994604	7·4491097	9·2363277
6	− 7·3688308	− 7·3639110	1·2142782	1·3513812	7·8700645	9·1379058
7	− 8·9737425	− 8·9712024	0·4383491	0·5175454	8·0866231	8·9623855
8	− 10·6067292	− 10·6053681	− 0·4359436	− 0·3893618	8·1152388	8·7099144
9	− 12·2624142	− 12·2616617	− 1·3867016	− 1·3588101	7·9828432	8·3831192
10	− 13·9369800	− 13·9365525	− 2·3991424	− 2·3821582	7·7173698	7·9860691
12	− 17·3320660	− 17·3319184	− 4·5701329	− 4·5635399	6·8787369	7·0005668
14	− 20·7760553	− 20·7760004	− 6·8934005	− 6·8907007	5·7363123	5·7926295
16	− 24·2586795	− 24·2586578	− 9·3352671	− 9·3341097	4·3712326	4·3978962
18	− 27·7728422	− 27·7728332	− 11·8732425	− 11·8727265	2·8330567	2·8459917
20	− 31·3133901	− 31·3133862	− 14·4913014	− 14·4910633	1·1542829	1·1607057
24	− 38·4589732	− 38·4589724	− 19·9225956	− 19·9225403	− 2·5397657	− 2·5380779
28	− 45·6733696	− 45·6733694	− 25·5617471	− 25·5617329	− 6·5880630	− 6·5875850
32	− 52·9422230	− 52·9422229	− 31·3651544	− 31·3651505	− 10·9143534	− 10·9142090
36	− 60·2555679	− 60·2555679	− 37·3026391	− 37·3026380	− 15·4667703	− 15·4667243
40	− 67·6061522	− 67·6061522	− 43·3522753	− 43·3522749	− 20·2079408	− 20·2079254

θ	a_3	b_4	a_4	b_5	a_5	b_6
0	9·0000000	16·0000000	16·0000000	25·0000000	25·0000000	36·0000000
1	9·0783688	16·0329701	16·0338323	25·0208408	25·0208543	36·0142899
2	9·3703225	16·1276880	16·1412038	25·0833490	25·0837778	36·0572070
3	9·9155063	16·2727012	16·3387207	25·1870798	25·1902855	36·1288712
4	10·6710271	16·4520353	16·6498189	25·3305449	25·3437576	36·2294114
5	11·5488320	16·6482199	17·0965817	25·5108160	25·5499717	36·3588668
6	12·4656007	16·8446016	17·6887830	25·7234107	25·8172720	36·5170667
7	13·3584213	17·0266608	18·4166087	25·9624472	26·1561202	36·7035027
8	14·1818804	17·1825278	19·2527051	26·2209995	26·5777533	36·9172131
9	14·9036797	17·3030110	20·1609264	26·4915472	27·0918661	37·1566950
10	15·5027844	17·3813807	21·1046337	26·7664264	27·7037687	37·4198588
12	16·3015349	17·3952497	22·9721275	27·3000124	29·2080550	38·0060087
14	16·5985405	17·2071153	24·6505951	27·7697667	31·0000508	38·6484719
16	16·4868843	16·8186837	26·0086783	28·1363559	32·9308951	39·3150108
18	16·0619754	16·2420804	26·9877664	28·3738582	34·8530587	39·9723511
20	15·3958109	15·4939776	27·5945782	28·4682213	36·6449897	40·5896641
24	13·5228427	13·5527965	27·8854408	28·2153594	39·5125519	41·6057099
28	11·1110798	11·1206227	27·2833082	27·4057488	41·2349503	42·2248415
32	8·2914962	8·2946721	26·0624482	26·1083526	41·9535112	42·3939428
36	5·1456363	5·1467375	24·3785094	24·3960665	41·9266646	42·1183561
40	1·7296491	1·7300456	22·3252763	22·3321485	41·3497544	41·4330052

Table II.—Coefficients of $ce_0(x, \theta)$.

$$ce_0(x, \theta) = A_0(\theta) + \sum_{r=1}^{\infty} A_{2r}(\theta) \cos 2rx.$$

$\theta =$	1	2	3	4	5
A_0	0·6729897	0·6225002	0·5856071	0·5597172	0·5406124
A_2	− ·3063036	− ·4712192	− ·5532800	− ·5989700	− ·6271154
A_4	186456	541409	892299	·1205112	·1479271
A_6	− 5117	− 28911	− 69171	− 120374	− 178481
A_8	79	883	3109	7068	12829
A_{10}	− 1	− 17	− 91	− 271	− 607
A_{12}			2	7	20
A_{14}					− 1

$\theta =$	6	7	8	9	10
A_0	0·5257758	0·5137771	0·5037681	0·4952174	0·4877754
A_2	− ·6457255	− ·6586433	− ·6679164	− ·6747290	− ·6798115
A_4	·1719724	·1931700	·2119732	·2287566	·2438259
A_6	− 240734	− 305248	− 370727	− 436280	− 501295
A_8	20334	29463	40065	51981	65057
A_{10}	− 1139	− 1898	− 2909	− 4187	− 5742
A_{12}	45	87	151	242	365
A_{14}	− 1	− 3	− 6	− 10	− 17
A_{16}					1

$\theta =$	12	14	16	18	20
A_0	0·4753235	0·4651824	0·4566602	0·4493312	0·4429163
A_2	− ·6865282	− ·6903326	− ·6923733	− ·6932892	− ·6934605
A_4	·2697750	·2913293	·3095268	·3251024	·3385895
A_6	− 628185	− 749490	− 864481	− 973029	− ·1075304
A_8	94116	126212	160511	196354	233222
A_{10}	− 9700	− 14776	− 20926	− 28079	− 36156
A_{12}	725	1262	2004	2968	4167
A_{14}	− 41	− 82	− 146	− 240	− 369
A_{16}	2	4	8	15	26
A_{18}					− 1

$\theta =$	24	28	32	36	40
A_0	0·4321113	0·4232506	0·4157666	0·4093056	0·4036330
A_2	− ·6924398	− ·6904030	− ·6878627	− ·6850817	− ·6822019
A_4	·3607893	·3783073	·3924804	·4041753	·4139802
A_6	− ·1262358	− ·1428645	− ·1577144	− ·1710465	− ·1830805
A_8	308518	384150	458781	531631	602265
A_{10}	− 54743	− 76033	− 99444	− 124482	− 150738
A_{12}	7300	11422	16508	22506	29352
A_{14}	− 754	− 1338	− 2151	− 3214	− 4540
A_{16}	62	125	225	370	568
A_{18}	− 4	− 10	− 19	− 35	− 59
A_{20}		1	1	3	5

TABLE III.—COEFFICIENTS OF $se_1(x, \theta)$.

$$se_1(x, \theta) = \sum_{r=0}^{\infty} B_{2r+1}(\theta) \sin (2r+1)x.$$

$\theta =$	1	2	3	4	5
B_1	0·9939680	0·9813463	0·9670314	0·9530304	0·9400190
B_3	− ·1095838	− ·1916945	− ·2531623	− ·3000099	− ·3365420
B_5	43676	145713	275065	414411	554775
B_7	− 890	− 5789	− 15968	− 31234	− 50896
B_9	11	141	572	1468	2939
B_{11}		− 2	− 14	− 47	− 116
B_{13}				1	3

$\theta =$	6	7	8	9	10
B_1	0·9281398	0·9173411	0·9075112	0·8985300	0·8902865
B_3	− ·3656733	− ·3893723	− ·4089890	− ·4254701	− ·4394946
B_5	691678	823001	947863	·1066035	·1177626
B_7	− 74168	100323	− 128735	− 158879	− 190324
B_9	5052	7838	11303	15433	20206
B_{11}	− 237	− 423	− 689	− 1046	− 1504
B_{13}	8	17	31	52	82
B_{15}			− 1	− 2	− 3

$\theta =$	12	14	16	18	20
B_1	0·8756412	0·8629656	0·8518254	0·8419080	0·8329838
B_3	− ·4620407	− ·4793158	− ·4929225	− ·5038741	− ·5128425
B_5	·1382270	·1564706	·1727959	·1874742	·2007372
B_7	− 255772	− 322984	− 390583	− 457672	− 523670
B_9	31549	45045	60392	77302	95515
B_{11}	− 2752	− 4481	− 6714	− 9458	12710
B_{13}	178	332	559	871	1279
B_{15}	− 9	− 19	− 36	− 62	− 100
B_{17}		1	2	3	6

$\theta =$	24	28	32	36	40
B_1	0·8174660	0·8043114	0·7929177	0·7828839	0·7739305
B_3	− ·5265493	− ·5364002	− ·5436993	− ·5492271	− ·5534793
B_5	·2237628	·2430744	·2595180	·2737005	·2860676
B_7	− 651072	− 771316	− 884072	− 989532	− ·1088111
B_9	134958	177223	221205	266102	311335
B_{11}	− 20676	− 30449	− 41823	− 54589	− 68545
B_{13}	2418	4026	6133	8748	11867
B_{15}	− 222	− 420	− 714	− 1119	− 1648
B_{17}	16	35	67	116	187
B_{19}	− 1	− 2	− 5	− 10	− 18
B_{21}				1	1

TABLE IV.—COEFFICIENTS OF $ce_1(x,\theta)$.

$$ce_1(x,\theta)=\sum_{r=0}^{\infty}A_{2r+1}(\theta)\cos(2r+1)x.$$

$\theta =$	1	2	3	4	5
A_1	0·9902021	0·9547182	0·8950977	0·8264490	0·7624637
A_3	− ·1395115	− ·2963446	− ·4418682	− ·5541324	− ·6315963
A_5	60343	263009	594792	992280	·1396848
A_7	− 1280	− 11295	− 38484	− 85397	− 149156
A_9	16	287	1473	4349	9448
A_{11}		− 5	− 37	− 147	− 397
A_{13}			1	4	12

$\theta =$	6	7	8	9	10
A_1	0·7081680	0·6633854	0·6264179	0·5955405	0·5693674
A_3	− ·6828771	− ·7166126	− ·7388555	− ·7534713	− ·7629036
A_5	·1779473	·2130985	·2450570	·2740242	·3002772
A_7	− 225569	− 311091	− 403013	− 499279	− 598302
A_9	17027	27168	39849	54985	72447
A_{11}	− 854	− 1581	− 2633	− 4059	− 5899
A_{13}	31	66	125	215	345
A_{15}	− 1	− 2	− 4	− 9	− 15

$\theta =$	12	14	16	18	20
A_1	0·5272423	0·4945478	0·4681857	0·4463064	0·4277390
A_3	− ·7719764	− ·7733810	− ·7706122	− ·7654959	− ·7590507
A_5	·3457429	·3834274	·4149006	·4413814	·4638155
A_7	− 799956	− ·1001051	− ·1197455	− ·1386799	− ·1567831
A_9	113717	162306	216859	276129	339026
A_{11}	− 10940	− 17924	− 26920	− 37922	− 50870
A_{13}	759	1434	2430	3803	5596
A_{15}	− 40	− 87	− 167	− 290	− 470
A_{17}	2	4	9	17	31
A_{19}				− 1	− 2

$\theta =$	24	28	32	36	40
A_1	0·3976564	0·3740668	0·3548770	0·3388395	0·3251558
A_3	− ·7443232	− ·7289191	− ·7138036	− ·6993518	− ·6856909
A_5	·4993336	·5256732	·5455227	·5606560	·5722811
A_7	− ·1903168	− ·2203293	− ·2470862	− ·2709356	− ·2922270
A_9	472135	610459	750122	888567	·1024146
A_{11}	− 82209	− 119979	− 163126	− 210638	− 261602
A_{13}	10576	17552	26589	37673	50728
A_{15}	− 1044	− 1981	− 3361	− 5251	− 7701
A_{17}	82	178	339	585	938
A_{19}	− 5	− 13	− 28	− 53	− 94
A_{21}		1	2	4	8

TABLE V.—COEFFICIENTS OF $se_2(x, \theta)$.

$$se_2(x, \theta) = \sum_{r=1}^{\infty} B_{2r}(\theta) \sin 2rx.$$

$\theta =$	1	2	3	4	5
B_2	0·9965719	0·9867860	0·9719337	0·9536390	0·9334294
B_4	− 826908	− ·1617181	− ·2342613	− ·2987558	− ·3548039
B_6	25787	100255	215746	362225	529637
B_8	− 429	− 3326	− 10675	− 23718	− 42959
B_{10}	4	69	331	977	2198
B_{12}		− 1	− 7	− 28	− 77
B_{14}					2

$\theta =$	6	7	8	9	10
B_2	0·9125038	0·8916871	0·8714883	0·8521861	0·8339074
B_4	− ·4028125	− ·4436086	− ·4781597	− ·5074115	− ·5322129
B_6	709372	894771	·1081032	·1264872	·1444148
B_8	− 68337	− 99438	− 135661	− 176343	− 220822
B_{10}	4167	7021	10859	15745	21714
B_{12}	− 175	− 343	− 603	− 978	− 1488
B_{14}	5	12	25	45	75
B_{16}			− 1	− 2	− 3

$\theta =$	12	14	16	18	20
B_2	0·8004987	0·7710182	0·7449519	0·7217741	0·7010247
B_4	− ·5712585	− ·5997806	− ·6208294	− ·6364735	− ·6481346
B_6	·1784261	·2096531	·2380581	·2637954	·2870910
B_8	− 318744	− 425171	− 536801	− 651158	− 766418
B_{10}	36924	56376	79784	106773	136938
B_{12}	− 2997	− 5265	− 8394	− 12455	− 17491
B_{14}	180	365	657	1085	1674
B_{16}	− 8	− 19	− 40	− 73	− 124
B_{18}		1	2	4	7

$\theta =$	24	28	32	36	40
B_2	0·6653541	0·6356586	0·6104189	0·5885991	0·5694704
B_4	− ·6632067	− ·6711132	− ·6746112	− ·6752971	− ·6741430
B_6	·3273154	·3605067	·3881142	·4112652	·4308276
B_8	− 994729	− ·1215088	− ·1424317	− ·1621148	− ·1805357
B_{10}	205187	281554	363551	449196	536955
B_{12}	− 30543	− 47497	− 68117	− 92070	− 118984
B_{14}	3428	6080	9741	14485	20346
B_{16}	− 300	− 609	− 1096	− 1800	− 2759
B_{18}	21	49	99	181	303
B_{20}	− 1	− 3	− 7	− 15	− 28
B_{22}					2

TABLE VI.—COEFFICIENTS OF $ce_2(x, \theta)$.

$$ce_2(x, \theta) = A_0(\theta) + \sum_{r=1}^{\infty} A_{2r}(\theta) \cos 2rx.$$

$\theta =$	1	2	3	4	5
A_0	0·2169279	0·3347511	0·3935458	0·4242881	0·4387372
A_2	·9482573	·8657777	·7930206	·7243738	·6536403
A_4	− 817670	− ·1618686	− ·2464638	− ·3362493	− ·4265789
A_6	25866	105249	248124	465555	758857
A_8	− 434	− 3581	− 12865	− 32671	− 67418
A_{10}	5	76	411	1404	3649
A_{12}		− 1	− 9	− 41	− 134
A_{14}				1	4

$\theta =$	6	7	8	9	10
A_0	0·4421011	0·4380469	0·4297751	0·4197503	0·4095096
A_2	·5798941	·5060457	·4359660	·3723112	·3160337
A_4	− ·5101644	− ·5806626	− ·6352872	− ·6747388	− ·7015377
A_6	·1113732	·1503831	·1901700	·2287428	·2650241
A_8	− 119889	− 190090	− 275685	− 373416	− 480201
A_{10}	7830	14534	24127	36762	52455
A_{12}	− 346	− 750	− 1424	− 2440	− 3864
A_{14}	11	28	61	117	206
A_{16}		− 1	− 2	− 4	− 8

$\theta =$	12	14	16	18	20
A_0	0·3910054	0·3760320	0·3640737	0·3543635	0·3462987
A_2	·2241353	·1540741	·0994657	·0557740	·0199863
A_4	− ·7282419	− ·7329554	− ·7258396	− ·7123429	− ·6954411
A_6	·3294052	·3832706	·4280731	·4653026	·4962297
A_8	− 711495	− 955576	− ·1203744	− ·1450274	− ·1691330
A_{10}	92738	144105	205379	275241	352345
A_{12}	− 8161	− 14701	− 23763	− 35520	− 50057
A_{14}	519	1086	1995	3332	5180
A_{16}	− 25	− 61	− 127	− 238	− 409
A_{18}	1	3	6	13	25
A_{20}				− 1	− 1

$\theta =$	24	28	32	36	40
A_0	0·3335412	0·3237374	0·3158373	0·3092551	0·3036350
A_2	− ·0352965	− ·0761715	− ·1077238	− ·1328661	− ·1533959
A_4	− ·6574644	− ·6186709	− ·5814674	− ·5466638	− ·5144349
A_6	·5431814	·5752635	·5967806	·6106922	·6190617
A_8	− ·2147890	− ·2563062	− ·2934569	− ·3264017	− ·3554696
A_{10}	523191	708851	902235	·1098103	·1292724
A_{12}	− 87438	− 135333	− 192643	− 258049	− 330197
A_{14}	10691	18987	30362	44960	62801
A_{16}	999	− 2043	− 3683	− 6048	− 9252
A_{18}	74	174	355	647	1087
A_{20}	4	− 12	− 28	− 56	− 104
A_{22}		1	2	4	8

TABLE VII.—COEFFICIENTS OF $se_3(x, \theta)$.

$$se_3(x, \theta) = \sum_{r=0}^{\infty} B_{2r+1}(\theta) \sin (2r+1)x.$$

$\theta =$	1	2	3	4	5
B_1	0·1096425	0·1920125	0·2538421	0·3009313	0·3373722
B_3	·9920165	·9735636	·9496345	·9224633	·8931139
B_5	− 622843	− ·1235575	− ·1832105	− ·2406377	− ·2951587
B_7	15595	62083	138616	243591	374432
B_9	− 217	− 1729	− 5800	− 13609	− 26170
B_{11}	2	31	156	488	1172
B_{13}			− 3	− 12	− 37
B_{15}					1

$\theta =$	6	7	8	9	10
B_1	0·3659351	0·3885254	0·4065020	0·4208701	0·4323950
B_3	·8622593	·8304668	·7982640	·7661295	·7344692
B_5	− ·3461167	− ·3929880	− ·4354476	− ·4733824	− ·5068651
B_7	527632	699031	884209	·1078860	·1279076
B_9	− 44252	− 68342	− 98632	− 135056	− 177344
B_{11}	2378	4282	7052	10842	15779
B_{13}	− 89	− 188	− 353	− 609	− 983
B_{15}	2	6	13	25	45
B_{17}				− 1	− 2

$\theta =$	12	14	16	18	20
B_1	0·4491570	0·4601475	0·4674373	0·4723093	0·4755659
B_3	·6737567	·6176701	·5667063	·5207471	·4793872
B_5	− ·5614179	− ·6016544	− ·6304399	− ·6503472	− ·6634687
B_7	·1683437	·2077727	·2450680	·2796861	·3114441
B_9	− 277771	− 395822	− 527193	− 667992	− 814947
B_{11}	29470	48610	73326	103487	138797
B_{13}	− 2192	− 4193	− 7178	− 11311	− 16718
B_{15}	121	269	523	921	1503
B_{17}	− 5	− 13	− 29	− 58	− 105
B_{19}		1	1	3	6

$\theta =$	24	28	32	36	40
B_1	0·4790943	0·4803388	0·4803597	0·4796887	0·4786131
B_3	·4084662	·3501741	·3015127	·2602747	·2248533
B_5	− ·6754658	− ·6752805	− ·6679968	− ·6565795	− ·6428005
B_7	·3665766	·4116288	·4481929	·4777703	·5016386
B_9	− ·1117282	− ·1419141	− ·1711632	− ·1989840	− ·2251337
B_{11}	223215	322968	434425	554335	679943
B_{13}	− 31702	− 52526	− 79207	− 111499	− 148996
B_{15}	3373	6425	10903	16997	24837
B_{17}	− 279	− 613	− 1174	− 2033	− 3258
B_{19}	18	47	102	196	345
B_{21}	− 1	− 3	− 7	− 16	− 30
B_{23}				1	2

Table VIII.—Coefficients of $ce_3(x, \theta)$.

$$ce_3(x, \theta) = \sum_{r=0}^{\infty} A_{2r+1}(\theta) \cos (2r+1)x.$$

$\theta =$	1	2	3	4	5
A_1	0·1396157	0·2972983	0·4450961	0·5609389	0·6423435
A_3	·9882511	·9469431	·8776563	·7952749	·7128512
A_5	− 621676	− ·1219612	− ·1772628	− ·2287074	− ·2789559
A_7	15578	61637	136504	240106	376053
A_9	− 217	− 1722	− 5767	− 13684	− 27163
A_{11}	2	31	156	497	1243
A_{13}			− 3	− 13	− 39
A_{15}					1

$\theta =$	6	7	8	9	10
A_1	0·6966296	0·7310702	0·7504676	0·7577999	0·7552689
A_3	·6345832	·5596260	·4861042	·4128898	·3400813
A_5	− ·3300943	− ·3826293	− ·4356009	− ·4869589	− ·5341214
A_7	550057	767182	·1029437	·1333872	·1671853
A_9	− 48391	− 79933	− 124372	− 183760	− 259028
A_{11}	2681	5213	9351	15666	24706
A_{13}	− 103	− 235	484	− 917	− 1615
A_{15}	3	8	18	39	77
A_{17}				− 1	− 3

$\theta =$	12	14	16	18	20
A_1	0·7291928	0·6893133	0·6480800	0·6109664	0·5791210
A_3	·2006213	·0787068	− ·0207838	− ·0997241	− ·1622752
A_5	− ·6071225	− ·6465950	− ·6578054	− ·6500914	− ·6310151
A_7	·2394649	·3093176	·3707821	·4225315	·4652946
A_9	− 453886	− 692866	956497	− ·1230949	− ·1507773
A_{11}	52497	94078	148841	215541	292865
A_{13}	− 4144	− 8696	− 15742	− 25629	− 38616
A_{15}	239	586	1213	2221	3711
A_{17}	− 11	− 30	− 71	− 147	− 272
A_{19}		1	3	8	16
A_{21}					− 1 .

$\theta =$	24	28	32	36	40
A_1	0·5289289	0·4914763	0·4622401	0·4385408	0·4187706
A_3	− ·2529417	− ·3139993	− ·3569144	− ·3880399	− ·4111317
A_5	− ·5765962	− ·5151505	− ·4543378	− ·3969950	− ·3440438
A_7	·5286785	·5695309	·5941427	·6069867	·6112823
A_9	− ·2049043	− ·2555249	− ·3014954	− ·3424220	− ·3783443
A_{11}	474198	682676	908972	·1145191	·1385050
A_{13}	− 74517	− 123983	− 186574	− 261206	− 346440
A_{15}	8542	16449	28027	43690	63680
A_{17}	− 749	− 1671	− 3229	− 5614	− 9008
A_{19}	52	134	295	573	1013
A_{21}	− 3	− 9	− 22	− 48	− 93
A_{23}			1	3	7

TABLE IX.—COEFFICIENTS OF $se_4(x, \theta)$.

$$se_4(x, \theta) = \sum_{r=1}^{\infty} B_{2r}(\theta) \sin 2rx.$$

$\theta =$	1	2	3	4	5
B_2	0·0827163	0·1619097	0·2348454	0·2999611	0·3567821
B_4	·9953225	·9817952	·9607292	·9337815	·9025316
B_6	− 499004	− 992280	− ·1475147	− ·1944355	− ·2397743
B_8	10406	41497	92933	164232	254802
B_{10}	− 124	− 990	− 3333	− 7875	− 15321
B_{12}	1	15	78	247	602
B_{14}			− 1	− 6	− 17

$\theta =$	6	7	8	9	10
B_2	0·4055840	0·4470398	0·4819673	0·5111862	0·5354554
B_4	·8682607	·8319194	·7941934	·7555906	·7165133
B_6	− ·2833616	− ·3250257	− ·3645728	− ·4017926	− ·4364777
B_8	363898	490552	633490	791106	961479
B_{10}	− 26347	− 41579	− 61570	− 86770	− 117504
B_{12}	1245	2297	3895	6186	9321
B_{14}	− 42	− 90	− 175	− 312	− 523
B_{16}	1	3	6	12	22
B_{18}					− 1

$\theta =$	12	14	16	18	20
B_2	0·5717781	0·5954508	0·6099571	0·6179506	0·6214050
B_4	·6382592	·5617277	·4886779	·4202778	·3571208
B_6	− ·4975672	− ·5470172	− ·5849525	− ·6122983	− ·6304406
B_8	·1331669	·1725603	·2125820	·2518189	·2892707
B_{10}	− 196170	− 297395	− 419163	− 558320	− 711279
B_{12}	18713	33125	53335	79797	112661
B_{14}	− 1262	− 2606	− 4791	− 8051	− 12599
B_{16}	64	153	322	607	1053
B_{18}	− 2	− 7	− 17	− 36	− 68
B_{20}				2	4

$\theta =$	24	28	32	36	40
B_2	0·6199872	0·6127401	0·6030315	0·5924486	0·5817421
B_4	·2467755	·1558247	·0809320	·0188717	− ·0330132
B_6	− ·6451501	− ·6398947	− ·6225193	− ·5981380	− ·5699647
B_8	·3566335	·4127532	·4580399	·4937535	·5213298
B_{10}	− ·1044817	− ·1396101	− ·1748327	− ·2090565	− ·2416149
B_{12}	197061	304059	429943	570714	722573
B_{14}	− 26267	− 46870	− 74970	− 110698	− 153861
B_{16}	2620	5415	9815	16151	24693
B_{18}	− 204	− 488	− 1003	− 1843	− 3104
B_{20}	13	35	82	169	314
B_{22}		− 2	− 6	− 13	− 26
B_{24}				1	2

TABLE X.—COEFFICIENTS OF $ce_4(x, \theta)$.

$$ce_4(x, \theta) = A_0(\theta) + \sum_{r=1}^{\infty} A_{2r}(\theta) \cos 2rx.$$

$\theta =$	1	2	3	4	5
A_0	0·0052120	0·0208599	0·0467405	0·0817838	0·1233638
A_2	·0835684	·1683516	·2545600	·3404212	·4218199
A_4	·9952242	·9802760	·9535006	·9129992	·8581521
A_6	− 498976	− 991423	− ·1469032	− ·1921002	− ·2336131
A_8	10405	41473	92676	162942	250652
A_{10}	− 124	− 989	− 3326	− 7831	− 15153
A_{12}	1	15	78	246	598
A_{14}			− 1	− 6	− 17

$\theta =$	6	7	8	9	10
A_0	0·1673896	0·2095928	0·2470339	0·2785543	0·3042351
A_2	·4934863	·5514268	·5945088	·6239903	·6420769
A_4	·7910919	·7164866	·6394157	·5633649	·4897790
A_6	− ·2708225	− ·3040743	− ·3345300	− ·3635326	− ·3920627
A_8	354230	473216	608934	764150	942128
A_{10}	− 25911	− 40798	− 60717	− 86854	− 120665
A_{12}	1233	2279	3905	6336	9865
A_{14}	− 42	− 90	− 177	− 325	− 565
A_{16}	1	3	6	12	24
A_{18}					− 1

$\theta =$	12	14	16	18	20
A_0	0·3405440	0·3606977	0·3681547	0·3662513	0·3587450
A_2	·6519183	·6351009	·5984510	·5491281	·4949709
A_4	·3496018	·2154054	·0868854	− ·0312122	− ·1335586
A_6	− ·4487959	− ·5020019	− ·5441005	− ·5681810	− ·5723987
A_8	·1376362	·1915534	·2528823	·3156889	·3741212
A_{10}	− 217808	− 363917	− 563578	− 809496	− ·1086034
A_{12}	21746	43104	77418	126617	190524
A_{14}	− 1514	− 3539	− 7335	− 13599	− 22866
A_{16}	78	215	512	1074	2014
A_{18}	− 3	− 10	− 28	− 65	− 136
A_{20}			1	3	7

$\theta =$	24	28	32	36	40
A_0	0·3388323	0·3211298	0·3074120	0·2967922	0·2883296
A_2	·3936870	·3129101	·2503722	·2009820	·1609259
A_4	− ·2858568	− ·3820602	− ·4422045	− ·4798147	− ·5029338
A_6	− ·5352509	− ·4668709	− ·3894241	− ·3126523	− ·2404558
A_8	·4668287	·5274020	·5631396	·5807448	·5851380
A_{10}	− ·1672204	− ·2247168	− ·2782052	− ·3265137	− ·3691808
A_{12}	356307	561931	796671	·1051288	·1317625
A_{14}	− 51649	− 95215	− 154117	− 228102	− 316231
A_{16}	5481	11794	21777	36136	55407
A_{18}	− 447	− 1121	− 2361	− 4393	− 7452
A_{20}	29	85	203	424	797
A_{22}	− 2	− 5	− 14	− 33	− 69
A_{24}			1	2	5

TABLE XI.—COEFFICIENTS OF $se_5(x, \theta)$.

$$se_5(x, \theta) = \sum_{r=0}^{\infty} B_{2r+1}(\theta) \sin (2r+1)x.$$

$\theta =$	1	2	3	4	5
B_1	0·0024939	0·0095184	0·0203517	0·0342316	0·0503825
B_3	·0623983	·1241361	·1844346	·2424503	·2973655
B_5	·9971799	·9887433	·9748010	·9556047	·9315670
B_7	− 416163	− 829307	− ·1236461	− ·1634827	− ·2021936
B_9	7436	29684	66573	117821	183057
B_{11}	− 77	− 619	− 2086	− 4932	− 9603
B_{13}	1	9	44	137	335
B_{15}			− 1	− 3	− 8

$\theta =$	6	7	8	9	10
B_1	0·0680550	0·0865685	0·1053436	0·1239178	0·1419460
B_3	·3484801	·3952774	·4374524	·4749018	·5076902
B_5	·9032410	·8712704	·8363273	·7990564	·7600380
B_7	− ·2395777	− ·2754843	− ·3098080	− ·3424762	− ·3734350
B_9	261844	353703	458122	574558	702416
B_{11}	− 16533	− 26146	− 38853	− 55047	− 75102
B_{13}	693	1282	2182	3486	5299
B_{15}	− 21	− 45	− 88	− 158	− 268
B_{17}		1	3	5	10

$\theta =$	12	14	16	18	20
B_1	0·1754907	0·2049832	0·2302493	0·2515296	0·2692495
B_3	·5601081	·5969369	·6207572	·6340483	·6390398
B_5	·6786748	·5953287	·5121901	·4309127	·3527989
B_7	− ·4300281	− ·4791568	− ·5203565	− ·5532795	− ·5778606
B_9	989590	·1312863	·1663432	·2030885	·2404264
B_{11}	− 128132	− 200148	− 292378	− 404851	− 536407
B_{13}	10907	19982	33521	52439	77470
B_{15}	− 664	− 1424	− 2738	− 4831	− 7946
B_{17}	30	76	168	335	613
B_{19}	− 1	− 3	− 8	− 18	− 37
B_{21}				1	2

$\theta =$	24	28	32	36	40
B_1	0·2959659	0·3141014	0·3265576	0·3353093	0·3416141
B_3	·6315833	·6103186	·5827805	·5532237	·5237981
B_5	·2097049	·0870904	− ·0149790	− ·0987129	− ·1670302
B_7	− ·6034884	− ·6028358	− ·5833053	− ·5515677	− ·5126578
B_9	·3129321	·3778306	·4322546	·4756778	·5088173
B_{11}	− 847620	− ·1203623	− ·1581686	− ·1963555	− ·2336226
B_{13}	147600	244986	367730	512309	674587
B_{15}	− 18207	− 35255	− 60361	− 94274	− 137280
B_{17}	1687	3808	7434	13017	20968
B_{19}	− 122	− 321	− 716	− 1405	− 2505
B_{21}	7	22	55	122	241
B_{23}		− 1	− 4	− 9	− 19
B_{25}					1

Table XII.—Coefficients of $ce_5(x, \theta)$.

$$ce_5(x, \theta) = \sum_{r=0}^{\infty} A_{2r+1}(\theta) \cos (2r+1)x.$$

$\theta =$	1	2	3	4	5
A_1	0·0027111	0·0112611	0·0262522	0·0482426	0·0776858
A_3	·0624118	·1243439	·1854309	·2453591	·3037510
A_5	·9971785	·9886991	·9744740	·9542797	·9277284
A_7	− 416162	− 829284	− ·1236214	− ·1633489	− ·2017061
A_9	7436	29684	66564	117753	182746
A_{11}	− 77	− 619	− 2086	− 4930	− 9590
A_{13}	1	9	44	137	335
A_{15}				− 3	− 8

$\theta =$	6	7	8	9	10
A_1	0·1147954	0·1593268	0·2103223	0·2659504	0·3236196
A_3	·3600226	·4132509	·4621241	·5050654	·5405667
A_5	·8943042	·8534992	·8050657	·7493358	·6874439
A_7	− ·2382076	− ·2722869	− ·3033497	− ·3308976	− ·3546978
A_9	260795	350851	451571	561475	679287
A_{11}	− 16483	− 25989	− 38443	− 54141	− 73368
A_{13}	692	1276	2164	3444	5211
A_{15}	− 21	− 45	− 87	− 157	− 265
A_{17}		1	3	5	10

$\theta =$	12	14	16	18	20
A_1	0·4338421	0·5242150	0·5900856	0·6337428	0·6584944
A_3	·5859780	·5991047	·5875370	·5581534	·5151069
A_5	·5529476	·4172375	·2886823	·1679224	·0535119
A_7	− ·3920752	− ·4202871	− ·4444427	− ·4662340	− ·4839496
A_9	937133	·1231302	·1576800	·1985101	·2454483
A_{11}	− 123910	− 194630	− 292783	− 426902	− 603934
A_{13}	10693	19891	34770	58024	92761
A_{15}	− 657	− 1441	− 2912	− 5530	− 9934
A_{17}	30	78	182	393	791
A_{19}	− 1	− 3	− 9	− 22	− 49
A_{21}					2

$\theta =$	24	28	32	36	40
A_1	0·6626349	0·6292498	0·5860324	0·5468653	0·5144765
A_3	·4006885	·2749586	·1639702	·0748395	·0044985
A_5	− ·1532171	− ·3127042	− ·4171763	− ·4784149	− ·5108384
A_7	− ·4933373	− ·4562706	− ·3849891	− ·2997831	− ·2133006
A_9	·3482384	·4392387	·5019520	·5373167	·5516334
A_{11}	− ·1086428	− ·1675275	− ·2274941	− ·2834046	− ·3335094
A_{13}	206377	380056	600046	851763	·1124693
A_{15}	− 27042	− 58933	− 107363	− 172519	− 254090
A_{17}	2620	6727	14091	25562	41898
A_{19}	− 196	− 593	− 1425	− 2914	− 5310
A_{21}	12	42	115	265	536
A_{23}		− 2	− 8	− 20	− 44
A_{25}				1	3

TABLE XIII.—COEFFICIENTS OF $se_6(x, \theta)$.

$$se_6(x, \theta) = \sum_{r=1}^{\infty} B_{2r}(\theta) \sin 2rx.$$

$\theta =$	1	2	3	4	5
B_2	0·0015602	0·0062128	0·0138737	0·0244018	0·0375992
B_4	·0499480	·0995822	·1485817	·1966140	·2433333
B_6	·9981127	·9924571	·9830539	·9699444	·9531988
B_8	− 356850	− 711944	− ·1063526	− ·1409849	− ·1749190
B_{10}	5578	22281	50019	88639	137930
B_{12}	− 52	− 413	− 1392	− 3293	− 6416
B_{14}		5	26	82	201
B_{16}				− 1	− 5
$\theta =$	6	7	8	9	10
B_2	0·0532125	0·0709384	0·0904323	0·1113210	0·1332192
B_4	·2883858	·3314194	·3720973	·4101152	·4452167
B_6	·9329261	·9092819	·8824725	·8527547	·8204286
B_8	− ·2079883	− ·2400362	− ·2709204	− ·3005177	− ·3287274
B_{10}	197622	267392	346873	435662	533334
B_{12}	− 11055	− 17495	− 26012	− 36875	− 50339
B_{14}	416	770	1311	2094	3184
B_{16}	− 11	− 25	− 48	− 86	− 146
B_{18}			1	3	5
$\theta =$	12	14	16	18	20
B_2	0·1785411	0·2236735	0·2665419	0·3058132	0·3407965
B_4	·5059560	·5535676	·5883081	·6111567	·6234816
B_6	·7492982	·6718593	·5907338	·5081236	·4257636
B_8	− ·3806978	− ·4264676	− ·4659151	− ·4990208	− ·5257760
B_{10}	753564	·1003981	·1280856	·1580040	·1896661
B_{12}	− 86057	− 135023	− 198893	− 279020	− 376302
B_{14}	6564	12083	20465	32503	49016
B_{16}	− 362	− 781	− 1517	− 2723	− 4581
B_{18}	15	38	85	173	325
B_{20}		− 1	− 4	− 9	− 18
B_{22}					1
$\theta =$	24	28	32	36	40
B_2	0·3973269	0·4374027	0·4642172	0·4812296	0·4913961
B_4	·6225734	·5971304	·5569728	·5095467	·4598608
B_6	·2668995	·1218704	− ·0048201	− ·1115484	− ·1990050
B_8	− ·5602333	− ·5700366	− ·5579359	− ·5285048	− ·4868907
B_{10}	·2558517	·3214381	·3815324	·4327860	·4736965
B_{12}	− 622783	− 932182	− ·1288947	− ·1673387	− ·2066839
B_{14}	98540	173939	277327	407902	562770
B_{16}	− 11138	− 23085	− 42273	− 70186	− 107804
B_{18}	952	2312	4853	9081	15508
B_{20}	− 64	− 182	− 437	− 921	− 1748
B_{22}	3	12	32	75	159
B_{24}			− 2	− 5	− 12
B_{26}					1

TABLE XIV.

$ce_0(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta = 1$ $a_0 = -0{\cdot}45514$	Δ^2	$\theta = 2$ $a_0 = -1{\cdot}51396$	Δ^2	$\theta = 3$ $a_0 = -2{\cdot}83439$	Δ^2	$\theta = 4$ $a_0 = -4{\cdot}28052$	Δ^2	$\theta = 5$ $a_0 = -5{\cdot}80005$	Δ^2
0°	0·38483	29	0·20262	34	0·11494	31	0·06990	26	0·04480	22
1	·38497	29	·20279	34	·11510	31	·07003	26	·04491	22
2	·38540	29	·20330	34	·11556	31	·07042	26	·04523	22
3	·38612	29	·20415	34	·11634	31	·07108	27	·04577	22
4	·38713	29	·20534	34	·11742	31	·07200	27	·04653	22
5	·38842	29	·20688	34	·11882	32	·07319	27	·04752	23
6	·39000	29	·20876	35	·12054	32	·07465	28	·04873	23
7	·39187	29	·21098	35	·12258	32	·07638	28	·05017	24
8	·39403	29	·21355	35	·12494	33	·07840	29	·05185	24
9	·39646	28	·21647	35	·12763	33	·08070	29	·05377	25
10	·39919	28	·21974	35	·13064	34	·08329	30	·05594	26
11	·40219	28	·22337	36	·13400	34	·08619	31	·05838	27
12	·40548	28	·22735	36	·13770	35	·08939	32	·06108	28
13	·40906	28	·23168	36	·14175	36	·09290	32	·06405	29
14	·41291	28	·23638	36	·14615	36	·09674	33	·06732	30
15	·41704	28	·24144	37	·15092	37	·10092	34	·07089	31
16	·42145	28	·24687	37	·15605	38	·10544	36	·07477	33
17	·42614	27	·25266	37	·16157	38	·11031	37	·07897	34
18	·43110	27	·25883	37	·16746	39	·11555	38	·08351	35
19	·43633	27	·26537	38	·17375	40	·12117	39	·08841	37
20	·44183	27	·27229	38	·18044	41	·12719	40	·09368	38
21	·44760	26	·27959	38	·18754	42	·13360	42	·09933	40
22	·45364	26	·28727	38	·19505	42	·14043	43	·10538	42
23	·45993	26	·29534	39	·20299	43	·14769	44	·11184	43
24	·46649	25	·30379	39	·21136	44	·15539	46	·11874	45
25	·47330	25	·31263	39	·22017	45	·16355	47	·12610	47
26	·48036	25	·32185	39	·22943	46	·17218	48	·13392	49
27	·48767	24	·33147	39	·23915	46	·18129	50	·14223	51
28	·49522	24	·34148	39	·24933	47	·19090	51	·15105	52
29	·50300	23	·35188	39	·25998	48	·20101	52	·16039	54
30	·51102	23	·36267	39	·27111	48	·21165	53	·17027	56
31	·51927	22	·37384	39	·28272	49	·22282	55	·18071	58
32	·52773	21	·38540	38	·29481	49	·23454	56	·19173	59
33	·53641	21	·39735	38	·30740	49	·24681	57	·20335	61
34	·54530	20	·40968	38	·32048	50	·25965	58	·21558	63
35	·55439	19	·42238	37	·33406	50	·27306	58	·22843	64
36	·56367	18	·43545	36	·34813	50	·28706	59	·24192	66
37	·57313	18	·44889	36	·36270	50	·30165	60	·25607	67
38	·58277	17	·46268	35	·37776	49	·31683	60	·27089	68
39	·59258	16	·47682	34	·39332	49	·33262	60	·28639	69
40	·60254	15	·49131	33	·40937	48	·34900	60	·30257	69
41	·61265	14	·50613	32	·42590	48	·36599	60	·31945	70
42	·62290	13	·52126	31	·44291	47	·38358	60	·33702	70
43	·63327	11	·53670	29	·46038	46	·40177	59	·35530	70
44	·64376	10	·55244	28	·47831	44	·42055	58	·37428	70
45	0·65435	9	0·56845	26	0·49669	43	0·43991	57	0·39397	70

TABLE XIV—*continued.*

$ce_0(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta=1$ $a_0 = -0{\cdot}45514$		$\theta=2$ $a_0 = -1{\cdot}51396$		$\theta=3$ $a_0 = -2{\cdot}83439$		$\theta=4$ $a_0 = -4{\cdot}28052$		$\theta=5$ $a_0 = -5{\cdot}80005$	
°		Δ^2		Δ^2		Δ^2		Δ^2		Δ^2
45	0·65435	9	0·56845	26	0·49669	43	0·43991	57	0·39397	70
46	·66503	8	·58472	24	·51549	41	·45985	56	·41435	69
47	·67579	6	·60124	23	·53470	39	·48035	54	·43541	68
48	·68662	5	·61799	21	·55431	37	·50139	53	·45715	66
49	·69749	4	·63494	19	·57429	35	·52296	50	·47956	64
50	·70841	2	·65207	16	·59462	32	·54503	48	·50261	62
51	·71934	1	·66937	14	·61527	30	·56758	45	·52628	60
52	·73029	-1	·68681	11	·63623	27	·59058	42	·55055	57
53	·74123	-2	·70436	9	·65745	24	·61401	39	·57538	53
54	·75215	-4	·72200	6	·67890	20	·63782	35	·60074	50
55	·76303	-5	·73971	3	·70056	17	·66199	31	·62661	45
56	·77385	-7	·75744	0	·72238	13	·68646	27	·65292	41
57	·78461	-9	·77518	-3	·74434	9	·71120	22	·67965	36
58	·79528	-10	·79289	-6	·76638	5	·73617	17	·70673	30
59	·80585	-12	·81054	-9	·78847	0	·76131	12	·73412	25
60	·81630	-14	·82810	-12	·81056	-4	·78657	7	·76175	19
61	·82662	-15	·84555	-16	·83262	-9	·81190	1	·78957	12
62	·83678	-17	·86283	-19	·85458	-14	·83724	-5	·81751	5
63	·84678	-19	·87992	-22	·87641	-18	·86252	-11	·84550	-2
64	·85658	-20	·89679	-26	·89806	-24	·88770	-17	·87347	-10
65	·86619	-22	·91340	-29	·91947	-29	·91270	-24	·90135	-17
66	·87558	-24	·92972	-33	·94059	-34	·93747	-31	·92905	-25
67	·88473	-25	·94571	-36	·96138	-39	·96192	-37	·95650	-33
68	·89363	-27	·96133	-40	0·98177	-44	0·98601	-44	0·98362	-42
69	·90226	-28	·97656	-43	1·00172	-50	1·00965	-51	1·01031	-50
70	·91061	-30	0·99135	-47	1·02118	-55	1·03277	-58	1·03651	-59
71	·91866	-31	1·00567	-50	1·04009	-60	1·05532	-65	1·06212	-67
72	·92639	-33	1·01949	-53	1·05840	-65	1·07721	-72	1·08706	-76
73	·93380	-34	1·03277	-57	1·07605	-70	1·09839	-79	1·11123	-84
74	·94087	-36	1·04549	-60	1·09301	-75	1·11878	-85	1·13457	-93
75	·94758	-37	1·05761	-63	1·10921	-80	1·13832	-92	1·15698	-101
76	·95392	-38	1·06910	-66	1·12462	-84	1·15694	-98	1·17838	-109
77	·95988	-39	1·07994	-68	1·13918	-89	1·17458	-104	1·19869	-116
78	·96545	-40	1·09009	-71	1·15286	-93	1·19117	-110	1·21784	-124
79	·97061	-41	1·09953	-73	1·16561	-97	1·20667	-115	1·23576	-131
80	·97536	-42	1·10824	-76	1·17739	-101	1·22102	-120	1·25236	-137
81	·97969	-43	1·11618	-78	1·18816	-104	1·23416	-125	1·26760	-143
82	·98359	-44	1·12335	-80	1·19789	-107	1·24605	-129	1·28140	-149
83	·98704	-45	1·12972	-81	1·20656	-110	1·25665	-133	1·29371	-154
84	·99005	-45	1·13528	-83	1·21412	-112	1·26592	-137	1·30449	-158
85	·99261	-46	1·14001	-84	1·22056	-114	1·27381	-140	1·31369	-162
86	·99471	-46	1·14389	-85	1·22587	-116	1·28032	-142	1·32126	-165
87	·99635	-47	1·14693	-86	1·23001	-117	1·28540	-144	1·32719	-168
88	·99752	-47	1·14910	-87	1·23297	-118	1·28905	-145	1·33144	-169
89	·99822	-47	1·15041	-87	1·23476	-119	1·29124	-146	1·33400	-170
90	0·99846	-47	1·15084	-87	1·23535	-119	1·29197	-146	1·33485	-171

TABLE XIV—*continued.*

$ce_0(x, \theta)$. $\theta = 6$ to $\theta = 10$.

x	$\theta=6$ $a_0 = -7{\cdot}36883$	Δ^2	$\theta=7$ $a_0 = -8{\cdot}97374$	Δ^2	$\theta=8$ $a_0 = -10{\cdot}60673$	Δ^2	$\theta=9$ $a_0 = -12{\cdot}26241$	Δ^2	$\theta=10$ $a_0 = -13{\cdot}93698$	Δ^2
° 0	0·02987	18	0·02054	14	0·01448	12	0·01042	10	0·00763	8
1	·02996	18	·02062	14	·01454	12	·01047	10	·00767	8
2	·03023	18	·02083	15	·01472	12	·01061	10	·00778	8
3	·03067	18	·02119	15	·01501	12	·01085	10	·00798	8
4	·03129	18	·02170	15	·01543	12	·01120	10	·00827	9
5	·03210	19	·02236	16	·01597	13	·01164	11	·00863	9
6	·03309	19	·02318	16	·01664	13	·01219	11	·00909	9
7	·03428	20	·02416	17	·01744	14	·01286	12	·00963	10
8	·03567	21	·02530	17	·01839	15	·01363	12	·01028	10
9	·03726	21	·02661	18	·01947	15	·01454	13	·01103	11
10	·03906	22	·02811	19	·02072	16	·01557	14	·01189	12
11	·04109	23	·02980	20	·02211	17	·01674	15	·01287	13
12	·04335	24	·03168	21	·02369	18	·01806	16	·01398	14
13	·04585	25	·03377	22	·02544	19	·01953	17	·01522	15
14	·04860	27	·03609	23	·02740	21	·02118	18	·01661	16
15	·05162	28	·03864	25	·02956	22	·02300	20	·01816	17
16	·05492	29	·04144	26	·03193	24	·02503	21	·01989	19
17	·05852	31	·04451	28	·03455	25	·02726	23	·02180	20
18	·06242	32	·04785	30	·03741	27	·02972	24	·02392	22
19	·06665	34	·05149	31	·04054	29	·03242	26	·02625	24
20	·07121	36	·05544	33	·04396	31	·03538	28	·02882	26
21	·07614	38	·05972	35	·04769	33	·03862	30	·03165	28
22	·08145	40	·06436	37	·05174	35	·04217	32	·03476	30
23	·08715	42	·06937	40	·05614	37	·04604	35	·03816	32
24	·09327	44	·07478	42	·06091	40	·05025	37	·04189	35
25	·09983	46	·08060	44	·06608	42	·05484	40	·04597	37
26	·10684	48	·08687	47	·07167	45	·05983	43	·05043	40
27	·11434	50	·09360	49	·07770	47	·06524	45	·05529	43
28	·12234	52	·10082	52	·08421	50	·07110	48	·06058	46
29	·13086	55	·10856	54	·09122	53	·07745	51	·06633	50
30	·13993	57	·11684	57	·09876	56	·08432	55	·07258	53
31	·14957	59	·12568	60	·10686	59	·09173	58	·07936	56
32	·15980	61	·13513	62	·11555	62	·09972	61	·08670	60
33	·17065	64	·14519	65	·12486	65	·10832	65	·09465	64
34	·18214	66	·15591	68	·13483	68	·11757	68	·10323	67
35	·19428	68	·16730	70	·14547	71	·12749	72	·11248	71
36	·20710	70	·17939	73	·15683	74	·13814	75	·12245	75
37	·22062	72	·19220	75	·16892	77	·14953	78	·13316	79
38	·23486	73	·20577	77	·18180	80	·16171	82	·14467	83
39	·24983	75	·22011	80	·19547	83	·17471	85	·15700	87
40	·26555	76	·23525	82	·20997	86	·18856	88	·17020	90
41	·28204	78	·25121	84	·22533	88	·20329	91	·18430	94
42	·29930	79	·26800	85	·24157	90	·21893	94	·19934	97
43	·31735	79	·28565	87	·25871	92	·23552	97	·21535	101
44	·33619	80	·30416	88	·27678	94	·25308	99	·23237	104
45	0·35583	80	0·32354	88	0·29579	96	0·27163	101	0·25042	106

TABLE XIV—*continued.*

$ce_0(x, \theta)$. $\theta = 6$ to $\theta = 10$.

x	$\theta=6$ $a_0 = -7{\cdot}36883$	Δ^2	$\theta=7$ $a_0 = -8{\cdot}97374$	Δ^2	$\theta=8$ $a_0 = -10{\cdot}60673$	Δ^2	$\theta=9$ $a_0 = -12{\cdot}26241$	Δ^2	$\theta=10$ $a_0 = -13{\cdot}93698$	Δ^2
45°	0·35583	80	0·32354	88	0·29579	96	0·27163	101	0·25042	106
46	·37627	80	·34382	89	·31575	97	·29120	103	·26953	109
47	·39750	79	·36497	89	·33668	97	·31180	105	·28974	111
48	·41953	78	·38702	89	·35858	98	·33344	105	·31105	112
49	·44234	77	·40995	88	·38146	97	·35614	106	·33348	113
50	·46591	75	·43376	86	·40531	97	·37990	106	·35704	114
51	·49023	73	·45844	85	·43013	95	·40471	105	·38174	114
52	·51528	70	·48396	82	·45590	93	·43057	104	·40758	113
53	·54103	67	·51030	79	·48260	91	·45747	102	·43455	111
54	·56746	63	·53744	76	·51022	88	·48539	99	·46263	109
55	·59451	59	·56534	72	·53871	84	·51429	96	·49180	106
56	·62215	54	·59396	67	·56805	80	·54416	91	·52204	102
57	·65034	49	·62325	62	·59818	75	·57493	86	·55330	98
58	·67902	44	·65316	56	·62906	69	·60657	81	·58554	92
59	·70813	37	·68364	50	·66063	62	·63902	74	·61870	86
60	·73762	31	·71462	43	·69282	55	·67221	67	·65271	78
61	·76742	24	·74603	35	·72557	47	·70607	59	·68751	70
62	·79745	16	·77779	27	·75878	38	·74051	50	·72300	61
63	·82764	8	·80982	18	·79237	29	·77545	40	·75910	50
64	·85792	- 1	·84203	9	·82625	19	·81078	29	·79571	39
65	·88818	- 9	·87434	- 1	·86033	8	·84640	18	·83271	27
66	·91835	- 19	·90663	- 11	·89448	- 3	·88221	6	·86998	15
67	·94834	- 28	·93882	- 22	·92861	- 14	·91807	- 7	·90740	1
68	0·97805	- 38	0·97080	- 32	·96260	- 27	·95386	- 20	·94482	- 13
69	1·00738	- 48	1·00245	- 44	0·99632	- 39	0·98945	- 34	0·98212	- 28
70	1·03624	- 58	1·03366	- 55	1·02965	- 52	1·02471	- 48	1·01915	- 43
71	1·06452	- 68	1·06432	- 67	1·06246	- 65	1·05949	- 62	1·05574	- 59
72	1·09212	- 78	1·09431	- 78	1·09463	- 78	1·09365	- 77	1·09174	- 75
73	1·11894	- 88	1·12352	- 90	1·12601	- 91	1·12704	- 91	1·12700	- 91
74	1·14489	- 98	1·15183	- 102	1·15649	- 104	1·15952	- 106	1·16135	- 107
75	1·16986	- 108	1·17912	- 113	1·18592	- 117	1·19094	- 121	1·19463	- 123
76	1·19375	- 117	1·20528	- 124	1·21417	- 130	1·22115	- 135	1·22668	- 139
77	1·21646	- 127	1·23019	- 135	1·24113	- 143	1·25002	- 149	1·25734	- 155
78	1·23791	- 135	1·25376	- 146	1·26666	- 155	1·27739	- 163	1·28646	- 170
79	1·25801	- 144	1·27586	- 155	1·29064	- 166	1·30314	- 176	1·31387	- 184
80	1·27667	- 152	1·29641	- 165	1·31296	- 177	1·32712	- 188	1·33945	- 198
81	1·29380	- 159	1·31531	- 174	1·33351	- 187	1·34923	- 200	1·36304	- 211
82	1·30935	- 166	1·33247	- 182	1·35219	- 197	1·36935	- 210	1·38452	- 223
83	1·32323	- 172	1·34781	- 189	1·36890	- 205	1·38736	- 220	1·40377	- 234
84	1·33539	- 178	1·36126	- 196	1·38356	- 212	1·40317	- 228	1·42068	- 243
85	1·34577	- 182	1·37275	- 201	1·39610	- 219	1·41670	- 236	1·43517	- 252
86	1·35433	- 186	1·38223	- 206	1·40644	- 224	1·42788	- 242	1·44713	- 259
87	1·36103	- 189	1·38966	- 209	1·41455	- 229	1·43663	- 247	1·45651	- 264
88	1·36584	- 191	1·39498	- 212	1·42037	- 232	1·44292	- 250	1·46325	- 268
89	1·36873	- 193	1·39819	- 214	1·42387	- 233	1·44671	- 252	1·46730	- 270
90	1·36970	- 193	1·39926	- 214	1·42504	- 234	1·44797	- 253	1·46866	- 271

Table XV.

$se_1(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta=1$ $b_1 = -0{\cdot}11025$	Δ^2	$\theta=2$ $b_1 = -1{\cdot}39068$	Δ^2	$\theta=3$ $b_1 = -2{\cdot}78538$	Δ^2	$\theta=4$ $b_1 = -4{\cdot}25918$	Δ^2	$\theta=5$ $b_1 = -5{\cdot}79008$	Δ^2
0°	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0
1	·01198	1	·00830	1	·00584	2	·00418	2	·00305	1
2	·02397	2	·01661	3	·01169	3	·00838	3	·00612	3
3	·03598	2	·02494	4	·01758	5	·01262	5	·00921	4
4	·04800	3	·03332	5	·02351	6	·01689	6	·01235	6
5	·06006	4	·04175	7	·02951	8	·02124	8	·01555	7
6	·07216	5	·05025	8	·03558	9	·02566	9	·01882	9
7	·08430	5	·05883	9	·04175	11	·03017	11	·02218	10
8	·09650	6	·06751	11	·04802	13	·03479	13	·02565	12
9	·10875	7	·07629	12	·05443	14	·03955	14	·02924	14
10	·12107	7	·08519	13	·06097	16	·04444	16	·03296	15
11	·13347	8	·09423	15	·06767	17	·04949	18	·03684	17
12	·14594	9	·10341	16	·07454	19	·05472	19	·04088	19
13	·15850	9	·11276	17	·08160	20	·06015	21	·04511	20
14	·17115	10	·12227	18	·08886	22	·06578	23	·04955	22
15	·18390	10	·13197	20	·09634	23	·07164	24	·05420	24
16	·19676	11	·14186	21	·10406	25	·07774	26	·05909	26
17	·20972	11	·15196	22	·11202	26	·08411	28	·06424	28
18	·22279	12	·16228	23	·12025	28	·09075	30	·06967	29
19	·23599	12	·17282	24	·12876	29	·09769	31	·07539	31
20	·24930	12	·18361	25	·13756	31	·10495	33	·08142	33
21	·26274	13	·19464	26	·14668	32	·11253	35	·08779	35
22	·27630	13	·20593	27	·15611	34	·12047	37	·09451	37
23	·29000	13	·21749	28	·16589	35	·12877	39	·10161	39
24	·30383	13	·22933	28	·17601	36	·13746	40	·10910	41
25	·31779	14	·24145	29	·18650	38	·14656	42	·11700	44
26	·33189	14	·25386	30	·19737	39	·15607	44	·12534	46
27	·34612	14	·26657	30	·20863	40	·16602	45	·13413	48
28	·36049	13	·27958	31	·22029	41	·17642	47	·14341	50
29	·37499	13	·29290	31	·23236	42	·18729	48	·15318	52
30	·38963	13	·30654	32	·24485	43	·19864	50	·16346	54
31	·40440	13	·32049	32	·25777	44	·21050	51	·17429	56
32	·41929	13	·33476	32	·27114	45	·22287	53	·18567	58
33	·43432	12	·34935	32	·28495	45	·23576	54	·19762	59
34	·44946	12	·36427	32	·29921	46	·24920	55	·21017	61
35	·46473	11	·37950	32	·31393	46	·26319	56	·22333	63
36	·48010	11	·39505	32	·32912	47	·27773	57	·23712	64
37	·49559	10	·41092	31	·34477	47	·29285	58	·25155	65
38	·51117	9	·42710	31	·36089	47	·30854	58	·26663	67
39	·52684	8	·44358	30	·37747	46	·32482	59	·28238	68
40	·54260	8	·46037	29	·39451	46	·34168	59	·29881	68
41	·55844	7	·47745	28	·41202	45	·35913	59	·31592	69
42	·57434	6	·49481	27	·42998	45	·37716	59	·33372	69
43	·59029	4	·51245	26	·44839	44	·39578	58	·35221	70
44	·60629	3	·53034	25	·46723	43	·41499	57	·37141	69
45	0·62233	2	0·54848	23	0·48650	41	0·43476	56	0·39129	69

TABLE XV—*continued.*

$se_1(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta=1$ $b_1 = -0{\cdot}11025$	Δ^2	$\theta=2$ $b_1 = -1{\cdot}39068$	Δ^2	$\theta=3$ $b_1 = -2{\cdot}78538$	Δ^2	$\theta=4$ $b_1 = -4{\cdot}25918$	Δ^2	$\theta=5$ $b_1 = -5{\cdot}79008$	Δ^2
45°	0·62233	2	0·54848	23	0·48650	41	0·43476	56	0·39129	69
46	·63838	1	·56686	22	·50618	40	·45510	55	·41187	68
47	·65445	−1	·58545	20	·52626	38	·47599	54	·43313	67
48	·67051	−2	·60424	18	·54672	36	·49742	52	·45506	66
49	·68654	−4	·62320	16	·56754	34	·51936	50	·47765	64
50	·70254	−5	·64233	14	·58869	31	·54181	47	·50087	62
51	·71850	−7	·66159	11	·61016	29	·56472	45	·52472	59
52	·73438	−8	·68097	9	·63192	26	·58809	42	·54916	56
53	·75018	−10	·70043	6	·65393	23	·61186	38	·57416	53
54	·76588	−12	·71995	3	·67616	19	·63603	35	·59969	49
55	·78146	−14	·73951	0	·69859	16	·66053	31	·62572	45
56	·79691	−16	·75907	−3	·72117	12	·68535	26	·65219	41
57	·81219	−17	·77861	−6	·74387	8	·71042	22	·67907	36
58	·82731	−19	·79809	−9	·76665	4	·73572	17	·70631	30
59	·84223	−21	·81748	−12	·78946	−1	·76118	12	·73385	24
60	·85694	−23	·83675	−16	·81227	−5	·78676	6	·76164	18
61	·87142	−25	·85587	−19	·83502	−10	·81240	0	·78961	12
62	·88564	−27	·87479	−23	·85768	−15	·83804	−5	·81769	5
63	·89960	−29	·89349	−26	·88018	−20	·86364	−12	·84583	−2
64	·91326	−31	·91193	−30	·90249	−25	·88911	−18	·87394	−10
65	·92661	−33	·93007	−33	·92454	−30	·91440	−25	·90195	−18
66	·93963	−35	·94788	−37	·94630	−35	·93945	−31	·92979	−26
67	·95229	−37	·96531	−41	·96770	−41	·96418	−38	·95737	−34
68	·96459	−39	·98234	−44	0·98869	−46	0·98854	−45	0·98462	−42
69	·97649	−41	0·99892	−48	1·00922	−51	1·01244	−52	1·01145	−51
70	·98799	−43	1·01502	−52	1·02924	−57	1·03582	−59	1·03776	−59
71	0·99906	−45	1·03061	−55	1·04868	−62	1·05861	−66	1·06349	−67
72	1·00968	−46	1·04564	−59	1·06751	−67	1·08075	−73	1·08854	−76
73	1·01984	−48	1·06008	−62	1·08567	−72	1·10215	−80	1·11283	−85
74	1·02951	−50	1·07390	−65	1·10310	−77	1·12276	−86	1·13627	−93
75	1·03869	−51	1·08707	−69	1·11976	−82	1·14250	−93	1·15878	−101
76	1·04736	−53	1·09955	−72	1·13559	−87	1·16132	−99	1·18028	−109
77	1·05550	−54	1·11131	−75	1·15056	−91	1·17914	−105	1·20068	−117
78	1·06310	−56	1·12233	−77	1·16461	−96	1·19592	−111	1·21992	−124
79	1·07014	−57	1·13257	−80	1·17771	−100	1·21158	−117	1·23791	−131
80	1·07661	−58	1·14201	−82	1·18981	−103	1·22607	−122	1·25459	−138
81	1·08250	−59	1·15063	−85	1·20088	−107	1·23935	−126	1·26989	−144
82	1·08780	−60	1·15841	−87	1·21088	−110	1·25137	−131	1·28375	−149
83	1·09250	−61	1·16532	−88	1·21978	−113	1·26207	−135	1·29612	−154
84	1·09659	−62	1·17134	−90	1·22755	−115	1·27143	−138	1·30694	−159
85	1·10007	−62	1·17647	−91	1·23417	−117	1·27941	−141	1·31618	−163
86	1·10292	−63	1·18068	−92	1·23961	−119	1·28598	−143	1·32379	−166
87	1·10514	−63	1·18396	−93	1·24386	−121	1·29112	−145	1·32974	−168
88	1·10673	−64	1·18632	−94	1·24691	−122	1·29480	−147	1·33401	−170
89	1·10769	−64	1·18773	−94	1·24874	−122	1·29702	−148	1·33658	−171
90	1·10801	−64	1·18821	−94	1·24936	−122	1·29776	−148	1·33743	−171

TABLE XV—*continued.*

$se_1(x, \theta)$. $\theta=6$ to $\theta=10$.

x	$\theta=6$, $b_1=-7{\cdot}36391$	Δ^2	$\theta=7$, $b_1=-8{\cdot}97120$	Δ^2	$\theta=8$, $b_1=-10{\cdot}60537$	Δ^2	$\theta=9$, $b_1=-12{\cdot}26166$	Δ^2	$\theta=10$, $b_1=-13{\cdot}93655$	Δ^2
0°	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0
1	·00226	1	·00170	1	·00129	1	·00099	1	·00077	1
2	·00453	3	·00341	2	·00259	2	·00199	2	·00155	2
3	·00683	4	·00514	4	·00391	3	·00301	3	·00234	2
4	·00917	5	·00691	5	·00527	4	·00406	4	·00316	3
5	·01157	7	·00872	6	·00666	5	·00514	5	·00401	4
6	·01403	8	·01060	7	·00811	6	·00627	6	·00490	5
7	·01657	10	·01255	9	·00962	8	·00746	7	·00584	6
8	·01921	11	·01459	10	·01121	9	·00872	8	·00684	7
9	·02196	13	·01672	11	·01289	10	·01005	9	·00791	8
10	·02483	14	·01897	13	·01467	11	·01148	10	·00906	9
11	·02785	16	·02135	14	·01657	13	·01300	11	·01030	10
12	·03102	17	·02387	16	·01859	14	·01464	13	·01164	11
13	·03437	19	·02655	17	·02076	16	·01641	14	·01310	13
14	·03790	21	·02940	19	·02309	17	·01832	16	·01468	14
15	·04165	23	·03245	21	·02558	19	·02039	17	·01641	16
16	·04562	24	·03570	23	·02827	21	·02263	19	·01829	17
17	·04983	26	·03918	25	·03117	23	·02506	21	·02034	19
18	·05431	28	·04290	27	·03430	25	·02770	23	·02258	21
19	·05906	30	·04689	29	·03767	27	·03057	25	·02503	23
20	·06412	32	·05117	31	·04130	29	·03368	27	·02771	25
21	·06951	34	·05575	33	·04523	31	·03706	29	·03063	27
22	·07524	37	·06067	35	·04946	33	·04073	31	·03383	29
23	·08133	39	·06593	38	·05403	36	·04471	34	·03731	32
24	·08782	41	·07157	40	·05896	38	·04903	36	·04111	34
25	·09471	44	·07761	42	·06427	41	·05371	39	·04526	37
26	·10205	46	·08407	45	·06999	44	·05878	42	·04977	40
27	·10984	48	·09099	48	·07614	46	·06427	45	·05468	43
28	·11811	51	·09838	50	·08276	49	·07021	48	·06002	46
29	·12689	53	·10628	53	·08987	52	·07663	51	·06581	49
30	·13620	55	·11470	56	·09750	55	·08355	54	·07210	53
31	·14606	58	·12369	59	·10569	58	·09102	57	·07892	56
32	·15650	60	·13326	61	·11446	61	·09906	61	·08630	60
33	·16755	62	·14344	64	·12384	65	·10771	64	·09427	63
34	·17921	65	·15426	67	·13387	68	·11700	68	·10288	67
35	·19153	67	·16576	69	·14458	71	·12696	71	·11216	71
36	·20451	69	·17794	72	·15599	74	·13764	75	·12215	75
37	·21819	71	·19085	75	·16814	77	·14907	78	·13288	79
38	·23257	73	·20450	77	·18107	80	·16128	82	·14441	83
39	·24768	74	·21892	79	·19479	83	·17431	85	·15676	86
40	·26354	76	·23414	81	·20933	85	·18818	88	·16997	90
41	·28015	77	·25017	83	·22473	88	·20294	91	·18409	94
42	·29754	78	·26703	85	·24101	90	·21861	94	·19914	97
43	·31570	79	·28474	86	·25819	92	·23522	97	·21517	100
44	·33465	79	·30331	87	·27629	94	·25280	99	·23220	103
45	0·35440	79	0·32275	88	0·29534	95	0·27137	101	0·25026	106

TABLE XV—*continued.*

$se_1(x,\ \theta)$. $\theta=6$ to $\theta=10$.

x	$\theta=6$ $b_1=-7\cdot36391$		$\theta=7$ $b_1=-8\cdot97120$		$\theta=8$ $b_1=-10\cdot60537$		$\theta=9$ $b_1=-12\cdot26166$		$\theta=10$ $b_1=-13\cdot93655$	
		Δ^2		Δ^2		Δ^2		Δ^2		Δ^2
45°	0·35440	79	0·32275	88	0·29534	95	0·27137	101	0·25026	106
46	·37494	79	·34308	89	·31533	96	·29096	103	·26939	109
47	·39627	79	·36429	89	·33629	97	·31157	104	·28960	111
48	·41840	78	·38639	88	·35822	97	·33323	105	·31092	112
49	·44129	77	·40937	88	·38113	97	·35595	106	·33336	113
50	·46496	75	·43323	86	·40500	97	·37972	106	·35693	114
51	·48937	73	·45795	84	·42985	95	·40455	105	·38164	114
52	·51451	70	·48352	82	·45564	93	·43042	104	·40749	113
53	·54035	67	·50991	79	·48237	91	·45734	102	·43447	111
54	·56685	63	·53709	76	·51001	88	·48527	99	·46256	109
55	·59398	59	·56503	72	·53853	84	·51419	96	·49174	106
56	·62171	54	·59369	67	·56789	80	·54406	91	·52198	102
57	·64997	49	·62303	62	·59805	75	·57485	86	·55325	98
58	·67873	43	·65298	56	·62895	69	·60650	81	·58549	92
59	·70792	37	·68350	50	·66054	62	·63896	74	·61866	86
60	·73749	31	·71452	43	·69275	55	·67216	67	·65268	78
61	·76736	24	·74596	35	·72552	47	·70603	58	·68748	70
62	·79747	16	·77776	27	·75875	38	·74049	49	·72299	61
63	·82773	8	·80983	18	·79236	29	·77543	40	·75909	50
64	·85807	− 1	·84208	9	·82626	19	·81078	29	·79570	39
65	·88841	− 9	·87442	− 1	·86036	8	·84641	18	·83271	27
66	·91865	− 19	·90675	− 11	·89453	− 3	·88223	6	·86999	15
67	·94870	− 28	·93898	− 22	·92868	− 14	·91810	− 7	·90741	1
68	0·97847	− 38	0·97099	− 33	·96268	− 27	·95390	− 20	·94484	− 13
69	1·00787	− 48	1·00267	− 44	0·99642	− 39	0·98950	− 34	0·98215	− 28
70	1·03679	− 58	1·03391	− 55	1·02977	− 52	1·02477	− 48	1·01918	− 43
71	1·06513	− 68	1·06460	− 67	1·06260	− 65	1·05956	− 62	1·05577	− 59
72	1·09279	− 78	1·09463	− 79	1·09478	− 78	1·09373	− 77	1·09178	− 75
73	1·11967	− 88	1·12387	− 90	1·12618	− 91	1·12713	− 91	1·12705	− 91
74	1·14567	− 98	1·15220	− 102	1·15667	− 104	1·15962	− 106	1·16141	− 107
75	1·17069	− 108	1·17952	− 113	1·18612	− 117	1·19105	− 121	1·19469	− 123
76	1·19462	− 118	1·20570	− 124	1·21439	− 130	1·22127	− 135	1·22675	− 139
77	1·21739	− 127	1·23064	− 135	1·24136	− 143	1·25014	− 149	1·25741	− 155
78	1·23888	− 136	1·25423	− 146	1·26690	− 155	1·27752	− 163	1·28653	− 170
79	1·25902	− 144	1·27636	− 156	1·29089	− 166	1·30327	− 176	1·31395	− 184
80	1·27771	− 152	1·29693	− 165	1·31322	− 177	1·32727	− 188	1·33952	− 198
81	1·29488	− 160	1·31585	− 174	1·33378	− 187	1·34938	− 200	1·36312	− 211
82	1·31046	− 166	1·33302	− 182	1·35247	− 197	1·36950	− 210	1·38460	− 223
83	1·32437	− 173	1·34838	− 189	1·36919	− 205	1·38751	− 220	1·40386	− 234
84	1·33655	− 178	1·36184	− 196	1·38386	− 213	1·40333	− 228	1·42077	− 243
85	1·34696	− 183	1·37334	− 201	1·39640	− 219	1·41687	− 236	1·43526	− 252
86	1·35553	− 187	1·38283	− 206	1·40675	− 224	1·42804	− 242	1·44722	− 259
87	1·36224	− 190	1·39026	− 210	1·41486	− 229	1·43680	− 247	1·45660	− 264
88	1·36706	− 192	1·39559	− 212	1·42069	− 232	1·44309	− 250	1·46334	− 268
89	1·36996	− 193	1·39880	− 214	1·42419	− 234	1·44688	− 252	1·46740	− 270
90	1·37093	− 194	1·39987	− 214	1·42536	− 234	1·44814	− 253	1·46876	− 271

Table XVI.

$ce_1(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta=1$ $a_1=1{\cdot}85911$	Δ^2	$\theta=2$ $a_1=2{\cdot}37920$	Δ^2	$\theta=3$ $a_1=2{\cdot}51904$	Δ^2	$\theta=4$ $a_1=2{\cdot}31801$	Δ^2	$\theta=5$ $a_1=1{\cdot}85819$	Δ^2
0°	0·85660	4	0·68357	34	0·50900	54	0·36343	63	0·25654	64
1	·85662	4	·68374	34	·50927	54	·36374	63	·25686	64
2	·85667	4	·68425	34	·51008	54	·36468	63	·25782	64
3	·85676	3	·68509	33	·51143	54	·36626	63	·25941	64
4	·85689	3	·68626	33	·51332	54	·36846	63	·26164	64
5	·85704	3	·68777	33	·51574	53	·37129	63	·26451	64
6	·85723	3	·68960	32	·51869	53	·37475	63	·26803	65
7	·85744	2	·69176	32	·52217	53	·37884	63	·27219	65
8	·85767	2	·69423	31	·52618	52	·38355	63	·27701	65
9	·85791	1	·69701	30	·53070	52	·38890	63	·28247	66
10	·85817	1	·70009	29	·53575	51	·39487	63	·28860	66
11	·85844	0	·70347	28	·54130	50	·40146	62	·29539	67
12	·85870	− 1	·70713	27	·54735	49	·40868	62	·30285	67
13	·85895	− 2	·71107	26	·55390	48	·41652	62	·31097	68
14	·85919	− 2	·71527	25	·56093	47	·42498	61	·31978	68
15	·85941	− 3	·71972	24	·56844	46	·43406	61	·32926	68
16	·85959	− 4	·72441	22	·57641	45	·44374	60	·33942	68
17	·85972	− 5	·72932	21	·58483	44	·45403	60	·35027	69
18	·85981	− 6	·73444	19	·59369	42	·46491	59	·36181	69
19	·85983	− 7	·73975	17	·60297	41	·47638	58	·37403	69
20	·85978	− 9	·74524	16	·61266	39	·48843	57	·38694	68
21	·85964	− 10	·75088	14	·62273	37	·50105	55	·40053	68
22	·85940	− 11	·75666	11	·63317	35	·51422	54	·41480	67
23	·85906	− 12	·76255	9	·64396	32	·52792	52	·42975	67
24	·85859	− 14	·76853	7	·65507	30	·54215	50	·44536	66
25	·85798	− 15	·77459	5	·66648	27	·55688	48	·46163	64
26	·85723	− 16	·78069	2	·67816	24	·57209	45	·47854	63
27	·85631	− 18	·78681	− 1	·69009	21	·58775	43	·49607	61
28	·85521	− 19	·79292	− 3	·70222	18	·60384	40	·51421	58
29	·85392	− 21	·79900	− 6	·71454	14	·62033	36	·53294	56
30	·85243	− 22	·80501	− 9	·72700	11	·63718	33	·55223	53
31	·85070	− 24	·81093	− 12	·73956	7	·65435	29	·57204	49
32	·84874	− 25	·81673	− 16	·75219	3	·67181	24	·59235	46
33	·84653	− 27	·82237	− 19	·76485	− 2	·68951	20	·61311	41
34	·84405	− 29	·82782	− 22	·77749	− 6	·70741	15	·63429	36
35	·84128	− 30	·83306	− 26	·79006	− 11	·72546	9	·65583	31
36	·83821	− 32	·83803	− 29	·80252	− 16	·74360	3	·67768	25
37	·83482	− 33	·84271	− 33	·81482	− 21	·76177	− 3	·69978	19
38	·83110	− 35	·84707	− 36	·82690	− 27	·77992	− 9	·72208	12
39	·82703	− 36	·85106	− 40	·83872	− 32	·79797	− 16	·74450	5
40	·82260	− 38	·85465	− 44	·85020	− 38	·81587	− 23	·76697	− 3
41	·81779	− 39	·85780	− 48	·86131	− 44	·83353	− 31	·78941	− 11
42	·81259	− 41	·86047	− 51	·87198	− 50	·85089	− 38	·81173	− 20
43	·80698	− 42	·86263	− 55	·88214	− 56	·86787	− 47	·83386	− 30
44	·80094	− 44	·86424	− 59	·89174	− 63	·88437	− 55	·85569	− 39
45	0·79447	− 45	0·86526	− 63	0·90070	− 69	0·90033	− 64	0·87713	− 50

TABLE XVI—*continued.*

$ce_1(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta=1$ $a_1=1{\cdot}85911$	Δ^2	$\theta=2$ $a_1=2{\cdot}37920$	Δ^2	$\theta=3$ $a_1=2{\cdot}51904$	Δ^2	$\theta=4$ $a_1=2{\cdot}31801$	Δ^2	$\theta=5$ $a_1=1{\cdot}85819$	Δ^2
45°	0·79447	−45	0·86526	−63	0·90070	−69	0·90033	−64	0·87713	−50
46	·78755	−46	·86565	−66	·90898	−76	·91566	−72	·89807	−60
47	·78017	−47	·86538	−70	·91651	−82	·93025	−82	·91840	−72
48	·77231	−49	·86441	−74	·92321	−88	·94404	−91	·93802	−83
49	·76396	−50	·86270	−77	·92903	−95	·95691	−100	·95681	−95
50	·75512	−51	·86022	−81	·93390	−101	·96879	−109	·97466	−107
51	·74577	−52	·85693	−84	·93775	−108	·97957	−119	0·99143	−119
52	·73590	−53	·85281	−87	·94053	−114	·98917	−128	1·00702	−131
53	·72551	−53	·84781	−90	·94218	−120	0·99748	−137	1·02129	−144
54	·71459	−54	·84192	−93	·94262	−126	1·00442	−147	1·03413	−156
55	·70312	−54	·83510	−95	·94181	−131	1·00989	−155	1·04541	−168
56	·69111	−55	·82733	−98	·93969	−136	1·01381	−164	1·05501	−180
57	·67856	−55	·81858	−100	·93621	−141	1·01609	−172	1·06281	−192
58	·66545	−55	·80883	−102	·93131	−146	1·01664	−180	1·06869	−203
59	·65178	−56	·79806	−103	·92495	−150	1·01539	−188	1·07254	−214
60	·63756	−56	·78626	−105	·91708	−154	1·01226	−195	1·07425	−224
61	·62279	−55	·77341	−106	·90768	−158	1·00718	−201	1·07371	−234
62	·60746	−55	·75950	−107	·89670	−160	1·00009	−207	1·07084	−243
63	·59158	−55	·74452	−107	·88412	−163	0·99094	−212	1·06553	−251
64	·57516	−54	·72847	−107	·86990	−165	·97966	−216	1·05772	−258
65	·55819	−53	·71135	−107	·85405	−166	·96623	−220	1·04732	−264
66	·54069	−53	·69315	−107	·83653	−166	·95060	−222	1·03428	−269
67	·52266	−52	·67389	−106	·81735	−166	·93275	−224	1·01855	−273
68	·50411	−51	·65357	−105	·79651	−166	·91266	−224	1·00009	−276
69	·48506	−49	·63220	−103	·77400	−164	·89033	−224	0·97887	−277
70	·46551	−48	·60980	−101	·74986	−162	·86575	−222	·95488	−277
71	·44549	−47	·58639	−99	·72409	−160	·83896	−220	·92812	−275
72	·42499	−45	·56200	−96	·69672	−156	·80996	−217	·89862	−272
73	·40405	−43	·53664	−93	·66778	−152	·77879	−212	·86639	−268
74	·38268	−41	·51035	−90	·63733	−148	·74550	−207	·83148	−262
75	·36089	−39	·48316	−86	·60539	−142	·71014	−200	·79396	−254
76	·33870	−37	·45512	−82	·57204	−136	·67278	−192	·75389	−245
77	·31614	−35	·42625	−78	·53732	−129	·63350	−183	·71137	−235
78	·29323	−33	·39661	−73	·50131	−122	·59239	−174	·66650	−223
79	·26999	−31	·36624	−68	·46407	−114	·54954	−163	·61940	−210
80	·24645	−28	·33520	−63	·42570	−106	·50506	−151	·57020	−195
81	·22262	−26	·30352	−57	·38626	−97	·45907	−139	·51905	−180
82	·19854	−23	·27128	−51	·34586	−87	·41170	−125	·46610	−163
83	·17423	−20	·23852	−45	·30459	−77	·36306	−111	·41152	−145
84	·14972	−17	·20530	−39	·26255	−67	·31332	−97	·35550	−126
85	·12503	−15	·17169	−33	·21983	−56	·26260	−82	·29821	−106
86	·10020	−12	·13775	−27	·17655	−45	·21107	−66	·23986	−86
87	·07526	−9	·10355	−20	·13281	−34	·15889	−50	·18066	−65
88	·05022	−6	·06915	−13	·08873	−23	·10620	−33	·12080	−44
89	·02512	−3	·03461	−7	·04442	−12	·05318	−17	·06051	−22
90	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0

TABLE XVI—*continued.*

$ce_1(x, \theta)$. $\theta=6$ to $\theta=10$.

x	$\theta=6$ $a_1=1{\cdot}21428$	Δ^2	$\theta=7$ $a_1=0{\cdot}43835$	Δ^2	$\theta=8$ $a_1=-0{\cdot}43594$	Δ^2	$\theta=9$ $a_1=-1{\cdot}38670$	Δ^2	$\theta=10$ $a_1=-2{\cdot}39914$	Δ^2
0°	0·18230	60	0·13133	54	0·09605	48	0·07128	42	0·05360	37
1	·18260	60	·13160	54	·09629	48	·07149	42	·05378	37
2	·18350	60	·13241	55	·09701	48	·07212	43	·05433	37
3	·18500	60	·13377	55	·09822	49	·07318	43	·05525	38
4	·18711	61	·13568	56	·09992	49	·07467	44	·05655	38
5	·18982	61	·13815	56	·10211	50	·07659	45	·05822	39
6	·19314	62	·14118	57	·10481	51	·07896	46	·06029	40
7	·19709	63	·14477	58	·10802	53	·08179	47	·06277	42
8	·20166	63	·14895	59	·11175	54	·08509	48	·06566	43
9	·20687	64	·15372	60	·11602	55	·08887	50	·06898	45
10	·21272	65	·15909	62	·12085	57	·09316	52	·07276	47
11	·21922	66	·16508	63	·12625	59	·09796	54	·07700	49
12	·22638	67	·17169	65	·13223	61	·10330	56	·08173	51
13	·23421	68	·17896	66	·13882	63	·10921	58	·08698	54
14	·24273	69	·18689	68	·14604	65	·11569	61	·09277	57
15	·25194	70	·19549	70	·15390	67	·12279	64	·09913	60
16	·26186	71	·20479	71	·16244	69	·13052	66	·10608	63
17	·27249	72	·21481	73	·17167	72	·13892	69	·11366	66
18	·28384	73	·22555	75	·18161	74	·14800	72	·12190	69
19	·29593	74	·23704	76	·19230	76	·15781	75	·13082	72
20	·30876	75	·24930	78	·20375	79	·16836	78	·14047	76
21	·32235	76	·26234	80	·21598	81	·17969	81	·15088	79
22	·33668	76	·27618	81	·22903	83	·19183	84	·16208	83
23	·35178	76	·29082	82	·24291	85	·20481	87	·17411	86
24	·36765	76	·30629	83	·25765	87	·21865	89	·18700	90
25	·38427	76	·32259	84	·27326	89	·23339	92	·20079	93
26	·40166	76	·33973	85	·28976	91	·24905	95	·21552	97
27	·41980	75	·35772	85	·30717	92	·26565	97	·23121	100
28	·43869	73	·37656	85	·32550	93	·28322	99	·24789	103
29	·45831	72	·39624	84	·34476	94	·30178	100	·26561	105
30	·47865	70	·41677	83	·36495	94	·32135	102	·28437	107
31	·49969	67	·43813	82	·38609	93	·34193	102	·30421	109
32	·52139	64	·46031	80	·40815	93	·36353	103	·32514	111
33	·54375	61	·48328	77	·43115	91	·38616	102	·34718	111
34	·56670	57	·50703	74	·45505	89	·40982	101	·37033	112
35	·59023	52	·53152	70	·47985	86	·43449	100	·39459	111
36	·61427	47	·55672	66	·50551	83	·46016	97	·41996	110
37	·63878	41	·58257	61	·53199	78	·48680	94	·44644	108
38	·66370	34	·60903	55	·55926	73	·51438	90	·47398	104
39	·68895	27	·63604	48	·58727	67	·54286	85	·50257	100
40	·71448	19	·66352	40	·61594	60	·57219	79	·53217	95
41	·74019	10	·69141	32	·64522	52	·60230	71	·56271	89
42	·76601	1	·71961	22	·67502	43	·63313	63	·59414	81
43	·79183	−9	·74804	12	·70526	33	·66458	53	·62638	72
44	·81757	−20	·77659	1	·73582	22	·69657	43	·65935	62
45	0·84310	−31	0·80515	−11	0·76661	10	0·72899	31	0·69294	51

TABLE XVI—*continued.*

$ce_1(x, \theta)$. $\theta=6$ to $\theta=10$.

x	$\theta=6$ $a_1=1\cdot21428$	Δ^2	$\theta=7$ $a_1=0\cdot43835$	Δ^2	$\theta=8$ $a_1=-0\cdot43594$	Δ^2	$\theta=9$ $a_1=-1\cdot38670$	Δ^2	$\theta=10$ $a_1=-2\cdot39914$	Δ^2
° 45	0·84310	− 31	0·80515	− 11	0·76661	10	0·72899	31	0·69294	51
46	·86832	− 43	·83361	− 24	·79750	− 3	·76171	17	·72703	38
47	·89311	− 56	·86182	− 37	·82836	− 17	·79461	3	·76150	23
48	·91734	− 69	·88967	− 52	·85904	− 32	·82754	− 13	·79620	7
49	·94089	− 83	·91700	− 67	·88941	− 49	·86034	− 29	·83097	− 10
50	·96360	− 97	·94366	− 83	·91928	− 66	·89285	− 47	·86565	− 29
51	0·98535	− 111	·96949	− 99	·94850	− 84	·92488	− 66	·90004	− 48
52	1·00598	− 126	0·99434	− 116	0·97688	− 102	·95625	− 87	·93394	− 69
53	1·02535	− 141	1·01803	− 133	1·00424	− 122	0·98676	− 108	·96716	− 92
54	1·04330	− 156	1·04038	− 151	1·03039	− 142	1·01619	− 129	0·99945	− 115
55	1·05970	− 172	1·06123	− 169	1·05512	− 162	1·04432	− 152	1·03059	− 139
56	1·07437	− 187	1·08038	− 187	1·07823	− 183	1·07094	− 175	1·06034	− 165
57	1·08718	− 202	1·09767	− 205	1·09951	− 203	1·09581	− 198	1·08844	− 190
58	1·09797	− 217	1·11290	− 223	1·11876	− 224	1·11870	− 222	1·11464	− 216
59	1·10660	− 231	1·12591	− 240	1·13577	− 245	1·13938	− 245	1·13868	− 242
60	1·11291	− 245	1·13651	− 257	1·15033	− 265	1·15760	− 268	1·16029	− 269
61	1·11678	− 258	1·14453	− 274	1·16224	− 285	1·17314	− 291	1·17922	− 294
62	1·11808	− 270	1·14982	− 289	1·17131	− 304	1·18576	− 313	1·19520	− 320
63	1·11668	− 281	1·15222	− 304	1·17734	− 322	1·19525	− 335	1·20798	− 344
64	1·11246	− 291	1·15157	− 318	1·18015	− 338	1·20140	− 355	1·21732	− 367
65	1·10533	− 300	1·14774	− 330	1·17958	− 354	1·20400	− 373	1·22299	− 389
66	1·09520	− 308	1·14062	− 341	1·17547	− 368	1·20286	− 390	1·22476	− 410
67	1·08199	− 315	1·13009	− 350	1·16769	− 380	1·19782	− 405	1·22244	− 428
68	1·06562	− 319	1·11607	− 357	1·15611	− 390	1·18873	− 418	1·21584	− 444
69	1·04607	− 323	1·09847	− 363	1·14063	− 398	1·17545	− 429	1·20480	− 457
70	1·02328	− 324	1·07725	− 366	1·12117	− 404	1·15788	− 437	1·18919	− 468
71	0·99726	− 324	1·05236	− 368	1·09768	− 407	1·13594	− 443	1·16890	− 475
72	·96799	− 322	1·02380	− 367	1·07012	− 408	1·10958	− 445	1·14385	− 480
73	·93550	− 318	0·99157	− 364	1·03849	− 406	1·07876	− 445	1·11401	− 481
74	·89984	− 312	·95571	− 358	1·00280	− 401	1·04349	− 441	1·07936	− 478
75	·86105	− 304	·91627	− 350	0·96310	− 393	1·00382	− 434	1·03992	− 472
76	·81922	− 295	·87332	− 340	·91946	− 383	0·95981	− 424	0·99570	− 463
77	·77445	− 283	·82696	− 328	·87199	− 371	·91156	− 410	·94697	− 449
78	·72685	− 269	·77733	− 313	·82082	− 354	·85921	− 394	·89369	− 432
79	·67655	− 254	·72457	− 296	·76611	− 336	·80291	− 374	·83610	− 411
80	·62371	− 237	·66885	− 277	·70804	− 315	·74288	− 351	·77440	− 386
81	·56850	− 219	·61036	− 256	·64682	− 291	·67933	− 325	·70883	− 359
82	·51110	− 198	·54931	− 232	·58270	− 265	·61254	− 297	·63967	− 328
83	·45172	− 177	·48595	− 207	·51592	− 237	·54277	− 266	·56724	− 294
84	·39057	− 154	·42050	− 181	·44677	− 207	·47035	− 232	·49187	− 257
85	·32788	− 130	·35325	− 153	·37555	− 175	·39560	− 197	·41394	− 218
86	·26389	− 105	·28447	− 124	·30258	− 142	·31889	− 160	·33382	− 177
87	·19885	− 80	·21445	− 94	·22820	− 108	·24058	− 121	·25193	− 134
88	·13301	− 53	·14349	− 63	·15273	− 72	·16107	− 81	·16871	− 90
89	·06664	− 27	·07190	− 32	·07655	− 36	·08074	− 41	·08458	− 45
90	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0

Table XVII.

$se_2(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta=1$ $b_2=3{\cdot}91702$		$\theta=2$ $b_2=3{\cdot}67223$		$\theta=3$ $b_2=3{\cdot}27692$		$\theta=4$ $b_2=2{\cdot}74688$		$\theta=5$ $b_2=2{\cdot}09946$	
°		Δ^2		Δ^2		Δ^2		Δ^2		Δ^2
0	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0
1	·02928	−2	·02416	0	·01969	2	·01591	3	·01280	3
2	·05853	−3	·04832	0	·03940	3	·03185	5	·02563	6
3	·08776	−5	·07249	1	·05914	5	·04784	8	·03853	9
4	·11693	−7	·09666	1	·07893	6	·06391	10	·05151	12
5	·14603	−9	·12085	1	·09878	8	·08008	13	·06462	15
6	·17505	−10	·14504	1	·11871	9	·09637	15	·07788	18
7	·20396	−12	·16924	1	·13873	11	·11281	17	·09132	21
8	·23275	−14	·19346	1	·15887	12	·12942	19	·10498	24
9	·26140	−16	·21768	1	·17912	13	·14623	22	·11887	27
10	·28989	−18	·24191	1	·19951	14	·16325	24	·13304	30
11	·31819	−20	·26615	0	·22003	15	·18052	26	·14749	32
12	·34630	−22	·29039	0	·24072	16	·19803	28	·16227	35
13	·37419	−24	·31463	−1	·26156	17	·21583	29	·17740	37
14	·40183	−26	·33886	−1	·28257	17	·23391	31	·19290	40
15	·42922	−29	·36308	−2	·30376	18	·25231	32	·20880	42
16	·45631	−31	·38728	−3	·32512	18	·27102	33	·22511	44
17	·48310	−33	·41144	−4	·34666	18	·29007	34	·24186	46
18	·50955	−36	·43555	−6	·36838	18	·30946	35	·25907	47
19	·53565	−38	·45961	−7	·39028	17	·32920	36	·27675	49
20	·56137	−41	·48360	−9	·41235	17	·34930	36	·29491	50
21	·58668	−43	·50749	−11	·43459	16	·36976	36	·31358	51
22	·61155	−46	·53128	−13	·45698	14	·39057	36	·33275	52
23	·63596	−49	·55494	−15	·47952	13	·41175	35	·35244	52
24	·65988	−52	·57845	−18	·50218	11	·43327	34	·37265	52
25	·68329	−55	·60178	−20	·52496	9	·45515	33	·39339	52
26	·70614	−58	·62491	−23	·54784	7	·47735	32	·41464	51
27	·72842	−61	·64781	−26	·57078	4	·49987	30	·43640	50
28	·75009	−64	·67045	−29	·59376	1	·52268	27	·45866	49
29	·77112	−67	·69279	−33	·61676	−2	·54578	25	·48141	47
30	·79148	−70	·71481	−36	·63974	−5	·56911	22	·50463	45
31	·81114	−74	·73647	−40	·66266	−9	·59267	18	·52830	42
32	·83006	−77	·75772	−44	·68550	−14	·61641	14	·55238	38
33	·84821	−80	·77852	−49	·70819	−18	·64029	10	·57685	34
34	·86556	−84	·79885	−53	·73071	−23	·66427	5	·60166	30
35	·88207	−87	·81864	−57	·75300	−28	·68830	0	·62678	25
36	·89772	−90	·83786	−62	·77500	−34	·71232	−6	·65214	20
37	·91246	−94	·85645	−67	·79667	−39	·73629	−12	·67770	14
38	·92627	−97	·87438	−72	·81795	−46	·76013	−19	·70340	7
39	·93911	−100	·89159	−77	·83877	−52	·78379	−26	·72917	0
40	·95094	−103	·90802	−82	·85907	−59	·80719	−33	·75493	−8
41	·96175	−107	·92363	−88	·87878	−65	·83025	−41	·78060	−17
42	·97149	−110	·93836	−93	·89785	−72	·85290	−50	·80611	−26
43	·98013	−113	·95217	−98	·91619	−80	·87505	−58	·83136	−36
44	·98764	−116	·96499	−104	·93373	−87	·89662	−67	·85625	−46
45	0·99399	−119	0·97677	−109	0·95039	−95	0·91751	−77	0·88069	−56

TABLE XVII—*continued.*

$se_2(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta=1$ $b_2=3·91702$	Δ^2	$\theta=2$ $b_2=3·67223$	Δ^2	$\theta=3$ $b_2=3·27692$	Δ^2	$\theta=4$ $b_2=2·74688$	Δ^2	$\theta=5$ $b_2=2·09946$	Δ^2
° 45	0·99399	−119	0·97677	−109	0·95039	−95	0·91751	−77	0·88069	−50
46	0·99916	−121	·98746	−115	·96611	−103	·93764	−86	·90456	−68
47	1·00312	−124	0·99700	−120	·98080	−110	·95690	−96	·92775	−79
48	1·00584	−126	1·00534	−125	0·99439	−118	·97520	−106	·95016	−91
49	1·00729	−129	1·01243	−130	1·00680	−126	0·99244	−117	·97166	−103
50	1·00745	−131	1·01821	−135	1·01794	−134	1·00850	−127	0·99212	−116
51	1·00631	−133	1·02264	−140	1·02775	−142	1·02330	−137	1·01142	−129
52	1·00384	−135	1·02567	−145	1·03614	−149	1·03672	−148	1·02943	−142
53	1·00002	−136	1·02725	−149	1·04303	−157	1·04866	−158	1·04603	−155
54	0·99484	−137	1·02734	−154	1·04836	−164	1·05902	−168	1·06108	−168
55	·98829	−138	1·02588	−157	1·05206	−171	1·06770	−178	1·07446	−181
56	·98036	−139	1·02286	−161	1·05404	−177	1·07460	−188	1·08602	−193
57	·97103	−140	1·01822	−164	1·05425	−184	1·07961	−197	1·09566	−206
58	·96030	−140	1·01194	−167	1·05263	−189	1·08265	−206	1·10323	−218
59	·94816	−140	1·00399	−170	1·04911	−195	1·08363	−215	1·10863	−229
60	·93463	−140	0·99434	−172	1·04364	−200	1·08247	−222	1·11173	−240
61	·91970	−139	·98297	−173	1·03618	−204	1·07908	−230	1·11243	−251
62	·90337	−139	·96987	−175	1·02668	−207	1·07339	−236	1·11062	−260
63	·88566	−137	·95503	−175	1·01511	−210	1·06534	−242	1·10622	−269
64	·86657	−136	·93843	−175	1·00144	−213	1·05488	−246	1·09912	−276
65	·84613	−134	·92008	−175	0·98564	−214	1·04195	−250	1·08926	−283
66	·82434	−132	·89997	−174	·96770	−215	1·02652	−253	1·07657	−288
67	·80124	−129	·87813	−173	·94760	−215	1·00856	−255	1·06100	−292
68	·77684	−127	·85457	−170	·92537	−214	0·98805	−256	1·04251	−295
69	·75117	−124	·82930	−168	·90099	−212	·96497	−255	1·02107	−296
70	·72427	−120	·80235	−165	·87449	−210	·93935	−254	0·99667	−296
71	·69616	−116	·77375	−161	·84589	−206	·91119	−251	·96930	−295
72	·66689	−112	·74355	−156	·81523	−202	·88051	−247	·93899	−291
73	·63650	−108	·71178	−151	·78255	−197	·84736	−242	·90576	−287
74	·60502	−103	·67850	−146	·74791	−191	·81180	−236	·86967	−280
75	·57251	−98	·64376	−140	·71136	−184	·77387	−228	·83078	−272
76	·53902	−93	·60762	−133	·67297	−176	·73367	−219	·78916	−263
77	·50459	−88	·57015	−126	·63283	−167	·69128	−209	·74492	−251
78	·46929	−82	·53141	−119	·59102	−158	·64679	−198	·69817	−239
79	·43316	−76	·49149	−110	·54763	−147	·60033	−186	·64903	−225
80	·39627	−69	·45047	−102	·50277	−136	·55200	−173	·59764	−209
81	·35869	−64	·40842	−93	·45654	−125	·50196	−158	·54416	−192
82	·32047	−57	·36545	−84	·40907	−113	·45032	−143	·48875	−174
83	·28168	−50	·32164	−74	·36046	−100	·39726	−127	·43160	−155
84	·24238	−43	·27709	−64	·31086	−87	·34293	−110	·37290	−135
85	·20265	−36	·23190	−54	·26040	−73	·28749	−93	·31286	−114
86	·16256	−29	·18617	−43	·20920	−59	·23112	−75	·25167	−92
87	·12218	−22	·14001	−33	·15742	−44	·17401	−57	·18957	−70
88	·08157	−15	·09352	−22	·10519	−30	·11632	−38	·12677	−47
89	·04082	−7	·04681	−11	·05267	−15	·05826	−19	·06350	−23
90	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0

TABLE XVII—*continued.*

$se_2(x, \theta)$. $\theta=6$ to $\theta=10$.

x	$\theta=6$ $b_2=1\cdot35138$	Δ^2	$\theta=7$ $b_2=0\cdot51755$	Δ^2	$\theta=8$ $b_2=-0\cdot38936$	Δ^2	$\theta=9$ $b_2=-1\cdot35881$	Δ^2	$\theta=10$ $b_2=-2\cdot38216$	Δ^2
°										
0	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0
1	·01028	3	·00826	3	·00665	3	·00537	3	·00435	3
2	·02059	7	·01655	7	·01333	7	·01077	6	·00873	6
3	·03097	10	·02491	10	·02008	10	·01623	10	·01316	9
4	·04145	13	·03337	14	·02692	13	·02179	13	·01769	12
5	·05207	17	·04197	17	·03391	17	·02747	16	·02233	15
6	·06284	20	·05074	20	·04105	20	·03331	19	·02713	18
7	·07382	23	·05971	24	·04840	23	·03935	23	·03211	21
8	·08503	26	·06892	27	·05598	27	·04561	26	·03730	25
9	·09650	30	·07840	31	·06384	30	·05214	29	·04273	28
10	·10827	33	·08819	34	·07199	34	·05895	33	·04845	31
11	·12037	36	·09831	37	·08048	37	·06609	36	·05447	35
12	·13282	39	·10881	41	·08935	41	·07360	40	·06085	38
13	·14566	42	·11972	44	·09863	44	·08151	44	·06761	42
14	·15892	45	·13106	47	·10834	48	·08985	47	·07478	46
15	·17263	48	·14288	51	·11854	51	·09866	51	·08242	49
16	·18682	50	·15521	54	·12925	55	·10799	55	·09055	53
17	·20150	53	·16807	57	·14052	58	·11786	58	·09921	57
18	·21672	55	·18150	60	·15236	62	·12831	62	·10844	61
19	·23248	57	·19552	63	·16483	65	·13939	66	·11829	65
20	·24882	59	·21017	65	·17795	69	·15112	70	·12879	69
21	·26575	61	·22548	68	·19175	72	·16356	73	·13999	74
22	·28330	63	·24146	70	·20627	75	·17673	77	·15192	78
23	·30147	64	·25815	72	·22154	78	·19066	81	·16463	82
24	·32029	65	·27556	74	·23758	80	·20541	84	·17815	86
25	·33976	66	·29371	76	·25443	83	·22099	87	·19253	89
26	·35988	66	·31262	77	·27210	85	·23744	90	·20780	93
27	·38067	66	·33231	78	·29062	87	·25479	93	·22401	96
28	·40211	66	·35277	79	·31001	88	·27307	95	·24117	100
29	·42421	65	·37402	79	·33028	89	·29230	97	·25934	103
30	·44696	63	·39605	78	·35144	90	·31250	99	·27852	105
31	·47035	61	·41887	77	·37350	90	·33368	100	·29876	107
32	·49434	59	·44246	76	·39646	89	·35586	100	·32007	109
33	·51892	56	·46680	74	·42031	88	·37904	100	·34247	110
34	·54406	52	·49188	71	·44505	86	·40322	99	·36596	110
35	·56972	48	·51767	67	·47065	84	·42840	98	·39056	110
36	·59586	43	·54413	63	·49708	81	·45456	96	·41625	108
37	·62243	37	·57122	58	·52433	77	·48168	93	·44302	106
38	·64936	31	·59889	52	·55234	72	·50972	89	·47086	103
39	·67661	24	·62709	46	·58107	66	·53865	84	·49973	99
40	·70408	16	·65574	38	·61045	59	·56841	78	·52960	94
41	·73172	7	·68477	30	·64043	51	·59896	70	·56042	88
42	·75942	−2	·71410	21	·67091	42	·63020	62	·59211	81
43	·78710	−12	·74364	10	·70181	32	·66207	53	·62461	72
44	·81466	−23	·77328	−1	·73303	21	·69446	42	·65782	62
45	0·84198	−35	0·80291	−13	0·76447	9	0·72727	30	0·69166	50

TABLE XVII—*continued.*

$se_2(x, \theta)$. $\theta=6$ to $\theta=10$.

x	$\theta=6$ $b_2=1{\cdot}35138$		$\theta=7$ $b_2=0{\cdot}51755$		$\theta=8$ $b_2=-0{\cdot}38936$		$\theta=9$ $b_2=-1{\cdot}35881$		$\theta=10$ $b_2=-2{\cdot}38216$	
°		Δ^2		Δ^2		Δ^2		Δ^2		Δ^2
45	0·84198	− 35	0·80291	− 13	0·76447	9	0·72727	30	0·69166	50
46	·86896	− 47	·83242	− 26	·79599	− 4	·76038	17	·72599	37
47	·89547	− 60	·86166	− 39	·82747	− 18	·79366	2	·76069	23
48	·92138	− 73	·89052	− 54	·85877	− 34	·82696	− 13	·79563	7
49	·94656	− 87	·91884	− 69	·88973	− 50	·86013	− 30	·83063	− 10
50	·97087	− 102	·94646	− 85	·92020	− 67	·89300	− 48	·86553	− 29
51	0·99416	− 117	·97324	− 102	·94999	− 85	·92539	− 67	·90014	− 49
52	1·01629	− 132	0·99900	− 119	0·97893	− 104	·95711	− 87	·93426	− 70
53	1·03709	− 147	1·02357	− 136	1·00684	− 123	0·98795	− 108	0·96768	− 92
54	1·05643	− 163	1·04677	− 154	1·03350	− 143	1·01770	− 130	1·00018	− 116
55	1·07414	− 178	1·06844	− 173	1·05874	− 164	1·04616	− 153	1·03152	− 140
56	1·09006	− 194	1·08837	− 191	1·08234	− 185	1·07308	− 176	1·06146	− 165
57	1·10405	− 210	1·10639	− 209	1·10408	− 206	1·09824	− 199	1·08974	− 191
58	1·11593	− 225	1·12232	− 227	1·12377	− 227	1·12141	− 223	1·11612	− 217
59	1·12557	− 239	1·13598	− 245	1·14120	− 248	1·14235	− 247	1·14033	− 243
60	1·13282	− 254	1·14718	− 263	1·15614	− 268	1·16081	− 270	1·16210	− 270
61	1·13753	− 267	1·15576	− 279	1·16841	− 288	1·17658	− 293	1·18117	− 296
62	1·13957	− 280	1·16154	− 295	1·17780	− 307	1·18942	− 315	1·19729	− 321
63	1·13881	− 291	1·16437	− 310	1·18412	− 325	1·19910	− 337	1·21020	− 345
64	1·13513	− 302	1·16410	− 324	1·18719	− 342	1·20542	− 357	1·21966	− 369
65	1·12844	− 311	1·16059	− 336	1·18683	− 358	1·20816	− 376	1·22542	− 391
66	1·11863	− 320	1·15372	− 347	1·18290	− 372	1·20715	− 393	1·22728	− 411
67	1·10562	− 326	1·14337	− 357	1·17526	− 384	1·20221	− 408	1·22503	− 429
68	1·08935	− 331	1·12946	− 364	1·16378	− 394	1·19320	− 421	1·21848	− 445
69	1·06977	− 334	1·11191	− 370	1·14836	− 402	1·17997	− 432	1·20748	− 459
70	1·04685	− 336	1·09065	− 373	1·12891	− 408	1·16242	− 440	1·19189	− 470
71	1·02057	− 336	1·06567	− 375	1·10539	− 411	1·14048	− 445	1·17161	− 477
72	0·99092	− 334	1·03694	− 374	1·07775	− 412	1·11408	− 448	1·14655	− 482
73	·95795	− 330	1·00447	− 371	1·04600	− 410	1·08320	− 447	1·11668	− 483
74	·92167	− 323	0·96829	− 365	1·01014	− 405	1·04785	− 444	1·08198	− 480
75	·88216	− 315	·92846	− 357	0·97024	− 398	1·00807	− 437	1·04248	− 474
76	·83950	− 305	·88505	− 347	·92635	− 387	0·96392	− 426	0·99824	− 464
77	·79378	− 293	·83818	− 334	·87859	− 374	·91550	− 413	·94936	− 450
78	·74513	− 279	·78796	− 319	·82709	− 358	·86295	− 396	·89597	− 433
79	·69369	− 264	·73455	− 302	·77200	− 339	·80644	− 376	·83825	− 412
80	·63961	− 246	·67813	− 282	·71352	− 318	·74617	− 353	·77640	− 388
81	·58307	− 227	·61887	− 261	·65186	− 294	·68236	− 327	·71067	− 360
82	·52427	− 206	·55702	− 237	·58726	− 268	·61528	− 299	64135	− 329
83	·46340	− 183	·49279	− 212	·51998	− 240	·54522	− 267	56873	− 295
84	·40071	− 160	·42645	− 184	·45030	− 209	·47248	− 234	·49317	− 258
85	·33642	− 135	·35826	− 156	·37853	− 177	·39740	− 198	·41504	− 219
86	·27078	− 109	·28851	− 126	·30499	− 143	·32034	− 161	·33471	− 177
87	·20405	− 83	·21750	− 96	·23001	− 109	·24168	− 122	·25261	− 135
88	·13650	− 55	·14554	− 64	·15395	− 73	·16180	− 82	·16916	− 90
89	·06839	− 28	·07293	− 32	·07716	− 37	·08111	− 41	·08481	− 45
90	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0

TABLE XVIII.

$ce_2(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta=1$ $a_2=4{\cdot}37130$		$\theta=2$ $a_2=5{\cdot}17267$		$\theta=3$ $a_2=6{\cdot}04520$		$\theta=4$ $a_2=6{\cdot}82907$		$\theta=5$ $a_2=7{\cdot}44911$	
		Δ^2		Δ^2		Δ^2		Δ^2		Δ^2
0°	1·08596	−78	1·04883	−37	0·96367	−1	0·85584	31	0·73529	57
1	1·08557	−78	1·04865	−38	·96366	−1	·85599	30	·73558	57
2	1·08439	−78	1·04808	−38	·96364	−2	·85645	30	·73644	57
3	1·08243	−79	1·04714	−38	·96360	−2	·85720	29	·73786	56
4	1·07968	−79	1·04582	−39	·96354	−3	·85825	29	·73984	55
5	1·07615	−79	1·04411	−39	·96345	−4	·85959	27	·74238	54
6	1·07183	−79	1·04201	−40	·96331	−5	·86120	26	·74545	53
7	1·06672	−79	1·03951	−41	·96313	−7	·86307	25	·74906	51
8	1·06082	−79	1·03660	−42	·96288	−8	·86519	23	·75318	50
9	1·05413	−79	1·03328	−43	·96255	−10	·86753	21	·75780	48
10	1·04665	−79	1·02952	−44	·96212	−12	·87008	18	·76289	45
11	1·03837	−80	1·02531	−46	·96156	−14	·87281	16	·76844	43
12	1·02929	−80	1·02065	−47	·96087	−17	·87570	13	·77441	40
13	1·01942	−80	1·01552	−49	·96001	−19	·87872	10	·78078	37
14	1·00875	−80	1·00990	−50	·95897	−22	·88183	6	·78752	33
15	0·99728	−80	1·00377	−52	·95770	−25	·88501	3	·79458	29
16	·98501	−80	0·99713	−54	·95618	−28	·88821	−1	·80194	25
17	·97193	−80	·98994	−56	·95439	−31	·89140	−5	·80955	21
18	·95805	−80	·98219	−58	·95228	−35	·89453	−10	·81737	16
19	·94337	−80	·97386	−60	·94983	−38	·89757	−14	·82535	11
20	·92788	−80	·96493	−62	·94700	−42	·90046	−19	·83344	5
21	·91160	−80	·95539	−64	·94375	−46	·90317	−24	·84158	−1
22	·89451	−80	·94520	−66	·94004	−50	·90562	−30	·84971	−7
23	·87662	−80	·93435	−68	·93584	−54	·90779	−35	·85778	−13
24	·85793	−79	·92282	−70	·93111	−58	·90960	−41	·86571	−20
25	·83846	−79	·91059	−72	·92579	−62	·91100	−47	·87345	−27
26	·81819	−78	·89764	−74	·91986	−66	·91193	−53	·88091	−35
27	·79714	−78	·88394	−76	·91328	−70	·91233	−59	·88803	−43
28	·77532	−77	·86949	−78	·90599	−74	·91215	−65	·89472	−51
29	·75273	−76	·85426	−79	·89795	−78	·91130	−72	·90091	−59
30	·72938	−75	·83823	−81	·88914	−82	·90974	−78	·90650	−68
31	·70528	−74	·82140	−82	·87950	−86	·90740	−85	·91142	−76
32	·68044	−72	·80374	−84	·86899	−90	·90420	−91	·91557	−86
33	·65488	−71	·78524	−85	·85758	−94	·90010	−98	·91887	−95
34	·62861	−69	·76590	−86	·84523	−98	·89501	−104	·92122	−104
35	·60164	−68	·74569	−86	·83191	−101	·88887	−111	·92254	−113
36	·57400	−66	·72463	−87	·81757	−104	·88163	−117	·92272	−123
37	·54570	−63	·70269	−87	·80218	−107	·87322	−123	·92167	−132
38	·51677	−61	·67989	−87	·78573	−110	·86357	−129	·91931	−141
39	·48723	−59	·65621	−87	·76817	−112	·85265	−134	·91554	−150
40	·45710	−56	·63167	−86	·74949	−114	·84037	−139	·91027	−158
41	·42641	−53	·60626	−85	·72967	−116	·82671	−144	·90342	−167
42	·39519	−50	·58001	−84	·70870	−117	·81161	−148	·89490	−175
43	·36347	−47	·55291	−82	·68655	−118	·79503	−152	·88464	−182
44	·33128	−43	·52499	−80	·66323	−118	·77693	−155	·87256	−189
45	0·29865	−40	0·49626	−78	0·63872	−118	0·75727	−157	0·85859	−195

TABLE XVIII—*continued.*

$ce_2(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta=1$ $a_2=4{\cdot}37130$		$\theta=2$ $a_2=5{\cdot}17267$		$\theta=3$ $a_2=6{\cdot}04520$		$\theta=4$ $a_2=6{\cdot}82907$		$\theta=5$ $a_2=7{\cdot}44911$	
°		Δ^2		Δ^2		Δ^2		Δ^2		Δ^2
45	0·29865	-40	0·49626	-78	0·63872	-118	0·75727	-157	0·85859	-195
46	·26563	-36	·46675	-75	·61305	-117	·73605	-159	·84267	-200
47	·23225	-32	·43649	-72	·58620	-115	·71323	-160	·82475	-205
48	·19855	-28	·40551	-69	·55820	-113	·68881	-161	·80479	-208
49	·16457	-23	·37383	-65	·52907	-111	·66278	-160	·78274	-211
50	·13036	-19	·34150	-61	·49883	-108	·63514	-159	·75859	-212
51	·09596	-14	·30856	-56	·46751	-104	·60592	-157	·73232	-212
52	·06143	-9	·27506	-51	·43516	-99	·57513	-153	·70392	-211
53	·02680	-4	·24104	-46	·40181	-94	·54281	-149	·67341	-209
54	- ·00787	1	·20656	-40	·36752	-88	·50899	-144	·64080	-206
55	- ·04253	7	·17168	-34	·33234	-82	·47373	-138	·60614	-201
56	- ·07712	12	·13646	-28	·29635	-75	·43709	-131	·56947	-194
57	- ·11159	18	·10096	-21	·25961	-67	·39915	-123	·53086	-186
58	- ·14588	23	·06525	-14	·22220	-59	·35998	-113	·49039	-177
59	- ·17994	28	·02941	-6	·18420	-50	·31967	-103	·44816	-166
60	- ·21371	35	- ·00650	1	·14570	-40	·27834	-92	·40427	-153
61	- ·24713	41	- ·04240	9	·10681	-30	·23609	-80	·35884	-139
62	- ·28014	47	- ·07820	18	·06761	-19	·19304	-66	·31203	-124
63	- ·31268	53	- ·11382	26	·02822	-8	·14934	-52	·26397	-107
64	- ·34469	59	- ·14918	35	- ·01125	3	·10510	-38	·21485	-89
65	- ·37612	65	- ·18420	43	- ·05069	15	·06049	-22	·16483	-70
66	- ·40690	71	- ·21878	52	- ·08997	28	·01566	-6	·11412	-49
67	- ·43697	77	- ·25284	61	- ·12898	40	- ·02922	11	·06292	-28
68	- ·46627	83	- ·28628	70	- ·16759	53	- ·07400	28	·01144	-5
69	- ·49475	88	- ·31902	79	- ·20567	66	- ·11849	46	- ·04008	18
70	- ·52234	94	- ·35098	88	- ·24309	79	- ·16252	64	- ·09143	42
71	- ·54900	99	- ·38205	97	- ·27972	92	- ·20591	82	- ·14235	66
72	- ·57466	105	- ·41215	106	- ·31543	105	- ·24848	101	- ·19261	91
73	- ·59927	110	- ·44119	114	- ·35010	118	- ·29003	119	- ·24196	116
74	- ·62279	115	- ·46910	122	- ·38359	130	- ·33040	137	- ·29014	141
75	- ·64515	120	- ·49578	130	- ·41578	142	- ·36940	155	- ·33692	165
76	- ·66631	125	- ·52116	138	- ·44655	154	- ·40686	172	- ·38205	189
77	- ·68623	129	- ·54516	146	- ·47577	166	- ·44259	189	- ·42528	213
78	- ·70486	133	- ·56770	153	- ·50334	177	- ·47643	205	- ·46638	236
79	- ·72215	137	- ·58871	159	- ·52914	187	- ·50822	220	- ·50513	257
80	- ·73808	141	- ·60814	165	- ·55307	197	- ·53780	235	- ·54131	278
81	- ·75260	144	- ·62590	171	- ·57504	206	- ·56504	248	- ·57470	297
82	- ·76569	147	- ·64196	176	- ·59494	214	- ·58979	261	- ·60514	314
83	- ·77730	149	- ·65626	181	- ·61271	221	- ·61193	272	·63242	330
84	- ·78743	152	- ·66874	185	- ·62826	228	- ·63136	282	- ·65641	344
85	- ·79603	154	- ·67937	188	- ·64153	234	- ·64797	290	- ·67695	357
86	- ·80310	155	- ·68812	191	- ·65246	238	- ·66167	297	- ·69393	367
87	- ·80861	157	- ·69496	194	- ·66102	242	- ·67241	303	- ·70724	375
88	- ·81256	158	- ·69986	195	- ·66715	244	- ·68011	307	- ·71680	380
89	- ·81494	158	- ·70280	196	- ·67085	246	- ·68475	309	- ·72256	384
90	-0·81573	158	-0·70379	197	-0·67208	247	-0·68630	310	-0·72449	385

TABLE XVIII—*continued.*

$ce_2(x, \theta)$. $\theta=6$ to $\theta=10$.

x	$\theta=6$ $a_2=7{\cdot}87006$	Δ^2	$\theta=7$ $a_2=8{\cdot}08662$	Δ^2	$\theta=8$ $a_2=8{\cdot}11524$	Δ^2	$\theta=9$ $a_2=7{\cdot}98284$	Δ^2	$\theta=10$ $a_2=7{\cdot}71737$	Δ^2
0°	0·61196	77	0·49619	89	0·39533	95	0·31217	95	0·24589	92
1	·61235	77	·49663	89	·39581	95	·31264	95	·24635	92
2	·61350	77	·49797	89	·39723	95	·31407	95	·24773	92
3	·61542	76	·50020	89	·39960	95	·31646	96	·25003	93
4	·61810	76	·50333	89	·40293	95	·31980	96	·25327	93
5	·62154	75	·50733	88	·40720	95	·32409	96	·25743	94
6	·62573	74	·51222	87	·41242	95	·32936	97	·26253	95
7	·63065	72	·51798	87	·41858	94	·33558	97	·26858	96
8	·63629	71	·52461	86	·42569	94	·34278	97	·27559	97
9	·64265	69	·53210	85	·43375	94	·35095	98	·28357	98
10	·64970	67	·54043	83	·44274	93	·36009	98	·29252	99
11	·65742	65	·54960	82	·45266	93	·37022	98	·30245	100
12	·66580	63	·55959	80	·46351	92	·38132	98	·31339	101
13	·67480	60	·57038	78	·47528	91	·39341	98	·32533	102
14	·68440	57	·58195	76	·48795	89	·40648	98	·33829	102
15	·69457	53	·59428	73	·50152	88	·42053	97	·35227	103
16	·70527	50	·60734	70	·51597	86	·43556	97	·36728	103
17	·71647	45	·62110	67	·53127	83	·45155	95	·38333	103
18	·72812	41	·63553	63	·54740	80	·46849	94	·40041	103
19	·74018	36	·65058	58	·56434	77	·48638	92	·41853	102
20	·75260	30	·66621	53	·58205	73	·50518	89	·43766	101
21	·76532	24	·68238	48	·60050	69	·52487	86	·45782	100
22	·77828	18	·69903	42	·61963	64	·54543	82	·47896	97
23	·79142	11	·71611	36	·63941	58	·56681	78	·50108	94
24	·80468	4	·73354	29	·65977	52	·58897	73	·52415	90
25	·81797	− 4	·75125	21	·68065	45	·61186	67	·54811	86
26	·83122	− 12	·76918	12	·70198	37	·63542	60	·57293	80
27	·84435	− 21	·78723	3	·72368	28	·65957	52	·59856	74
28	·85727	− 30	·80531	− 6	·74566	19	·68425	43	·62492	66
29	·86989	− 40	·82333	− 17	·76783	8	·70936	34	·65194	57
30	·88211	− 50	·84118	− 28	·79008	− 3	·73480	23	·67953	47
31	·89382	− 61	·85875	− 40	·81231	− 15	·76048	11	·70759	36
32	·90493	− 72	·87592	− 52	·83439	− 28	·78625	− 2	·73601	23
33	·91531	− 83	·89257	− 65	·85618	− 42	·81201	− 16	·76467	10
34	·92486	− 95	·90858	− 79	·87756	− 57	·83760	− 32	·79342	− 6
35	·93346	−107	·92379	− 93	·89836	− 72	·86288	− 48	·82211	− 22
36	·94099	−119	·93808	−107	·91845	− 89	·88767	− 66	·85059	− 40
37	·94733	−132	·95129	−123	·93764	−106	·91181	− 84	·87866	− 59
38	·95235	−144	·96328	−138	·95578	−124	·93511	−103	·90615	− 80
39	·95593	−157	·97389	−154	·97268	−142	·95737	−124	·93284	−101
40	·95794	−169	·98296	−169	0·98816	−161	·97840	−145	·95851	−124
41	·95827	−181	·99034	−185	1·00204	−180	0·99798	−167	0·98295	−148
42	·95678	−193	·99587	−201	1·01412	−199	1·01589	−189	1·00591	−172
43	·95337	−204	0·99938	−216	1·02420	−218	1·03191	−211	1·02714	−198
44	·94792	−215	1·00074	−232	1·03211	−238	1·04582	−234	1·04640	−224
45	0·94031	−225	0·99978	−246	1·03764	−256	1·05739	−257	1·06341	−250

TABLE XVIII—*continued.*

$ce_2(x, \theta)$. $\theta=6$ to $\theta=10$.

x	$\theta=6$ $a_2=7\cdot87006$	Δ^2	$\theta=7$ $a_2=8\cdot08662$	Δ^2	$\theta=8$ $a_2=8\cdot11524$	Δ^2	$\theta=9$ $a_2=7\cdot98284$	Δ^2	$\theta=10$ $a_2=7\cdot71737$	Δ^2
45°	0·94031	−225	0·99978	−246	1·03764	−256	1·05739	−257	1·06341	−250
46	·93045	−235	·99635	−260	1·04060	−275	1·06639	−280	1·07793	−276
47	·91824	−243	·99033	−273	1·04081	−293	1·07259	−302	1·08968	−302
48	·90360	−251	·98157	−285	1·03810	−309	1·07577	−323	1·09841	−328
49	·88645	−258	·96995	−296	1·03230	−325	1·07572	−344	1·10386	−353
50	·86672	−263	·95537	−306	1·02324	−339	1·07223	−363	1·10578	−377
51	·84437	−266	·93774	−314	1·01079	−352	1·06512	−380	1·10393	−399
52	·81935	−269	·91696	−320	0·99482	−363	1·05420	−396	1·09809	−420
53	·79164	−269	·89298	−325	·97521	−372	1·03932	−410	1·08805	−438
54	·76124	−268	·86575	−327	·95189	−378	1·02035	−421	1·07363	−454
55	·72816	−265	·83525	−327	·92479	−383	0·99717	−429	1·05466	−467
56	·69242	−261	·80148	−325	·89385	−384	·96970	−435	1·03102	−477
57	·65407	−254	·76445	−321	·85908	−382	·93788	−437	1·00261	−484
58	·61319	−245	·72422	−314	·82049	−378	·90169	−436	0·96936	−486
59	·56985	−234	·68085	−304	·77811	−370	·86114	−431	·93124	−485
60	·52418	−221	·63444	−291	·73204	−359	·81629	−422	·88828	−479
61	·47629	−206	·58512	−276	·68237	−345	·76722	−409	·84052	−469
62	·42633	−189	·53304	−258	·62926	−327	·71405	−392	·78808	−453
63	·37449	−170	·47837	−238	·57288	−305	·65697	−371	·73111	−433
64	·32094	−149	·42133	−214	·51345	−281	·59617	−346	·66980	−408
65	·26591	−126	·36215	−188	·45120	−253	·53191	−316	·60441	−378
66	·20961	−101	·30109	−160	·38644	−221	·46449	−283	·53524	−344
67	·15230	− 75	·23843	−129	·31946	−187	·39424	−246	·46263	−304
68	·09424	− 47	·17447	− 96	·25061	−150	·32153	−205	·38697	−260
69	·03571	− 18	·10955	− 62	·18026	−110	·24677	−160	·30872	−212
70	− ·02301	12	·04402	− 25	·10882	− 67	·17041	−113	·22835	−160
71	− ·08160	43	− ·02177	13	·03670	− 23	·09292	− 63	·14637	−104
72	− ·13977	75	− ·08742	52	− ·03564	23	·01481	− 10	·06336	− 46
73	− ·19718	107	− ·15256	92	− ·10776	70	− ·06340	44	− ·02012	15
74	− ·25353	139	− ·21679	132	− ·17917	118	− ·14117	100	− ·10345	78
75	− ·30848	172	− ·27969	172	− ·24940	167	− ·21794	157	− ·18600	142
76	− ·36171	203	− ·34088	212	− ·31797	215	− ·29315	213	− ·26712	206
77	− ·41291	235	− ·39994	252	− ·38437	263	− ·36622	269	− ·34619	271
78	− ·46177	265	− ·45648	290	− ·44815	310	− ·43660	325	− ·42254	334
79	− ·50797	294	− ·51012	327	− ·50882	356	− ·50373	378	− ·49555	396
80	− ·55124	321	− ·56049	363	− ·56593	399	− ·56707	430	− ·56460	456
81	− ·59130	347	− ·60724	396	− ·61906	440	− ·62612	478	− ·62910	512
82	− ·62788	371	− ·65003	426	− ·66779	478	− ·68039	524	− ·68847	565
83	− ·66076	393	− ·68855	454	− ·71175	512	− ·72941	565	·74219	613
84	− ·68971	412	− ·72253	479	− ·75058	543	− ·77279	602	− ·78979	656
85	− ·71454	428	− ·75173	501	− ·78398	570	− ·81014	634	− ·83083	693
86	− ·73509	442	− ·77591	518	− ·81168	592	− ·84116	661	− ·86493	725
87	− ·75121	453	− ·79491	533	− ·83346	610	− ·86557	682	− ·89179	749
88	− ·76281	461	− ·80858	543	− ·84915	622	− ·88315	697	− ·91116	767
89	− ·76980	465	− ·81683	549	− ·85861	630	− ·89377	707	− ·92285	778
90	−0·77214	467	−0·81958	551	−0·86178	633	−0·89732	710	−0·92676	782

TABLE XIX.

$se_3(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta=1$ $b_3=9·04774$		$\theta=2$ $b_3=9·14063$		$\theta=3$ $b_3=9·22313$		$\theta=4$ $b_3=9·26145$		$\theta=5$ $b_3=9·23633$	
°		Δ^2		Δ^2		Δ^2		Δ^2		Δ^2
0	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0
1	·04859	− 10	·04426	− 7	·03976	− 4	·03532	− 1	·03108	1
2	·09708	− 21	·08846	− 14	·07949	− 8	·07063	− 3	·06217	1
3	·14535	− 31	·13252	− 21	·11913	− 12	·10591	− 4	·09327	2
4	·19332	− 42	·17637	− 28	·15866	− 16	·14115	− 6	·12439	3
5	·24087	− 52	·21994	− 35	·19803	− 20	·17633	− 7	·15554	3
6	·28789	− 62	·26316	− 42	·23720	− 24	·21144	− 9	·18671	3
7	·33430	− 72	·30596	− 49	·27613	− 29	·24645	− 11	·21792	3
8	·37999	− 82	·34827	− 56	·31477	− 33	·28135	− 13	·24916	3
9	·42484	− 92	·39002	− 63	·35308	− 38	·31612	− 16	·28042	2
10	·46878	−102	·43114	− 71	·39101	− 43	·35072	− 19	·31171	1
11	·51169	−112	·47155	− 78	·42851	− 48	·38514	− 22	·34302	0
12	·55348	−122	·51118	− 85	·46554	− 53	·41935	− 25	·37432	− 1
13	·59405	−131	·54995	− 93	·50203	− 59	·45330	− 29	·40562	− 3
14	·63331	−140	·58780	−100	·53794	− 64	·48697	− 33	·43688	− 5
15	·67117	−150	·62464	−108	·57321	− 70	·52031	− 37	·46809	− 8
16	·70753	−158	·66040	−116	·60777	− 77	·55328	− 42	·49922	− 12
17	·74230	−167	·69500	−123	·64157	− 83	·58583	− 47	·53023	− 15
18	·77541	−175	·72838	−131	·67453	− 90	·61792	− 53	·56109	− 20
19	·80676	−184	·76044	−139	·70660	− 97	·64947	− 59	·59175	− 24
20	·83628	−191	·79111	−146	·73770	−104	·68045	− 65	·62217	− 30
21	·86388	−199	·82032	−154	·76776	−111	·71077	− 72	·65229	− 36
22	·88949	−206	·84799	−162	·79671	−119	·74037	− 79	·68205	− 42
23	·91304	−213	·87404	−169	·82446	−127	·76919	− 87	·71138	− 50
24	·93446	−219	·89840	−177	·85095	−135	·79713	− 95	·74022	− 57
25	·95368	−225	·92099	−184	·87608	−143	·82413	−103	·76848	− 66
26	·97065	−231	·94173	−192	·89978	−152	·85009	−112	·79609	− 75
27	·98531	−236	·96056	−199	·92197	−160	·87493	−122	·82294	− 84
28	0·99761	−241	·97740	−206	·94256	−168	·89855	−131	·84896	− 94
29	1·00750	−245	0·99219	−212	·96146	−177	·92086	−141	·87403	−105
30	1·01494	−249	1·00486	−219	·97859	−185	·94177	−151	·89806	−116
31	1·01989	−252	1·01534	−225	0·99387	−194	·96116	−161	·92092	−127
32	1·02232	−254	1·02357	−230	1·00721	−202	·97894	−172	·94251	−139
33	1·02221	−256	1·02951	−236	1·01852	−210	0·99501	−182	·96271	−152
34	1·01953	−258	1·03308	−240	1·02773	−218	1·00925	−193	·98139	−164
35	1·01428	−258	1·03425	−245	1·03476	−226	1·02157	−203	0·99843	−177
36	1·00644	−258	1·03298	−249	1·03953	−233	1·03186	−213	1·01370	−190
37	0·99602	−258	1·02921	−252	1·04196	−240	1·04001	−224	1·02707	−203
38	·98302	−256	1·02293	−255	1·04199	−247	1·04593	−233	1·03841	−216
39	·96746	−254	1·01410	−257	1·03956	−253	1·04952	−243	1·04760	−228
40	·94936	−252	1·00270	−258	1·03460	−258	1·05067	−252	1·05451	−241
41	·92873	−248	0·98873	−258	1·02706	−262	1·04931	−260	1·05900	−253
42	·90563	−244	·97217	−258	1·01690	−266	1·04534	−268	1·06097	−265
43	·88010	−239	·95303	−257	1·00408	−269	1·03870	−275	1·06029	−276
44	·85217	−233	·93131	−255	0·98856	−271	1·02929	−282	1·05685	−286
45	0·82191	−226	0·90705	−252	0·97034	−273	1·01708	−287	1·05055	−295

TABLE XIX—*continued.*

$se_3(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta=1$ $b_3=9{\cdot}04774$		$\theta=2$ $b_3=9{\cdot}14063$		$\theta=3$ $b_3=9{\cdot}22313$		$\theta=4$ $b_3=9{\cdot}26145$		$\theta=5$ $b_3=9{\cdot}23633$	
°		Δ^2		Δ^2		Δ^2		Δ^2		Δ^2
45	0·82191	−226	0·90705	−252	0·97034	−273	1·01708	−287	1·05055	−295
46	·78939	−219	·88025	−249	·94938	−273	1·00199	−291	1·04130	−304
47	·75468	−211	·85097	−244	·92570	−272	0·98399	−294	1·02901	−311
48	·71786	−202	·81925	−238	·89930	−270	·96305	−296	1·01360	−317
49	·67901	−193	·78515	−232	·87021	−267	·93915	−297	0·99502	−322
50	·63823	−183	·74873	−224	·83844	−262	·91229	−296	·97322	−325
51	·59563	−172	·71006	−216	·80406	−256	·88246	−294	·94817	−327
52	·55132	−160	·66924	−206	·76712	−249	·84970	−290	·91985	−326
53	·50540	−148	·62636	−195	·72768	−241	·81404	−284	·88827	−324
54	·45800	−135	·58152	−184	·68583	−231	·77554	−277	·85345	−320
55	·40926	−121	·53485	−171	·64167	−220	·73427	−268	·81542	−314
56	·35930	−107	·48647	−158	·59530	−208	·69032	−258	·77425	−306
57	·30827	− 93	·43651	−143	·54686	−194	·64379	−245	·73002	−296
58	·25632	− 77	·38512	−128	·49647	−179	·59481	−231	·68283	−283
59	·20359	− 62	·33245	−112	·44430	−163	·54351	−215	·63281	−268
60	·15024	− 46	·27867	− 95	·39049	−145	·49006	−198	·58011	−251
61	·09644	− 30	·22394	− 77	·33523	−127	·43464	−179	·52489	−232
62	·04233	− 13	·16844	− 58	·27871	−107	·37743	−158	·46736	−211
63	− ·01190	4	·11236	− 39	·22112	− 86	·31864	· 135	·40771	−188
64	− ·06610	21	·05589	− 20	·16267	− 64	·25849	−112	·34619	−162
65	− ·12009	38	− ·00078	0	·10358	− 41	·19723	− 86	·28304	−135
66	− ·17370	55	− ·05744	21	·04408	− 18	·13511	− 60	·21855	−106
67	− ·22676	72	− ·11390	41	− ·01560	6	·07238	− 33	·15300	− 75
68	− ·27911	89	− ·16995	62	− ·07521	31	·00933	− 4	·08669	− 43
69	− ·33056	106	− ·22537	83	− ·13452	56	− ·05376	25	·01995	− 10
70	− ·38095	123	− ·27996	104	− ·19326	81	− ·11660	55	− ·04689	24
71	− ·43011	139	− ·33351	124	− ·25119	107	− ·17890	85	− ·11348	59
72	− ·47788	155	− ·38582	145	− ·30805	132	− ·24035	115	− ·17949	95
73	− ·52410	171	− ·43666	166	− ·36359	157	− ·30064	146	− ·24455	131
74	− ·56861	186	− ·48585	185	− ·41756	182	− ·35949	176	− ·30830	166
75	− ·61126	201	− ·53319	205	− ·46971	206	− ·41657	205	− ·37038	202
76	− ·65190	215	− ·57848	223	− ·51979	230	− ·47160	234	− ·43045	237
77	− ·69039	228	− ·62154	241	− ·56758	253	− ·52428	263	− ·48816	271
78	− ·72661	241	− ·66219	258	− ·61284	274	− ·57434	290	− ·54315	304
79	− ·76042	252	− ·70025	274	− ·65536	295	− ·62151	316	− ·59510	335
80	− ·79170	263	− ·73558	289	− ·69492	314	− ·66551	340	− ·64370	365
81	− ·82035	274	− ·76802	303	− ·73134	332	− ·70611	363	− ·68865	393
82	− ·84627	283	− ·79743	315	− ·76444	349	− ·74309	384	− ·72967	419
83	− ·86936	291	− ·82368	327	− ·79405	364	− ·77623	40[illegible]	·76650	442
84	− ·88953	298	− ·84668	337	− ·82002	377	− ·80535	419	− ·79891	463
85	− ·90673	304	− ·86630	345	− ·84222	388	− ·83028	433	− ·82669	480
86	− ·92089	309	− ·88247	352	− ·86054	397	− ·85088	445	− ·84907	495
87	− ·93195	313	− ·89513	358	− ·87489	405	− ·86702	455	− ·86770	507
88	− ·93988	316	− ·90421	362	− ·88519	410	− ·87862	461	− ·88066	515
89	− ·94465	318	− ·90967	364	− ·89139	413	− ·88561	465	− ·88847	520
90	−0·94624	318	−0·91149	365	−0·89346	414	−0·88794	467	−0·89108	522

TABLE XIX—*continued.*

$se_3(x, \theta)$. $\theta = 6$ to $\theta = 10$.

x	$\theta=6$ $b_3=9{\cdot}13791$		$\theta=7$ $b_3=8{\cdot}96239$		$\theta=8$ $b_3=8{\cdot}70991$		$\theta=9$ $b_3=8{\cdot}38312$		$\theta=10$ $b_3=7{\cdot}98607$	
°		Δ^2		Δ^2		Δ^2		Δ^2		Δ^2
0	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0
1	·02713	2	·02352	4	·02028	4	·01741	5	·01490	5
2	·05428	5	·04708	7	·04061	9	·03488	10	·02986	11
3	·08148	7	·07070	11	·06102	13	·05244	15	·04493	16
4	·10875	9	·09444	14	·08157	18	·07016	20	·06016	22
5	·13611	11	·11832	17	·10230	22	·08808	25	·07560	27
6	·16358	13	·14237	20	·12324	26	·10625	30	·09132	32
7	·19118	15	·16662	23	·14445	30	·12471	34	·10736	37
8	·21892	16	·19111	26	·16595	34	·14353	39	·12377	42
9	·24683	17	·21586	29	·18780	37	·16273	43	·14060	47
10	·27491	18	·24090	31	·21001	40	·18236	47	·15791	52
11	·30316	18	·26624	33	·23263	43	·20247	51	·17573	56
12	·33160	18	·29191	34	·25568	46	·22309	55	·19413	61
13	·36022	18	·31792	35	·27919	48	·24426	58	·21312	65
14	·38902	17	·34428	36	·30319	50	·26601	61	·23277	69
15	·41800	16	·37099	36	·32768	51	·28837	63	·25311	72
16	·44713	14	·39807	35	·35269	52	·31136	65	·27416	75
17	·47641	12	·42549	34	·37822	52	·33500	67	·29596	77
18	·50580	9	·45326	33	·40427	52	·35931	68	·31854	79
19	·53528	5	·48135	30	·43085	51	·38429	68	·34191	81
20	·56481	1	·50975	27	·45793	49	·40995	67	·36609	82
21	·59435	− 4	·53842	24	·48551	47	·43629	66	·39109	82
22	·62385	− 10	·56732	19	·51356	44	·46329	64	·41690	81
23	·65325	− 16	·59642	14	·54205	40	·49093	62	·44353	80
24	·68249	− 23	·62565	8	·57093	35	·51919	58	·47096	77
25	·71150	− 31	·65496	1	·60016	29	·54803	53	·49915	74
26	·74021	− 40	·68428	− 7	·62968	22	·57740	47	·52809	70
27	·76851	− 49	·71352	− 16	·65941	14	·60724	41	·55773	64
28	·79633	− 59	·74260	− 26	·68928	5	·63749	33	·58800	57
29	·82356	− 70	·77143	− 36	·71920	− 5	·66806	23	·61884	49
30	·85009	− 81	·79990	− 48	·74907	− 16	·69887	13	·65018	40
31	·87581	− 94	·82788	− 60	·77878	− 29	·72980	1	·68191	29
32	·90059	− 106	·85526	− 74	·80820	− 42	·76075	− 12	·71394	17
33	·92431	− 120	·88191	− 88	·83721	− 56	·79159	− 26	·74613	3
34	·94683	− 134	·90767	− 103	·86565	− 72	·82217	− 41	·77836	− 12
35	·96801	− 148	·93241	− 119	·89337	− 88	·85234	− 58	·81046	− 28
36	0·98770	− 163	·95596	− 135	·92022	− 106	·88193	− 76	·84229	− 46
37	1·00576	− 179	·97816	− 152	·94600	− 124	·91076	− 95	·87365	− 66
38	1·02204	− 194	0·99884	− 170	·97055	− 143	·93863	− 115	·90435	− 87
39	1·03637	− 210	1·01782	− 188	0·99366	− 163	·96536	− 137	·93418	− 109
40	1·04861	− 125	1·03493	− 206	1·01515	− 183	0·99072	− 159	·96292	− 132
41	1·05859	− 241	1·04997	− 224	1·03480	− 204	1·01450	− 182	0·99033	− 157
42	1·06616	− 256	1·06278	− 243	1·05240	− 226	1·03645	− 205	1·01618	− 183
43	1·07118	− 271	1·07315	− 261	1·06775	− 247	1·05636	− 229	1·04020	− 209
44	1·07349	− 285	1·08092	− 279	1·08063	− 268	1·07397	− 254	1·06213	− 236
45	1·07294	− 299	1·08590	− 296	1·09083	− 289	1·08904	− 278	1·08170	− 263

TABLE XIX—*continued.*

$se_3(x, \theta)$. $\theta=6$ to $\theta=10$.

x	$\theta=6$ $b_3=9{\cdot}13791$	Δ^2	$\theta=7$ $b_3=8{\cdot}96239$	Δ^2	$\theta=8$ $b_3=8{\cdot}70091$	Δ^2	$\theta=9$ $b_3=8{\cdot}38312$	Δ^2	$\theta=10$ $b_3=7{\cdot}98607$	Δ^2
45°	1·07294	−299	1·08590	−296	1·09083	−289	1·08904	−278	1·08170	−263
46	1·06941	−311	1·08791	−313	1·09813	−310	1·10133	−302	1·09864	−291
47	1·06277	−323	1·08679	−329	1·10234	−330	1·11060	−326	1·11268	−318
48	1·05290	−333	1·08238	−344	1·10324	−349	1·11661	−349	1·12353	−345
49	1·03970	−342	1·07454	−357	1·10066	−367	1·11913	−371	1·13094	−371
50	1·02308	−350	1·06312	−369	1·09441	−383	1·11793	−392	1·13464	−396
51	1·00296	−355	1·04802	−379	1·08433	−397	1·11282	−411	1·13438	−419
52	0·97928	−359	1·02912	−387	1·07028	−410	1·10360	−428	1·12992	−441
53	·95202	−361	1·00636	−393	1·05213	−420	1·09011	−443	1·12106	−461
54	·92115	−360	0·97967	−396	1·02978	−428	1·07218	−455	1·10758	−478
55	·88668	−357	·94902	−397	1·00314	−433	1·04970	−465	1·08933	−492
56	·84863	−352	·91439	−395	0·97218	−435	1·02257	−471	1·06616	−502
57	·80706	−344	·87581	−391	·93686	−434	0·99073	−474	1·03796	−509
58	·76205	−334	·83332	−383	·89720	−429	·95416	−473	1·00468	−512
59	·71370	−321	·78700	−372	·85325	−421	·91286	−468	0·96626	−511
60	·66214	−305	·73696	−358	·80508	−409	·86689	−459	·92274	−505
61	·60752	−287	·68334	−341	·75282	−394	·81632	−445	·87417	−494
62	·55004	−265	·62631	−320	·69662	−374	·76131	−427	·82065	−479
63	·48991	−241	·56608	−296	·63668	−351	·70202	−405	·76235	−458
64	·42736	−215	·50289	−269	·57323	−324	·63867	−378	·69946	−432
65	·36267	−186	·43701	−239	·50654	−293	·57155	−347	·63225	−401
66	·29611	−155	·36874	−206	·43692	−258	·50095	−311	·56104	−365
67	·22801	−121	·29841	−170	·36472	−220	·42724	−272	·48617	−324
68	·15870	− 86	·22639	−131	·29032	−179	·35082	−228	·40807	−278
69	·08852	− 49	·15305	− 90	·21414	−134	·27212	−180	·32719	−227
70	·01787	− 10	·07882	− 47	·13661	− 87	·19162	−129	·24404	−173
71	− ·05289	30	·00411	− 2	·05822	− 38	·10982	− 75	·15916	−115
72	− ·12334	71	− ·07062	44	− ·02056	14	·02728	− 19	·07313	− 54
73	− ·19309	112	− ·14491	91	− ·09919	66	− ·05546	39	− ·01344	10
74	− ·26172	154	− ·21830	139	− ·17716	120	− ·13780	99	− ·09990	76
75	− ·32880	196	− ·29030	186	− ·25393	175	− ·21915	160	− ·18561	143
76	− ·39393	237	− ·36043	234	− ·32896	229	− ·29889	221	− ·26988	211
77	− ·45669	277	− ·42823	281	− ·40169	282	− ·37643	282	− ·35204	278
78	− ·51668	316	− ·49321	327	− ·47160	335	− ·45115	341	− ·43142	345
79	− ·57351	354	− ·55493	371	− ·53816	386	− ·52246	399	− ·50736	410
80	− ·62680	390	− ·61294	413	− ·60087	434	− ·58978	454	− ·57919	472
81	− ·67620	423	− ·66682	452	− ·65923	480	− ·65256	507	− ·64630	531
82	− ·72136	454	− ·71618	489	− ·71278	523	− ·71028	555	− ·70810	587
83	− ·76198	482	− ·76065	522	− ·76111	562	− ·76244	600	− ·76403	637
84	− ·79778	507	− ·79990	552	− ·80383	596	− ·80860	640	− ·81359	682
85	− ·82851	529	− ·83364	577	− ·84058	626	− ·84836	674	− ·85633	722
86	− ·85396	547	− ·86160	599	− ·87108	651	− ·88138	703	− ·89186	754
87	− ·87394	561	− ·88357	616	− ·89506	671	− ·90737	726	− ·91983	780
88	− ·88832	571	− ·89939	628	− ·91233	685	− ·92610	743	− ·94001	799
89	− ·89698	577	− ·90893	635	− ·92275	693	− ·93740	752	− ·95219	811
90	−0·89988	579	−0·91211	638	−0·92624	697	−0·94118	756	−0·95626	815

TABLE XX.

$ce_3(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta=1$ $a_3=9{\cdot}07837$	Δ^2	$\theta=2$ $a_3=9{\cdot}37032$	Δ^2	$\theta=3$ $a_3=9{\cdot}91551$	Δ^2	$\theta=4$ $a_3=10{\cdot}67103$	Δ^2	$\theta=5$ $a_3=11{\cdot}54883$	Δ^2
0°	1·06724	−230	1·12827	−185	1·15858	−138	1·15020	− 94	1·11125	− 52
1	1·06609	−230	1·12735	−185	1·15789	−138	1·14973	− 94	1·11099	− 53
2	1·06264	−229	1·12458	−184	1·15581	−138	1·14832	− 94	1·11020	− 53
3	1·05689	−228	1·11997	−184	1·15236	−139	1·14598	− 95	1·10888	− 54
4	1·04887	−227	1·11352	−183	1·14751	−139	1·14268	− 96	1·10701	− 56
5	1·03858	−225	1·10524	−183	1·14128	−139	1·13843	− 97	1·10459	− 57
6	1·02604	−223	1·09513	−182	1·13366	−140	1·13321	− 98	1·10160	− 59
7	1·01127	−220	1·08319	−181	1·12463	−140	1·12701	−100	1·09802	− 62
8	0·99431	−217	1·06945	−180	1·11421	−141	1·11981	−102	1·09382	− 65
9	·97518	−213	1·05390	−179	1·10237	−141	1·11159	−104	1·08897	− 68
10	·95392	−209	1·03657	−177	1·08913	−142	1·10233	−106	1·08344	− 71
11	·93056	−205	1·01747	−175	1·07446	−142	1·09202	−108	1·07721	− 75
12	·90516	−200	0·99661	−174	1·05837	−143	1·08062	−111	1·07023	− 79
13	·87776	−195	·97402	−171	1·04085	−143	1·06811	−113	1·06246	− 83
14	·84842	−189	·94971	−169	1·02190	−144	1·05448	−116	1·05386	− 87
15	·81718	−183	·92372	−166	1·00151	−144	1·03968	−119	1·04439	− 92
16	·78411	−176	·89606	−163	0·97968	−144	1·02370	−121	1·03400	− 97
17	·74929	−169	·86677	−160	·95641	−144	1·00651	−124	1·02264	−102
18	·71277	−162	·83588	−156	·93170	−144	0·98808	−126	1·01027	−106
19	·67463	−154	·80344	−152	·90555	−143	·96838	−129	0·99684	−111
20	·63494	−146	·76947	−148	·87797	−142	·94740	−131	·98229	−116
21	·59380	−137	·73402	−143	·84897	−141	·92511	−133	·96657	−121
22	·55129	−128	·69714	−138	·81856	−140	·90148	−135	·94965	−126
23	·50749	−119	·65889	−132	·78675	−138	·87651	−137	·93146	−131
24	·46251	−109	·61931	−126	·75357	−135	·85017	−138	·91197	−135
25	·41643	− 99	·57847	−120	·71903	−133	·82245	−138	·89113	−139
26	·36937	− 88	·53643	−113	·68317	−129	·79335	−139	·86890	−143
27	·32143	− 77	·49326	−105	·64601	−126	·76286	−139	·84524	−146
28	·27271	− 66	·44904	− 98	·60759	−121	·73098	−138	·82012	−149
29	·22333	− 55	·40384	− 89	·56796	−116	·69772	−137	·79352	−151
30	·17340	− 43	·35775	− 80	·52717	−111	·66310	−135	·76541	−153
31	·12305	− 30	·31086	− 71	·48527	−105	·62713	−132	·73576	−154
32	·07239	− 18	·26326	− 61	·44231	− 98	·58984	−129	·70459	−154
33	·02155	− 5	·21505	− 51	·39838	− 91	·55127	−124	·67187	−153
34	− ·02934	7	·16633	− 40	·35354	− 83	·51145	−119	·63763	−151
35	− ·08015	21	·11722	− 29	·30787	− 74	·47043	−114	·60187	−149
36	− ·13077	34	·06782	− 17	·26147	− 64	·42828	−107	·56462	−145
37	− ·18104	47	·01825	− 5	·21443	− 54	·38506	− 99	·52592	−141
38	− ·23085	60	− ·03137	8	·16684	− 43	·34084	− 91	·48581	−135
39	− ·28005	74	− ·08090	21	·11883	− 31	·29572	− 81	·44435	−128
40	− ·32851	87	− ·13022	34	·07050	− 19	·24979	− 71	·40161	−120
41	− ·37609	101	− ·17920	48	·02199	− 6	·20316	− 59	·35767	−111
42	− ·42267	114	− ·22770	62	− ·02659	8	·15593	− 47	·31262	−100
43	− ·46811	127	− ·27558	76	− ·07509	22	·10824	− 34	·26658	− 88
44	− ·51227	141	− ·32269	91	− ·12337	36	·06021	− 19	·21965	− 75
45	−0·55503	153	−0·36889	105	−0·17129	52	0·01200	− 4	0·17198	− 60

TABLE XX—*continued.*

$ce_3(x, \theta)$. $\theta=1$ to $\theta=5$.

x	$\theta=1$ $a_3=9{\cdot}07837$	Δ^2	$\theta=2$ $a_3=9{\cdot}37032$	Δ^2	$\theta=3$ $a_3=9{\cdot}91551$	Δ^2	$\theta=4$ $a_3=10{\cdot}67103$	Δ^2	$\theta=5$ $a_3=11{\cdot}54883$	Δ^2
° 45	−0·55503	153	−0·36889	105	−0·17129	52	0·01200	− 4	0·17198	− 60
46	− ·59625	166	− ·41405	120	− ·21869	67	− ·03626	12	·12370	− 45
47	− ·63581	179	− ·45800	135	− ·26541	84	− ·08439	29	·07498	− 28
48	− ·67359	191	− ·50061	149	− ·31130	100	− ·13223	46	·02597	− 10
49	− ·70946	202	− ·54172	164	− ·35619	117	− ·17961	65	− ·02313	9
50	− ·74331	213	− ·58120	178	− ·39992	133	− ·22634	83	− ·07214	29
51	− ·77503	224	− ·61889	192	− ·44230	150	− ·27225	102	− ·12086	50
52	− ·80450	234	− ·65466	206	− ·48319	167	− ·31712	122	− ·16907	72
53	− ·83163	244	− ·68837	220	− ·52240	184	− ·36079	142	− ·21657	94
54	− ·85632	253	− ·71989	233	− ·55977	201	− ·40303	161	− ·26312	117
55	− ·87849	261	− ·74908	245	− ·59513	217	− ·44366	181	− ·30849	141
56	− ·89804	269	− ·77582	257	− ·62833	233	− ·48248	201	− ·35247	164
57	− ·91490	276	− ·79999	268	− ·65919	248	− ·51929	220	− ·39479	188
58	− ·92901	282	− ·82148	278	− ·68758	263	− ·55390	239	− ·43524	211
59	− ·94030	287	− ·84019	288	− ·71334	277	− ·58612	258	− ·47358	234
60	− ·94873	291	− ·85602	296	− ·73633	290	− ·61576	275	− ·50958	257
61	− ·95424	295	− ·86889	304	− ·75643	302	− ·64265	292	− ·54301	279
62	− ·95680	297	− ·87871	311	− ·77351	313	− ·66662	307	− ·57365	299
63	− ·95640	299	− ·88543	316	− ·78746	322	− ·68752	322	− ·60129	319
64	− ·95300	299	− ·88899	320	− ·79819	331	− ·70520	335	− ·62575	337
65	− ·94662	299	− ·88935	323	− ·80562	338	− ·71953	346	− ·64683	354
66	− ·93725	297	− ·88647	325	− ·80966	343	− ·73040	356	− ·66438	369
67	− ·92490	295	− ·88034	326	− ·81027	347	− ·73770	365	− ·67823	382
68	− ·90961	291	− ·87096	325	− ·80741	350	− ·74136	371	− ·68826	393
69	− ·89140	287	− ·85832	323	− ·80104	351	− ·74131	375	− ·69437	401
70	− ·87032	281	− ·84246	319	− ·79117	350	− ·73751	377	− ·69646	407
71	− ·84644	275	− ·82341	314	− ·77781	347	− ·72994	377	− ·69448	411
72	− ·81980	267	− ·80122	308	− ·76098	342	− ·71859	375	− ·68839	412
73	− ·79050	258	− ·77596	300	− ·74072	336	− ·70349	371	− ·67819	410
74	− ·75861	249	− ·74769	291	− ·71711	328	− ·68469	364	− ·66389	405
75	− ·72423	238	− ·71653	280	− ·69022	318	− ·66225	355	− ·64554	397
76	− ·68747	227	− ·68256	268	− ·66015	306	− ·63626	344	− ·62322	387
77	− ·64844	215	− ·64591	255	− ·62703	292	− ·60683	330	− ·59703	373
78	− ·60726	202	− ·60670	241	− ·59098	277	− ·57411	314	− ·56711	357
79	− ·56406	188	− ·56510	225	− ·55217	260	− ·53824	296	− ·53362	338
80	− ·51899	173	− ·52124	208	− ·51075	242	− ·49941	277	− ·49675	317
81	− ·47218	158	− ·47530	191	− ·46691	222	− ·45781	255	− ·45671	293
82	− ·42380	142	− ·42745	172	− ·42085	201	− ·41367	231	− ·41374	267
83	− ·37400	125	− ·37788	152	− ·37279	179	− ·36721	206	− ·36811	238
84	− ·32294	109	− ·32679	132	− ·32293	155	− ·31869	179	− ·32010	208
85	− ·27079	91	− ·27438	111	− ·27153	131	− ·26838	152	− ·27000	176
86	− ·21774	73	− ·22085	90	− ·21881	106	− ·21655	123	− ·21815	142
87	− ·16395	55	− ·16643	68	− ·16504	80	− ·16350	93	− ·16488	108
88	− ·10961	37	− ·11133	45	− ·11047	53	− ·10952	62	− ·11052	72
89	− ·05490	19	− ·05578	23	− ·05537	27	− ·05491	31	− ·05544	36
90	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0

TABLE XX—*continued*.

$ce_3(x, \theta)$. $\theta=6$ to $\theta=10$.

x	$\theta=6$ $a_3=12{\cdot}46560$		$\theta=7$ $a_3=13{\cdot}35842$		$\theta=8$ $a_3=14{\cdot}18188$		$\theta=9$ $a_3=14{\cdot}90368$		$\theta=10$ $a_3=15{\cdot}50278$	
°		Δ^2		Δ^2		Δ^2		Δ^2		Δ^2
0	1·05154	− 15	0·97729	19	0·89237	49	0·80022	75	0·70483	97
1	1·05147	− 15	·97739	19	·89261	49	·80060	75	·70531	96
2	1·05124	− 16	·97767	18	·89335	48	·80173	74	·70676	96
3	1·05086	− 17	·97813	17	·89457	47	·80360	73	·70916	95
4	1·05030	− 19	·97876	15	·89627	45	·80621	72	·71251	93
5	1·04956	− 21	·97955	13	·89841	43	·80953	70	·71679	92
6	1·04861	− 23	·98046	10	·90099	40	·81355	67	·72199	89
7	1·04742	− 26	·98147	7	·90397	37	·81824	64	·72809	87
8	1·04598	− 30	·98254	3	·90732	33	·82357	60	·73504	83
9	1·04424	− 34	·98365	− 1	·91100	29	·82950	56	·74283	80
10	1·04216	− 38	·98474	− 6	·91496	24	·83599	51	·75142	75
11	1·03970	− 42	·98577	− 11	·91917	18	·84299	46	·76076	70
12	1·03682	− 48	·98669	− 17	·92355	12	·85045	40	·77080	65
13	1·03347	− 53	·98743	− 23	·92806	6	·85831	33	·78149	59
14	1·02958	− 59	·98795	− 30	·93263	− 2	·86649	26	·79278	52
15	1·02511	− 65	·98816	− 37	·93717	− 9	·87494	18	·80458	44
16	1·01999	− 71	·98800	− 45	·94163	− 18	·88358	10	·81682	36
17	1·01416	− 78	·98739	− 53	·94591	− 26	·89231	0	·82943	27
18	1·00755	− 85	·98626	− 61	·94992	− 36	·90104	− 9	·84231	17
19	1·00010	− 92	·98451	− 70	·95358	− 46	·90968	− 20	·85536	7
20	0·99173	− 99	·98207	− 79	·95677	− 56	·91812	− 31	·86848	− 5
21	·98237	− 106	·97883	− 88	·95941	− 67	·92624	− 43	·88155	− 17
22	·97195	− 114	·97472	− 98	·96137	− 78	·93394	− 56	·89445	− 30
23	·96039	− 121	·96963	− 107	·96256	− 90	·94108	− 69	·90704	− 45
24	·94762	− 128	·96347	− 117	·96284	− 102	·94753	− 83	·91919	− 59
25	·93358	− 135	·95614	− 127	·96210	− 114	·95315	− 97	·93074	− 75
26	·91818	− 142	·94753	− 137	·96023	− 127	·95781	− 112	·94154	− 92
27	·90136	− 149	·93756	− 146	·95708	− 139	·96135	− 127	·95143	− 109
28	·88306	− 155	·92613	− 156	·95254	− 152	·96362	− 142	·96023	− 126
29	·86321	− 161	·91313	− 165	·94648	− 164	·96448	− 158	·96777	− 145
30	·84176	− 166	·89848	− 174	·93878	− 177	·96375	− 173	·97386	− 163
31	·81864	− 170	·88210	− 182	·92931	− 189	·96130	− 189	·97832	− 182
32	·79383	− 174	·86389	− 190	·91796	− 200	·95695	− 204	·98095	− 201
33	·76727	− 177	·84378	− 197	·90459	− 211	·95056	− 220	·98157	− 220
34	·73894	− 179	·82170	− 203	·88912	− 222	·94198	− 234	·97999	− 239
35	·70882	− 180	·79759	− 208	·87143	− 231	·93105	− 248	·97602	− 257
36	·67689	− 180	·77140	− 212	·85142	− 239	·91765	− 261	·96948	− 275
37	·64316	− 179	·74309	− 215	·82903	− 247	·90163	− 273	·96018	− 292
38	·60763	− 177	·71263	− 216	·80416	− 252	·88289	− 284	·94796	− 308
39	·57034	− 173	·68000	− 216	·77677	− 257	·86131	− 293	·93266	− 322
40	·53131	− 168	·64521	− 215	·74682	− 259	·83680	− 300	·91414	− 335
41	·49061	− 161	·60828	− 211	·71427	− 260	·80929	− 305	·89228	− 346
42	·44829	− 153	·56923	− 206	·67912	− 259	·77873	− 309	·86695	− 354
43	·40445	− 143	·52813	− 199	·64138	− 255	·74508	− 310	·83809	− 360
44	·35917	− 132	·48503	− 190	·60110	− 249	·70833	− 308	·80563	− 363
45	0·31258	− 119	0·44003	− 179	0·55832	− 241	0·66851	− 303	0·76954	− 363

TABLE XX—*continued.*

$ce_8(x, \theta)$. $\theta = 6$ to $\theta = 10$.

x	$\theta=6$ $a_8 = 12·46560$	Δ^2	$\theta=7$ $a_8 = 13·35842$	Δ^2	$\theta=8$ $a_8 = 14·18·88$	Δ^2	$\theta=9$ $a_8 = 14·90368$	Δ^2	$\theta=10$ $a_8 = 15·50278$	Δ^2
° 45	0·31258	−119	0·44003	−179	0·55832	−241	0·66851	−303	0·76954	−363
46	·26480	−104	·39325	−166	·51313	−230	·62565	−296	·72981	−360
47	·21598	− 87	·34480	−150	·46564	−217	·57984	−285	·68649	−353
48	·16629	− 69	·29486	−133	·41599	−201	·53117	−271	·63964	−342
49	·11590	− 50	·24358	−113	·36432	−182	·47979	−254	·58936	−328
50	·06502	− 29	·19117	− 92	·31084	−160	·42587	−234	·53580	−309
51	·01385	− 6	·13784	− 68	·25575	−136	·36962	−210	·47915	−287
52	− ·03738	18	·08383	− 43	·19931	−109	·31126	−182	·41963	−260
53	− ·08844	43	·02940	− 15	·14176	− 80	·25109	−152	·35752	−229
54	− ·13907	69	− ·02519	14	·08342	− 48	·18939	−118	·29312	−193
55	− ·18901	95	− ·07964	44	·02459	− 15	·12652	− 81	·22679	−154
56	− ·23800	123	− ·13365	76	− ·03438	21	·06284	− 41	·15892	−111
57	− ·28576	151	− ·18690	109	− ·09314	59	− ·00125	1	·08994	− 64
58	− ·33201	179	− ·23907	142	− ·15131	98	− ·06533	46	·02031	− 15
59	− ·37646	208	− ·28982	176	− ·20850	138	− ·12896	92	− ·04947	38
60	− ·41885	236	− ·33881	210	− ·26432	179	− ·19167	140	− ·11887	93
61	− ·45887	263	− ·38569	244	− ·31835	220	− ·25298	188	− ·18734	149
62	− ·49627	290	− ·43014	278	− ·37018	261	− ·31241	238	− ·25432	207
63	− ·53076	315	− ·47181	310	− ·41941	301	− ·36946	287	− ·31924	265
64	− ·56210	340	− ·51037	342	− ·46562	341	− ·42364	335	− ·38150	323
65	− ·59005	363	− ·54553	371	− ·50842	379	− ·47447	382	− ·44053	380
66	− ·61437	383	− ·57697	399	− ·54744	415	− ·52148	428	− ·49576	436
67	− ·63485	402	− ·60441	425	− ·58231	448	− ·56421	471	− ·54663	489
68	− ·65132	418	− ·62761	448	− ·61269	479	− ·60223	511	− ·59261	539
69	− ·66360	432	− ·64633	468	− ·63828	507	− ·63514	547	− ·63319	585
70	− ·67156	443	− ·66038	484	− ·65881	530	− ·66259	579	− ·66792	627
71	− ·67509	451	− ·66959	497	− ·67403	550	− ·68425	606	− ·69638	663
72	− ·67412	455	− ·67382	506	− ·68376	565	− ·69985	628	− ·71822	693
73	− ·66859	456	− ·67299	511	− ·68784	575	− ·70917	644	− ·73313	716
74	− ·65851	454	− ·66704	512	− ·68617	580	− ·71206	654	− ·74088	732
75	− ·64388	448	− ·65598	509	− ·67871	579	− ·70841	657	− ·74131	741
76	− ·62478	439	− ·63982	501	− ·66545	573	− ·69818	654	− ·73434	741
77	− ·60129	426	− ·61866	489	− ·64647	562	− ·68141	645	− ·71995	734
78	− ·57354	409	− ·59261	472	− ·62186	545	− ·65819	628	− ·69823	718
79	− ·54170	389	− ·56184	450	− ·59180	523	− ·62869	604	− ·66934	693
80	− ·50598	366	− ·52657	425	− ·55651	495	− ·59315	574	− ·63351	661
81	− ·46659	339	− ·48705	395	− ·51628	462	− ·55186	538	− ·59106	621
82	− ·42382	310	− ·44357	362	− ·47143	424	− ·50520	495	− ·54241	573
83	− ·37795	277	− ·39648	325	− ·42233	382	− ·45359	447	− ·48802	518
84	− ·32930	243	− ·34613	285	− ·36942	335	− ·39750	393	− ·42845	457
85	− ·27823	206	− ·29294	242	− ·31315	285	− ·33749	335	− ·36431	390
86	− ·22510	167	− ·23732	197	− ·25403	232	− ·27412	273	− ·29627	318
87	− ·17031	126	− ·17974	149	− ·19259	176	− ·20802	208	− ·22504	242
88	− ·11425	85	− ·12066	100	− ·12938	119	− ·13985	140	− ·15139	163
89	− ·05734	43	− ·06058	51	− ·06499	60	− ·07028	70	− ·07611	82
90	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0

Table XXI.

$se_4(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta=1$ $b_4=16{\cdot}03297$	Δ^2	$\theta=2$ $b_4=16{\cdot}12769$	Δ^2	$\theta=3$ $b_4=16{\cdot}27270$	Δ^2	$\theta=4$ $b_4=16{\cdot}45204$	Δ^2	$\theta=5$ $b_4=16{\cdot}64822$	Δ^2
° 0	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0
1	·06724	− 29	·06433	− 24	·06103	− 19	·05744	− 15	·05364	− 11
2	·13420	− 57	·12841	− 47	·12187	− 38	·11472	− 30	·10717	− 22
3	·20058	− 86	·19203	− 71	·18233	− 57	·17172	− 44	·16048	− 33
4	·26611	−114	·25493	− 94	·24221	− 76	·22826	− 59	·21346	− 44
5	·33049	−142	·31689	−118	·30134	− 95	·28422	− 74	·26601	− 55
6	·39346	−169	·37767	−140	·35951	−114	·33943	− 89	·31800	− 67
7	·45475	−195	·43705	−163	·41654	−133	·39375	−104	·36933	− 78
8	·51408	−221	·49479	−185	·47225	−151	·44703	−119	·41988	− 90
9	·57120	−246	·55069	−207	·52644	−169	·49911	−134	·46952	−102
10	·62587	−270	·60452	−228	·57895	−188	·54985	−150	·51815	−114
11	·67784	−293	·65607	−248	·62957	−205	·59909	−165	·56563	−127
12	·72688	−314	·70514	−268	·67814	−223	·64669	−180	·61184	−140
13	·77278	−335	·75153	−287	·72449	−240	·69248	−195	·65665	−153
14	·81532	−354	·79506	−305	·76843	−257	·73632	−211	·69992	−167
15	·85433	−372	·83553	−322	·80981	−273	·77805	−226	·74153	−180
16	·88961	−388	·87278	−339	·84845	−289	·81753	−241	·78134	−194
17	·92101	−403	·90665	−354	·88420	−304	·85460	−256	·81920	−209
18	·94838	−416	·93698	−368	·91691	−319	·88911	−270	·85497	−223
19	·97159	−428	·96363	−381	·94643	−333	·92092	−285	·88852	−237
20	0·99052	−437	0·98647	−392	·97262	−346	·94989	−299	·91969	−252
21	1·00507	−445	1·00539	−403	0·99536	−358	·97586	−312	·94835	−266
22	1·01517	−451	1·02028	−412	1·01451	−369	0·99872	−325	·97434	−281
23	1·02076	−455	1·03105	−419	1·02997	−380	1·01832	−338	0·99753	−295
24	1·02180	−457	1·03764	−425	1·04163	−389	1·03454	−350	1·01777	−309
25	1·01827	−457	1·03997	−429	1·04941	−397	1·04726	−361	1·03492	−322
26	1·01017	−455	1·03801	−432	1·05321	−403	1·05638	−371	1·04885	−335
27	0·99751	−451	1·03173	−433	1·05298	−409	1·06179	−380	1·05943	−348
28	·98034	−445	1·02112	−432	1·04867	−413	1·06340	−388	1·06653	−359
29	·95871	−437	1·00619	−429	1·04023	−415	1·06113	−395	1·07004	−370
30	·93272	−427	0·98697	−425	1·02764	−415	1·05491	−400	1·06986	−380
31	·90245	−415	·96350	−418	1·01090	−414	1·04469	−404	1·06588	−388
32	·86804	−401	·93586	−410	0·99001	−411	1·03044	−406	1·05801	−395
33	·82963	−384	·90411	−399	·96502	−406	1·01212	−407	1·04620	−401
34	·78736	−366	·86838	−387	·93596	−400	0·98974	−405	1·03038	−405
35	·74144	−347	·82878	−372	·90290	−391	·96330	−402	1·01051	−407
36	·69204	−325	·78545	−356	·86593	−380	·93283	−397	0·98657	−407
37	·63941	−301	·73856	−338	·82516	−367	·89840	−390	·95856	−405
38	·58375	−276	·68829	−318	·78072	−352	·86007	−380	·92650	−401
39	·52533	−250	·63485	−296	·73275	−335	·81794	−368	·89042	−395
40	·46442	−222	·57845	−272	·68144	−316	·77212	−354	·85039	−386
41	·40128	−192	·51933	−246	·62696	−295	·72277	−338	·80650	−375
42	·33623	−162	·45775	−219	·56954	−271	·67003	−319	·75886	−360
43	·26955	−130	·39398	−190	·50940	−246	·61412	−297	·70762	−344
44	·20157	− 98	·32831	−160	·44681	−218	·55523	−273	·65294	−324
45	0·13260	− 65	0·26104	−128	0·38203	−189	0·49361	−247	0·59503	−302

TABLE XXI—*continued.*

$se_4(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta=1$ $b_4=16{\cdot}03297$	Δ^2	$\theta=2$ $b_4=16{\cdot}12769$	Δ^2	$\theta=3$ $b_4=16{\cdot}27270$	Δ^2	$\theta=4$ $b_4=16{\cdot}45204$	Δ^2	$\theta=5$ $b_4=16{\cdot}64822$	Δ^2
45°	0·13260	−65	0·26104	−128	0·38203	−189	0·49361	−247	0·59503	−302
46	·06300	−31	·19249	−95	·31536	−158	·42952	−219	·53409	−276
47	−·00692	3	·12299	−61	·24710	−126	·36324	−188	·47040	−248
48	−·07681	38	·05287	−27	·17759	−91	·29508	−155	·40422	−218
49	−·14631	73	−·01751	9	·10717	−56	·22536	−120	·33587	−184
50	−·21508	107	−·08781	45	·03619	−19	·15445	−84	·26567	−149
51	−·28279	142	−·15765	81	−·03498	19	·08269	−46	·19399	−110
52	−·34907	176	−·22668	118	−·10596	57	·01048	−6	·12120	−70
53	−·41361	209	−·29453	155	−·17637	96	−·06179	35	·04771	−28
54	−·47605	241	−·36083	191	−·24581	136	−·13370	77	−·02606	16
55	−·53608	273	−·42522	227	−·31390	175	−·20485	120	−·09968	61
56	−·59338	303	−·48735	262	−·38024	214	−·27479	163	−·17268	107
57	−·64764	332	−·54687	296	−·44443	253	−·34311	206	−·24461	154
58	−·69859	360	−·60342	329	−·50609	291	−·40937	249	−·31500	202
59	−·74594	385	−·65669	360	−·56483	328	−·47314	291	−·38337	249
60	−·78943	409	−·70636	390	−·62030	364	−·53400	333	−·44924	296
61	−·82883	431	−·75213	418	−·67212	398	−·59153	373	−·51216	342
62	−·86391	451	−·79373	444	−·71996	430	−·64534	411	−·57165	387
63	−·89449	469	−·83088	467	−·76350	460	−·69503	448	−·62727	430
64	−·92037	484	−·86336	489	−·80243	488	−·74025	482	−·67858	471
65	−·94142	496	−·89095	507	−·83649	513	−·78065	513	−·72519	510
66	−·95750	506	−·91347	523	−·86542	535	−·81592	542	−·76670	545
67	−·96852	514	−·93076	536	−·88901	553	−·84577	567	−·80276	577
68	−·97440	518	−·94269	545	−·90706	569	−·86995	588	−·83306	605
69	−·97510	520	−·94917	552	−·91942	580	−·88825	606	−·85731	628
70	−·97060	519	−·95012	555	−·92599	588	−·90050	619	−·87527	648
71	−·96090	515	−·94553	555	−·92667	592	−·90655	628	−·88676	662
72	−·94606	508	−·93538	551	−·92142	593	−·90632	633	−·89163	671
73	−·92613	499	−·91972	544	−·91025	589	−·89977	632	−·88978	675
74	−·90121	486	−·89862	534	−·89319	581	−·88690	627	−·88117	674
75	−·87142	471	−·87218	520	−·87032	569	−·86775	618	−·86583	667
76	−·83693	454	−·84053	503	−·84177	553	−·84243	603	−·84382	654
77	−·79789	433	−·80385	483	−·80768	533	−·81107	584	−·81526	636
78	−·75453	410	−·76235	459	−·76827	509	−·77388	560	−·78034	612
79	−·70706	385	−·71625	433	−·72377	481	−·73109	531	−·73929	583
80	−·65574	358	−·66583	403	−·67446	450	−·68299	498	−·69242	549
81	−·60085	328	−·61138	371	−·62065	415	−·62990	461	−·64005	510
82	−·54267	297	−·55321	336	−·56269	378	−·57220	420	−·58259	466
83	−·48153	264	−·49169	300	−·50095	337	−·51030	376	−·52047	417
84	−·41775	229	−·42716	261	−·43585	294	−·44463	329	·45417	365
85	−·35169	193	−·36004	220	−·36780	248	−·37568	278	−·38422	310
86	−·28369	156	−·29071	178	−·29727	201	−·30395	226	−·31118	251
87	−·21414	117	−·21960	134	−·22473	152	−·22996	171	−·23561	191
88	−·14342	79	−·14715	90	−·15067	102	−·15426	115	−·15814	128
89	−·07191	39	−·07380	45	−·07559	51	−·07742	58	−·07939	64
90	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0

TABLE XXI—*continued.*

$se_4(x, \theta)$. $\theta=6$ to $\theta=10$.

x	$\theta=6$ $b_4=16{\cdot}84460$	Δ^2	$\theta=7$ $b_4=17{\cdot}02666$	Δ^2	$\theta=8$ $b_4=17{\cdot}18253$	Δ^2	$\theta=9$ $b_4=17{\cdot}30301$	Δ^2	$\theta=10$ $b_4=17{\cdot}38138$	Δ^2
° 0	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0
1	·04973	− 7	·04581	− 4	·04194	− 2	·03817	1	·03457	3
2	·09939	− 15	·09158	− 9	·08386	− 3	·07635	2	·06916	5
3	·14891	− 22	·13726	− 13	·12575	− 5	·11455	2	·10381	8
4	·19820	− 30	·18281	− 18	·16759	− 7	·15277	2	·13854	10
5	·24719	− 38	·22819	− 23	·20936	− 9	·19101	2	·17337	12
6	·29580	− 46	·27334	− 28	·25104	− 12	·22927	2	·20832	14
7	·34395	− 55	·31821	− 33	·29261	− 15	·26756	1	·24341	15
8	·39156	− 63	·36275	− 39	·33402	− 18	·30586	0	·27865	16
9	·43853	− 73	·40689	− 46	·37525	− 22	·34416	− 2	·31404	16
10	·48477	− 82	·45057	− 53	·41626	− 27	·38244	− 5	·34959	15
11	·53020	− 92	·49372	− 61	·45699	− 33	·42067	− 8	·38529	14
12	·57470	−103	·53627	− 69	·49740	− 39	·45882	− 12	·42113	11
13	·61817	−114	·57811	− 78	·53742	− 46	·49686	− 17	·45708	8
14	·66050	−126	·61918	− 88	·57697	− 54	·53472	− 23	·49311	4
15	·70157	−138	·65937	− 99	·61599	− 62	·57235	− 30	·52918	− 1
16	·74126	−151	·69857	−110	·65439	− 72	·60968	− 38	·56525	− 7
17	·77944	−164	·73667	−122	·69206	− 83	·64663	− 47	·60124	− 15
18	·81599	−177	·77356	−134	·72891	− 94	·68311	− 57	·63708	− 23
19	·85076	−192	·80910	−148	·76481	−107	·71903	− 68	·67268	− 33
20	·88362	−206	·84316	−162	·79965	−120	·75425	− 81	·70796	− 45
21	·91442	−221	·87561	−177	·83329	−134	·78867	− 94	·74279	− 57
22	·94301	−236	·90629	−192	·86558	−150	·82215	−109	·77704	− 71
23	·96924	−251	·93504	−208	·89638	−166	·85453	−125	·81059	− 86
24	0·99296	−267	·96172	−224	·92552	−183	·88566	−142	·84328	−103
25	1·01401	−282	0·98615	−241	·95283	−200	·91537	−160	·87493	−121
26	1·03224	−297	1·00817	−258	0·97815	−219	·94349	−179	·90538	−140
27	1·04750	−312	1·02761	−275	1·00127	−237	·96981	−199	·93443	−160
28	1·05963	−327	1·04430	−293	1·02202	−256	0·99415	−219	·96188	−182
29	1·06849	−341	1·05806	−310	1·04021	−276	1·01630	−240	0·98751	−204
30	1·07395	−355	1·06872	−326	1·05564	−295	1·03604	−262	1·01110	−227
31	1·07585	−367	1·07612	−343	1·06812	−315	1·05316	−284	1·03242	−251
32	1·07408	−379	1·08009	−358	1·07745	−334	1·06744	−306	1·05122	−276
33	1·06852	−389	1·08049	−373	1·08345	−352	1·07866	−328	1·06727	−301
34	1·05907	−398	1·07715	−386	1·08592	−370	1·08660	−349	1·08030	−325
35	1·04564	−406	1·06994	−399	1·08469	−387	1·09105	−370	1·09009	−350
36	1·02815	−411	1·05875	−409	1·07960	−402	1·09179	−390	1·09637	−374
37	1·00654	−415	1·04347	−418	1·07048	−416	1·08863	−409	1·09892	−397
38	0·98079	−416	1·02400	−425	1·05719	−429	1·08138	−426	1·09749	−419
39	·95088	−415	1·00028	−430	1·03962	−439	1·06986	−442	1·09187	−440
40	·91681	−412	0·97226	−432	1·01767	−446	1·05393	−455	1·08185	−458
41	·87862	−406	·93992	−431	0·99125	−451	1·03344	−466	1·06725	−474
42	·83637	−397	·90326	−428	·96032	−453	1·00830	−473	1·04791	−488
43	·79015	−385	·86233	−421	·92486	−452	0·97843	−478	1·02369	−498
44	·74008	−370	·81718	−411	·88487	−448	·94378	−479	0·99449	−505
45	0·68632	−352	0·76792	−398	0·84040	−440	0·90433	−476	0·96023	−508

TABLE XXI—*continued.*

$se_4(x, \theta)$. $\theta = 6$ to $\theta = 10$.

x	$\theta=6$ $b_4=16{\cdot}84460$		$\theta=7$ $b_4=17{\cdot}02666$		$\theta=8$ $b_4=17{\cdot}18253$		$\theta=9$ $b_4=17{\cdot}30301$		$\theta=10$ $b_4=17{\cdot}38138$	
°		Δ^2		Δ^2		Δ^2		Δ^2		Δ^2
45	0·68632	−352	0·76792	−398	0·84040	−440	0·90433	−476	0·96023	−508
46	·62903	−331	·71467	−381	·79154	−427	·86013	−469	·92090	−507
47	·56843	−306	·65762	−360	·73840	−411	·81122	−458	·87650	−501
48	·50478	−278	·59697	−336	·68115	−391	·75774	−442	·82709	−490
49	·43835	−247	·53295	−308	·62000	−366	·69983	−422	·77278	−474
50	·36945	−213	·46586	−276	·55518	−337	·63771	−396	·71372	−453
51	·29842	−176	·39601	−240	·48698	−304	·57161	−366	·65014	−426
52	·22563	−136	·32376	−201	·41575	−266	·50186	−331	·58229	−394
53	·15149	−93	·24949	−158	·34186	−225	·42880	−290	·51051	−356
54	·07643	−48	·17365	−113	·26572	−179	·35284	−245	·43517	−312
55	·00088	0	·09667	−64	·18780	−129	·27443	−196	·35672	−263
56	− ·07467	49	·01906	−13	·10858	−76	·19405	−142	·27563	−208
57	− ·14973	99	− ·05869	41	·02859	−20	·11226	−84	·19247	−149
58	− ·22380	151	− ·13602	96	− ·05160	38	·02963	−22	·10781	−86
59	− ·29636	203	− ·21239	153	− ·13140	99	− ·05323	42	·02229	−18
60	− ·36690	255	− ·28724	210	− ·21022	161	− ·13566	109	− ·06341	53
61	− ·43488	307	− ·35998	268	− ·28742	225	− ·21701	178	− ·14857	127
62	− ·49979	358	− ·43005	326	− ·36238	288	− ·29658	247	− ·23246	202
63	− ·56111	408	− ·49685	382	− ·43445	352	− ·37368	317	− ·31434	279
64	− ·61835	456	− ·55984	437	− ·50300	414	− ·44761	387	− ·39342	356
65	− ·67103	502	− ·61846	490	− ·56741	475	− ·51766	455	− ·46894	432
66	− ·71868	544	− ·67217	540	− ·62708	532	− ·58317	521	− ·54015	506
67	− ·76090	583	− ·72048	587	− ·68142	587	− ·64347	584	− ·60630	577
68	− ·79729	618	− ·76293	629	− ·72990	638	− ·69792	643	− ·66667	645
69	− ·82749	649	− ·79908	667	− ·77200	683	− ·74595	697	− ·72060	707
70	− ·85120	675	− ·82856	700	− ·80726	723	− ·78701	745	− ·76745	764
71	− ·86816	695	− ·85104	727	− ·83529	757	− ·82063	787	− ·80666	814
72	− ·87817	710	− ·86625	748	− ·85575	785	− ·84637	821	− ·83774	856
73	− ·88109	719	− ·87398	762	− ·86836	805	− ·86391	847	− ·86026	889
74	− ·87682	721	− ·87410	769	− ·87292	817	− ·87298	865	− ·87389	913
75	− ·86534	717	− ·86653	769	− ·86931	821	− ·87339	874	− ·87838	928
76	− ·84668	707	− ·85127	761	− ·85749	817	− ·86506	874	− ·87360	932
77	− ·82095	690	− ·82839	747	− ·83750	805	− ·84799	864	− ·85950	925
78	− ·78832	667	− ·79805	724	− ·80946	783	− ·82228	844	− ·83615	907
79	− ·74901	638	− ·76047	695	− ·77359	754	− ·78813	815	− ·80374	879
80	− ·70333	602	− ·71594	658	− ·73018	716	− ·74582	777	− ·76253	839
81	− ·65163	560	− ·66484	614	− ·67962	670	− ·69575	729	− ·71293	790
82	− ·59432	513	− ·60759	564	− ·62235	617	− ·63838	672	− ·65544	730
83	− ·53188	461	− ·54471	507	− ·55891	556	− ·57430	608	− ·59064	661
84	− ·46483	404	− ·47676	446	− ·48991	490	− ·50414	536	− ·51923	584
85	− ·39374	343	− ·40434	379	− ·41601	417	− ·42862	457	− ·44198	499
86	− ·31921	279	− ·32814	308	− ·33795	340	− ·34854	373	− ·35975	407
87	− ·24188	212	− ·24885	234	− ·25649	258	− ·26472	283	− ·27345	310
88	− ·16244	142	− ·16721	158	− ·17244	174	− ·17808	191	− ·18404	209
89	− ·08158	72	− ·08400	79	− ·08666	87	− ·08952	96	− ·09255	105
90	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0

TABLE XXII.

$ce_4(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta=1$, $a_4=16{\cdot}03383$	Δ^2	$\theta=2$, $a_4=16{\cdot}14120$	Δ^2	$\theta=3$, $a_4=16{\cdot}33872$	Δ^2	$\theta=4$, $a_4=16{\cdot}64982$	Δ^2	$\theta=5$, $a_4=17{\cdot}09658$	Δ^2
0°	1·03514	-442	1·07440	-397	1·11684	-352	1·15864	-305	1·19333	-258
1	1·03292	-441	1·07241	-397	1·11508	-351	1·15711	-305	1·19204	-258
2	1·02630	-439	1·06646	-395	1·10981	-350	1·15254	-304	1·18817	-258
3	1·01528	-434	1·05656	-391	1·10104	-348	1·14492	-303	1·18173	-257
4	0·99993	-428	1·04275	-387	1·08879	-345	1·13427	-302	1·17271	-257
5	·98029	-420	1·02507	-381	1·07310	-341	1·12060	-299	1·16112	-256
6	·95646	-410	1·00358	-374	1·05399	-336	1·10394	-297	1·14696	-256
7	·92853	-398	0·97835	-365	1·03153	-330	1·08431	-293	1·13025	-255
8	·89661	-385	·94947	-356	1·00576	-324	1·06175	-290	1·11100	-253
9	·86084	-370	·91704	-345	0·97675	-316	1·03629	-285	1·08921	-252
10	·82136	-354	·88116	-332	·94458	-308	1·00798	-280	1·06491	-250
11	·77834	-336	·84196	-319	·90934	-298	0·97686	-275	1·03811	-247
12	·73197	-317	·79957	-304	·87110	-288	·94300	-268	1·00883	-245
13	·68243	-296	·75414	-288	·82999	-277	·90645	-261	0·97711	-241
14	·62992	-274	·70583	-271	·78611	-264	·86729	-253	·94298	-237
15	·57469	-250	·65481	-253	·73959	-251	·82560	-244	·90647	-233
16	·51695	-226	·60126	-233	·69056	-237	·78147	-235	·86763	-228
17	·45695	-200	·54537	-213	·63916	-221	·73499	-224	·82652	-222
18	·39495	-173	·48736	-192	·58556	-205	·68627	-213	·78319	-215
19	·33122	-146	·42743	-169	·52990	-187	·63542	-200	·73771	-207
20	·26603	-117	·36581	-146	·47237	-169	·58256	-187	·69016	-198
21	·19967	- 88	·30274	-121	·41315	-149	·52785	-172	·64063	-189
22	·13242	- 59	·23845	- 96	·35244	-129	·47141	-156	·58921	-178
23	·06458	- 29	·17320	- 70	·29044	-108	·41341	-140	·53602	-166
24	- ·00354	2	·10724	- 44	·22736	- 85	·35401	-122	·48117	-152
25	- ·07165	32	·04085	- 17	·16343	- 62	·29339	-103	·42480	-138
26	- ·13944	63	- ·02572	11	·09887	- 38	·23175	- 83	·36704	-122
27	- ·20660	93	- ·09217	39	·03394	- 13	·16928	- 62	·30807	-105
28	- ·27282	124	- ·15824	67	- ·03112	12	·10620	- 39	·24804	- 87
29	- ·33780	154	- ·22364	96	- ·09606	39	·04272	- 16	·18715	- 67
30	- ·40124	184	- ·28808	124	- ·16062	65	- ·02092	8	·12558	- 46
31	- ·46285	213	- ·35128	153	- ·22452	93	- ·08448	33	·06355	- 24
32	- ·52233	241	- ·41296	181	- ·28750	120	- ·14770	59	·00128	0
33	- ·57940	269	- ·47283	209	- ·34927	148	- ·21033	86	- ·06099	24
34	- ·63378	295	- ·53060	237	- ·40957	176	- ·27211	113	- ·12301	50
35	- ·68521	320	- ·58601	264	- ·46811	204	- ·33275	141	- ·18454	77
36	- ·73344	344	- ·63879	290	- ·52461	231	- ·39198	169	- ·24530	105
37	- ·77823	367	- ·68866	315	- ·57880	259	- ·44952	198	- ·30501	133
38	- ·81934	388	- ·73538	340	- ·63040	286	- ·50508	226	- ·36339	163
39	- ·85658	407	- ·77870	363	- ·67914	312	- ·55837	255	- ·42014	192
40	- ·88974	425	- ·81839	385	- ·72476	338	- ·60912	283	- ·47497	222
41	- ·91866	441	- ·85423	405	- ·76700	362	- ·65704	311	- ·52758	252
42	- ·94317	454	- ·88602	424	- ·80563	385	- ·70184	338	- ·57766	282
43	- ·96313	466	- ·91357	441	- ·84039	407	- ·74327	364	- ·62492	312
44	- ·97843	476	- ·93670	456	- ·87109	428	- ·78106	389	- ·66906	341
45	-0·98897	483	-0·95527	470	-0·89750	447	-0·81495	413	- ·70978	370

TABLE XXII—*continued.*

$ce_4(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta=1$ $a_4=16·03383$	Δ^2	$\theta=2$ $a_4=16·14120$	Δ^2	$\theta=3$ $a_4=16·33872$	Δ^2	$\theta=4$ $a_4=16·64982$	Δ^2	$\theta=5$ $a_4=17·09658$	Δ^2
45°	−0·98897	483	−0·95527	470	−0·89750	447	−0·81495	413	−0·70978	370
46	− ·99469	488	− ·96914	480	− ·91945	463	− ·84470	435	− ·74681	397
47	− ·99553	490	− ·97821	489	− ·93677	478	− ·87011	456	− ·77988	423
48	− ·99146	490	− ·98239	495	− ·94930	490	− ·89095	474	− ·80871	447
49	− ·98249	488	− ·98162	499	− ·95693	500	− ·90705	491	− ·83308	469
50	− ·96865	483	− ·97585	500	− ·95956	508	− ·91825	504	− ·85276	489
51	− ·94997	476	− ·96508	499	− ·95711	512	− ·92440	515	− ·86755	507
52	− ·92653	466	− ·94933	495	− ·94954	514	− ·92540	524	− ·87727	521
53	− ·89843	454	− ·92863	488	− ·93682	513	− ·92116	529	− ·88178	533
54	− ·86580	439	− ·90305	478	− ·91897	509	− ·91163	531	− ·88096	541
55	− ·82878	422	− ·87270	465	− ·89603	502	− ·89680	529	− ·87473	546
56	− ·78753	402	− ·83769	450	− ·86807	491	− ·87667	524	− ·86303	548
57	− ·74227	381	− ·79818	432	− ·83520	478	− ·85130	516	− ·84586	545
58	− ·69319	357	− ·75436	411	− ·79756	461	− ·82077	504	− ·82324	538
59	− ·64055	331	− ·70643	388	− ·75531	440	− ·78520	488	− ·79523	528
60	− ·58460	303	− ·65462	362	− ·70865	417	− ·74476	468	− ·76195	513
61	− ·52561	274	− ·59919	333	− ·65783	391	− ·69963	445	− ·72354	493
62	− ·46389	242	− ·54044	302	− ·60309	362	− ·65006	418	− ·68020	470
63	− ·39975	209	− ·47866	269	− ·54474	329	− ·59630	388	− ·63217	442
64	− ·33351	175	− ·41419	235	− ·48309	295	− ·53867	354	− ·57971	410
65	− ·26552	140	− ·34737	198	− ·41850	257	− ·47751	317	− ·52315	375
66	− ·19613	104	− ·27857	160	− ·35134	218	− ·41317	277	− ·46284	335
67	− ·12570	67	− ·20818	120	− ·28200	176	− ·34607	234	− ·39918	292
68	− ·05460	29	− ·13658	79	− ·21090	133	− ·27663	189	− ·33261	246
69	·01678	− 9	− ·06420	37	− ·13847	88	− ·20530	141	− ·26357	197
70	·08808	− 47	·00856	− 5	− ·06517	41	− ·13256	92	− ·19257	145
71	·15890	− 85	·08126	− 48	·00855	− 6	− ·05891	41	− ·12011	91
72	·22887	−123	·15349	− 91	·08221	− 53	·01516	− 11	− ·04674	36
73	·29761	−160	·22481	−133	·15534	−101	·08912	− 63	·02698	− 21
74	·36475	−197	·29480	−175	·22747	−148	·16245	−116	·10049	− 78
75	·42992	−233	·36304	−217	·29811	−195	·23462	−168	·17323	−136
76	·49276	−267	·42911	−257	·36679	−242	·30510	−220	·24460	−193
77	·55294	−300	·49261	−296	·43306	−287	·37339	−271	·31404	−249
78	·61011	−332	·55315	−333	·49646	−330	·43896	−320	·38099	−304
79	·66396	−362	·61035	−369	·55656	−371	·50133	−367	·44490	−357
80	·71420	−390	·66387	−402	·61296	−410	·56002	−412	·50523	−407
81	·76054	−415	·71336	−433	·66525	−446	·61460	−454	·56149	−455
82	·80273	−439	·75853	−462	·71307	−480	·66463	−492	·61320	−499
83	·84052	−460	·79907	−487	·75610	−510	·70975	−527	·65993	−538
84	·87372	−479	·83475	−510	·79403	−537	·74958	−558	·70127	−574
85	·90214	−495	·86533	−529	·82658	−560	·78383	−585	·73687	−604
86	·92560	−508	·89061	−545	·85354	−579	·81223	−608	·76643	−630
87	·94399	−518	·91045	−558	·87471	−594	·83456	−625	·78969	−650
88	·95721	−525	·92471	−567	·88994	−605	·85063	−638	·80645	−665
89	·96516	−530	·93330	−572	·89912	−611	·86032	−645	·81657	−673
90	0·96782	−531	0·93617	−574	0·90219	−614	0·86356	−648	0·81995	−676

TABLE XXII—*continued.*

$ce_4(x, \theta)$. $\theta=6$ to $\theta=10$.

x	$\theta=6$ $a_4=17\cdot68878$		$\theta=7$ $a_4=18\cdot41661$		$\theta=8$ $a_4=19\cdot25271$		$\theta=9$ $a_4=20\cdot16093$		$\theta=10$ $a_4=21\cdot10463$	
°		Δ^2		Δ^2		Δ^2		Δ^2		Δ^2
0	1·21410	−210	1·21689	−164	1·20162	−119	1·17071	−77	1·12711	−38
1	1·21304	−210	1·21607	−164	1·20103	−119	1·17032	−77	1·12692	−38
2	1·20989	−211	1·21362	−165	1·19924	−120	1·16916	−79	1·12634	−40
3	1·20462	−211	1·20951	−166	1·19625	−122	1·16722	−80	1·12537	−42
4	1·19725	−212	1·20375	−167	1·19204	−124	1·16447	−83	1·12398	−44
5	1·18775	−212	1·19632	−169	1·18659	−126	1·16089	−86	1·12215	−48
6	1·17614	−213	1·18721	−171	1·17987	−130	1·15645	−90	1·11984	−53
7	1·16239	−214	1·17638	−173	1·17187	−133	1·15110	−95	1·11700	−58
8	1·14649	−215	1·16382	−176	1·16253	−137	1·14482	−100	1·11358	−64
9	1·12845	−216	1·14951	−179	1·15182	−142	1·13753	−105	1·10952	−70
10	1·10826	−216	1·13341	−182	1·13969	−146	1·12919	−112	1·10475	−78
11	1·08590	−217	1·11549	−185	1·12610	−152	1·11974	−118	1·09921	−86
12	1·06136	−217	1·09572	−188	1·11099	−157	1·10910	−126	1·09281	−94
13	1·03466	−218	1·07408	−191	1·09432	−162	1·09720	−133	1·08547	−103
14	1·00577	−217	1·05053	−194	1·07602	−168	1·08397	−141	1·07709	−113
15	0·97472	−217	1·02504	−196	1·05604	−174	1·06934	−149	1·06758	−123
16	·94150	−215	0·99758	−199	1·03432	−179	1·05321	−157	1·05684	−133
17	·90612	−214	·96814	−201	1·01082	−184	1·03551	−165	1·04477	−144
18	·86861	−211	·93669	−202	0·98547	−189	1·01616	−173	1·03126	−155
19	·82899	−208	·90322	−203	·95822	−194	0·99508	−181	1·01620	−165
20	·78729	−204	·86771	−203	·92904	−198	·97219	−189	0·99948	−176
21	·74355	−199	·83018	−203	·89788	−201	·94741	−196	·98101	−186
22	·69783	−192	·79062	−201	·86470	−204	·92067	−202	·96067	−197
23	·65018	−185	·74905	−198	·82949	−206	·89191	−208	·93836	−206
24	·60069	−177	·70549	−194	·79222	−206	·86107	−213	·91399	−215
25	·54942	−167	·66000	−189	·75289	−206	·82810	−217	·88748	−223
26	·49649	−156	·61261	−183	·71150	−204	·79296	−219	·85873	−230
27	·44200	−143	·56339	−175	·66808	−200	·75564	−220	·82769	−236
28	·38608	−129	·51243	−165	·62265	−195	·71611	−220	·79429	−240
29	·32887	−113	·45981	−154	·57527	−189	·67438	−218	·75849	−243
30	·27052	−96	·40566	−141	·52601	−180	·63047	−214	·72026	−243
31	·21122	−77	·35010	−126	·47494	−170	·58441	−208	·67960	−242
32	·15114	−57	·29327	−110	·42218	−157	·53628	−200	·63652	−239
33	·09049	−35	·23535	−91	·36784	−143	·48614	−190	·59105	−233
34	·02948	−12	·17652	−71	·31208	−126	·43410	−177	·54324	−225
35	−·03164	13	·11699	−48	·25506	−107	·38029	−162	·49319	−214
36	−·09263	40	·05697	−24	·19697	−86	·32486	−144	·44099	−200
37	−·15322	67	−·00330	2	·13802	−62	·26799	−124	·38679	−183
38	−·21314	96	−·06355	29	·07845	−37	·20987	−101	·33076	−164
39	−·27210	126	−·12350	58	·01852	−9	·15075	−75	·27309	−141
40	−·32980	157	−·18287	89	−·04150	21	·09088	−47	·21401	−115
41	−·38593	188	−·24135	121	−·10132	53	·03054	−16	·15378	−86
42	−·44017	220	−·29862	154	−·16060	86	−·02996	17	·09270	−53
43	−·49222	253	−·35435	188	−·21903	121	−·09030	52	·03109	−18
44	−·54174	285	−·40819	223	−·27624	157	−·15011	89	−·03071	19
45	−0·58840	317	−0·45980	258	−0·33188	195	−0·20903	128	−0·09232	60

TABLE XXII—*continued.*

$ce_4(x, \theta)$. $\theta=6$ to $\theta=10$.

x	$\theta=6$ $a_4=17{\cdot}68878$		$\theta=7$ $a_4=18{\cdot}41661$		$\theta=8$ $a_4=19{\cdot}25271$		$\theta=9$ $a_4=20{\cdot}16093$		$\theta=10$ $a_4=21{\cdot}10463$	
°		Δ^2		Δ^2		Δ^2		Δ^2		Δ^2
45	−0·58840	317	−0·45980	258	−0·33188	195	−0·20903	128	−0·09232	60
46	− ·63190	348	− ·50883	293	− ·38557	233	− ·26666	169	− ·15333	102
47	− ·67191	379	− ·55494	328	− ·43694	271	− ·32261	211	− ·21332	146
48	− ·70813	408	− ·59776	362	− ·48559	309	− ·37644	253	− ·27184	192
49	− ·74027	436	− ·63697	395	− ·53115	347	− ·42776	295	− ·32845	239
50	− ·76804	462	− ·67222	427	− ·57323	385	− ·47611	338	− ·38266	287
51	− ·79119	486	− ·70321	457	− ·61147	420	− ·52109	379	− ·43401	334
52	− ·80948	508	− ·72963	484	− ·64551	454	− ·56228	420	− ·48203	381
53	− ·82269	526	− ·75121	509	− ·67500	486	− ·59927	458	− ·52623	427
54	− ·83064	541	− ·76769	532	− ·69962	515	− ·63168	495	− ·56617	470
55	− ·83318	553	− ·77886	550	− ·71910	541	− ·65914	528	− ·60140	512
56	− ·83019	561	− ·78452	565	− ·73315	563	− ·68131	558	− ·63152	550
57	− ·82160	565	− ·78454	576	− ·74158	582	− ·69791	585	− ·65614	584
58	− ·80736	564	− ·77879	582	− ·74419	595	− ·70867	605	− ·67491	614
59	− ·78748	559	− ·76723	584	− ·74085	604	− ·71338	621	− ·68756	638
60	− ·76201	549	− ·74983	580	− ·73147	607	− ·71187	632	− ·69382	657
61	− ·73105	535	− ·72662	571	− ·71603	604	− ·70405	636	− ·69351	669
62	− ·69473	516	− ·69771	557	− ·69454	596	− ·68986	635	− ·68651	675
63	− ·65326	492	− ·66322	538	− ·66709	582	− ·66932	626	− ·67276	673
64	− ·60686	463	− ·62335	513	− ·63382	561	− ·64253	611	− ·65228	663
65	− ·55584	430	− ·57835	483	− ·59494	535	− ·60962	589	− ·62518	646
66	− ·50051	392	− ·52853	447	− ·55071	502	− ·57083	559	− ·59161	621
67	− ·44127	349	− ·47423	406	− ·50146	463	− ·52644	523	− ·55183	588
68	− ·37853	303	− ·41588	361	− ·44757	419	− ·47682	480	− ·50618	547
69	− ·31277	253	− ·35392	310	− ·38950	369	− ·42240	431	− ·45506	498
70	− ·24446	200	− ·28886	256	− ·32774	314	− ·36367	376	− ·39896	442
71	− ·17417	144	− ·22124	198	− ·26283	255	− ·30118	315	− ·33844	380
72	− ·10243	85	− ·15163	137	− ·19538	191	− ·23554	249	− ·27412	311
73	− ·02984	25	− ·08065	74	− ·12601	125	− ·16742	179	− ·20670	237
74	·04301	− 37	− ·00894	8	− ·05539	55	− ·09751	105	− ·13690	158
75	·11548	− 99	·06286	− 58	·01577	− 16	− ·02655	29	− ·06552	76
76	·18697	−161	·13407	−120	·08678	− 88	·04470	− 49	·00662	− 8
77	·25685	−223	·20402	−193	·15690	−161	·11546	−128	·07868	− 94
78	·32450	−283	·27205	−258	·22542	−232	·18494	−206	·14981	−180
79	·38932	−342	·33749	−323	·29161	−303	·25236	−283	·21914	−264
80	·45072	−397	·39971	−384	·35477	−370	·31694	−358	·28583	−347
81	·50815	−450	·45808	−442	·41424	−435	·37795	−429	·34905	−426
82	·56108	−499	·51203	−497	·46936	−495	·43467	−496	·40800	−501
83	·60902	−544	·56102	−546	·51953	−550	·48644	−557	·46194	−569
84	·65152	−584	·60454	−591	·56420	−599	·53264	−612	·51019	−631
85	·68819	−618	·64215	−629	·60287	−642	·57271	−660	·55213	−686
86	·71867	−647	·67346	−662	·63513	−678	·60618	−701	·58721	−731
87	·74269	−670	·69816	−687	·66060	−707	·63264	−733	·61498	−767
88	·76000	−686	·71599	−706	·67900	−728	·65178	−756	·63509	−793
89	·77046	−696	·72676	−717	·69012	−740	·66336	−770	·64725	−809
90	0·77396	−699	0·73037	−721	0·69384	−744	0·66723	−775	0·65132	−815

Table XXIII.

$se_5(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta = 1$, $b_5 = 25{\cdot}02084$	Δ^2	$\theta = 2$, $b_5 = 25{\cdot}08335$	Δ^2	$\theta = 3$, $b_5 = 25{\cdot}18708$	Δ^2	$\theta = 4$, $b_5 = 25{\cdot}33054$	Δ^2	$\theta = 5$, $b_5 = 25{\cdot}51082$	Δ^2
0°	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0
1	·08526	−60	·08318	−53	·08090	−47	·07840	−41	·07568	−36
2	·16993	−119	·16583	−106	·16133	−94	·15639	−83	·15100	−71
3	·25340	−178	·24742	−159	·24081	−141	·23355	−124	·22561	−107
4	·33510	−235	·32741	−211	·31889	−187	·30948	−164	·29915	−142
5	·41444	−291	·40530	−261	·39510	−232	·38376	−204	·37127	−177
6	·49088	−345	·48058	−310	·46899	−276	·45601	−243	·44161	−212
7	·56387	−396	·55276	−357	·54011	−318	·52582	−281	·50984	−245
8	·63290	−445	·62138	−402	·60806	−360	·59282	−318	·57562	−279
9	·69748	−491	·68597	−444	·67241	−399	·65664	−354	·63861	−311
10	·75715	−533	·74612	−484	·73277	−436	·71691	−389	·69848	−343
11	·81148	−572	·80143	−522	·78877	−471	·77329	−422	·75493	−373
12	·86009	−607	·85152	−556	·84005	−504	·82546	−453	·80765	−403
13	·90263	−638	·89606	−586	·88630	−534	·87310	−482	·85633	−431
14	·93879	−665	·93473	−613	·92720	−562	·91591	−509	·90071	−458
15	·96830	−687	·96727	−637	·96249	−586	·95363	−534	·94052	−483
16	0·99094	−704	0·99344	−656	0·99192	−607	0·98600	−557	0·97549	−506
17	1·00655	−716	1·01306	−671	1·01528	−625	1·01281	−577	1·00541	−527
18	1·01500	−723	1·02596	−682	1·03239	−639	1·03385	−594	1·03006	−546
19	1·01621	−725	1·03203	−689	1·04311	−650	1·04896	−608	1·04924	−563
20	1·01018	−722	1·03121	−691	1·04733	−657	1·05798	−619	1·06279	−578
21	0·99692	−714	1·02349	−689	1·04499	−659	1·06082	−626	1·07056	−589
22	·97651	−701	1·00887	−682	1·03605	−658	1·05740	−630	1·07244	−598
23	·94910	−683	0·98743	−671	1·02052	−653	1·04768	−631	1·06834	−604
24	·91486	−660	·95928	−654	0·99847	−644	1·03165	−627	1·05820	−606
25	·87402	−632	·92459	−634	·96998	−630	1·00935	−620	1·04199	−605
26	·82687	−599	·88357	−608	·93519	−612	0·98084	−609	1·01973	−601
27	·77373	−562	·83646	−579	·89429	−590	·94623	−594	0·99146	−593
28	·71497	−520	·78356	−545	·84748	−563	·90569	−575	·95727	−581
29	·65101	−475	·72521	−507	·79505	−533	·85939	−552	·91727	−564
30	·58230	−426	·66180	−465	·73728	−498	·80758	−524	·87163	−544
31	·50934	−373	·59373	−419	·67454	−459	·75052	−493	·82054	−520
32	·43264	−318	·52146	−370	·60720	−417	·68854	−457	·76426	−492
33	·35276	−260	·44550	−318	·53570	−371	·62198	−418	·70306	−459
34	·27028	−200	·36635	−263	·46048	−322	·55124	−375	·63727	−422
35	·18580	−138	·28458	−205	·38205	−269	·47675	−328	·56726	−381
36	·09995	−74	·20075	−146	·30093	−214	·39898	−278	·49343	−337
37	·01335	−10	·11546	−84	·21768	−156	·31844	−224	·41624	−288
38	−·07334	55	·02933	−21	·13286	−96	·23566	−168	·33617	−236
39	−·15949	119	−·05701	42	·04709	−34	·15120	−109	·25373	−181
40	−·24444	184	−·14294	106	−·03903	29	·06565	−48	·16948	−123
41	−·32755	247	−·22780	170	−·12486	93	−·02037	15	·08402	−62
42	−·40820	308	−·31096	234	−·20976	157	−·10624	79	−·00207	2
43	−·48576	368	−·39178	296	−·29310	221	−·19132	144	−·08814	67
44	−·55965	425	−·46965	357	−·37422	285	−·27496	210	−·17354	133
45	−0·62928	479	−0·54394	415	−0·45249	347	−0·35650	275	−0·25761	200

TABLE XXIII—*continued.*

$se_6(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta = 1$ $b_6 = 25{\cdot}02084$	Δ^2	$\theta = 2$ $b_6 = 25{\cdot}08335$	Δ^2	$\theta = 3$ $b_6 = 25{\cdot}18708$	Δ^2	$\theta = 4$ $b_6 = 25{\cdot}33054$	Δ^2	$\theta = 5$ $b_6 = 25{\cdot}51082$	Δ^2
° 45	−0·62928	479	−0·54394	415	−0·45249	347	−0·35650	275	−0·25761	200
46	− ·69412	530	− ·61409	472	− ·52730	408	− ·43529	339	− ·33968	268
47	− ·75366	577	− ·67952	525	− ·59803	466	− ·51068	403	− ·41907	334
48	− ·80742	620	− ·73970	574	− ·66409	522	− ·58205	464	− ·49512	400
49	− ·85498	658	− ·79414	620	− ·72494	574	− ·64878	522	− ·56716	465
50	− ·89596	692	− ·84238	661	− ·78005	623	− ·71028	578	− ·63456	526
51	− ·93002	720	− ·88401	697	− ·82892	668	− ·76601	630	− ·69669	585
52	− ·95688	743	− ·91866	729	− ·87112	706	− ·81545	677	− ·75297	640
53	− ·97631	760	− ·94603	754	− ·90626	741	− ·85811	719	− ·80285	691
54	− ·98813	771	− ·96586	774	− ·93400	769	− ·89358	756	− ·84582	736
55	− ·99225	776	− ·97795	787	− ·95404	791	− ·92149	787	− ·88143	776
56	− ·98860	775	− ·98216	795	− ·96618	807	− ·94153	812	− ·90927	810
57	− ·97719	768	− ·97843	796	− ·97025	816	− ·95344	830	− ·92902	836
58	− ·95810	755	− ·96674	790	− ·96615	818	− ·95706	840	− ·94040	856
59	− ·93146	736	− ·94716	777	− ·95388	813	− ·95228	843	− ·94322	867
60	− ·89746	711	− ·91980	758	− ·93347	801	− ·93907	838	− ·93737	871
61	− ·85634	680	− ·88486	733	− ·90506	781	− ·91748	826	− ·92282	865
62	− ·80843	643	− ·84259	701	− ·86883	755	− ·88763	805	− ·89961	852
63	− ·75409	601	− ·79331	662	− ·82505	721	− ·84972	777	− ·86789	829
64	− ·69373	554	− ·73741	618	− ·77407	680	− ·80405	740	− ·82788	798
65	− ·62783	503	− ·67533	568	− ·71627	633	− ·75098	696	− ·77988	758
66	− ·55690	447	− ·60756	513	− ·65215	580	− ·69094	645	− ·72431	710
67	− ·48151	387	− ·53466	453	− ·58223	520	− ·62444	587	− ·66164	654
68	− ·40224	324	− ·45723	389	− ·50711	455	− ·55208	522	− ·59243	590
69	− ·31973	258	− ·37590	321	− ·42743	386	− ·47450	452	− ·51733	519
70	− ·23464	190	− ·29136	250	− ·34390	312	− ·39239	376	− ·43704	441
71	− ·14766	120	− ·20433	176	− ·25725	234	− ·30654	295	− ·35234	358
72	− ·05948	48	− ·11554	100	− ·16826	154	− ·21773	211	− ·26406	270
73	·02918	− 24	− ·02576	22	− ·07773	71	− ·12681	123	− ·17308	178
74	·11760	− 96	·06425	− 56	·01351	− 12	− ·03466	34	− ·08032	83
75	·20507	−167	·15370	−134	·10463	− 97	·05782	− 57	·01327	− 14
76	·29087	−237	·24181	−211	·19477	−181	·14974	−148	·10672	−112
77	·37429	−306	·32782	−286	·28311	−264	·24018	−238	·19905	−209
78	·45466	−372	·41096	−360	·36882	−344	·32824	−326	·28930	−305
79	·53132	−435	·49051	−430	·45108	−422	·41305	−412	·37649	−399
80	·60362	−494	·56576	−497	·52912	−496	·49373	−494	·45969	−488
81	·67099	−550	·63605	−559	·60220	−566	·56948	−571	·53802	−573
82	·73285	−601	·70074	−617	·66961	−631	·63952	−643	·61060	−653
83	·78871	−647	·75926	−669	·73071	−690	·70313	−708	·67666	−725
84	·83809	−688	·81108	−716	·78492	−742	·75966	−767	·73547	−790
85	·88058	−723	·85575	−756	·83171	−787	·80852	−817	·78638	−846
86	·91585	−753	·89286	−789	·87062	−825	·84921	−859	·82883	−893
87	·94358	−776	·92207	−816	·90129	−855	·88131	−893	·86234	−931
88	·96356	−792	·94313	−835	·92341	−876	·90448	−917	·88655	−957
89	·97561	−802	·95584	−846	·93677	−889	·91848	−932	·90118	−974
90	0·97964	−806	0·96009	−850	0·94123	−893	0·92316	−936	0·90608	−979

TABLE XXIII—*continued.*

$se_5(x, \theta)$. $\theta = 6$ to $\theta = 10$.

x	$\theta=6$ $b_5=25\cdot72341$	Δ^2	$\theta=7$ $b_5=25\cdot96245$	Δ^2	$\theta=8$ $b_5=26\cdot22100$	Δ^2	$\theta=9$ $b_5=26\cdot49155$	Δ^2	$\theta=10$ $b_5=26\cdot76643$	Δ^2
° 0	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0
1	·07275	− 30	·06962	− 25	·06634	− 21	·06293	− 16	·05945	− 12
2	·14519	− 61	·13899	− 51	·13247	− 41	·12571	− 33	·11877	− 25
3	·21702	− 91	·20785	− 76	·19819	− 62	·18815	− 49	·17785	− 37
4	·28795	−121	·27595	−102	·26329	− 83	·25010	− 66	·23655	− 50
5	·35766	−151	·34303	−127	·32755	−104	·31139	− 83	·29476	− 64
6	·42585	−181	·40884	−153	·39077	−126	·37185	−101	·35232	− 77
7	·49223	−211	·47313	−178	·45273	−148	·43130	−119	·40912	− 92
8	·55650	−240	·53562	−204	·51322	−170	·48957	−137	·46499	−107
9	·61836	−270	·59608	−230	·57201	−192	·54647	−156	·51980	−123
10	·67753	−298	·65424	−255	·62888	−214	·60181	−176	·57338	−139
11	·73372	−326	·70985	−281	·68361	−237	·65538	−196	·62557	−157
12	·78665	−354	·76265	−306	·73597	−260	·70700	−216	·67619	−175
13	·83604	−380	·81239	−331	·78573	−283	·75646	−238	·72505	−194
14	·88162	−406	·85882	−356	·83265	−307	·80354	−259	·77198	−214
15	·92315	−431	·90170	−380	·87651	−330	·84802	−282	·81676	−235
16	·96036	−455	·94077	−404	·91706	−353	·88969	−304	·85920	−257
17	0·99303	−477	0·97581	−427	·95408	−377	·92832	−327	·89906	−279
18	1·02092	−498	1·00657	−449	0·98734	−399	·96367	−350	·93614	−302
19	1·04384	−517	1·03286	−470	1·01660	−421	0·99553	−373	0·97020	−325
20	1·06159	−534	1·05444	−489	1·04165	−443	1·02365	−396	1·00100	−349
21	1·07399	−550	1·07114	−508	1·06226	−464	1·04780	−419	1·02832	−373
22	1·08089	−563	1·08275	−524	1·07824	−483	1·06778	−440	1·05190	−397
23	1·08217	−573	1·08913	−539	1·08939	−501	1·08335	−462	1·07152	−420
24	1·07773	−581	1·09012	−551	1·09553	−518	1·09430	−481	1·08694	−443
25	1·06747	−585	1·08561	−561	1·09649	−532	1·10044	−500	1·09792	−465
26	1·05136	−587	1·07548	−568	1·09213	−544	1·10158	−517	1·10426	−486
27	1·02938	−585	1·05968	−572	1·08233	−554	1·09755	−532	1·10573	−505
28	1·00155	−580	1·03816	−573	1·06699	−561	1·08820	−544	1·10215	−523
29	0·96793	−571	1·01090	−571	1·04603	−565	1·07341	−554	1·09334	−538
30	·92859	−558	0·97794	−565	1·01942	−566	1·05308	−561	1·07915	−551
31	·88368	−540	·93933	−555	0·98716	−562	1·02714	−564	1·05945	−561
32	·83337	−519	·89517	−540	·94928	−555	0·99556	−564	1·03414	−567
33	·77786	−494	·84562	−522	·90584	−544	·95834	−559	1·00317	−569
34	·71742	−464	·79084	−499	·85697	−528	·91553	−550	0·96651	−567
35	·65235	−429	·73108	−471	·80282	−507	·86722	−537	·92417	−561
36	·58298	−391	·66661	−439	·74360	−482	·81353	−518	·87624	−549
37	·50971	−348	·59775	−402	·67957	−451	·75467	−494	·82281	−532
38	·43296	−301	·52487	−361	·61102	−416	·69086	−465	·76405	−510
39	·35320	−250	·44838	−314	·53832	−375	·62239	−431	·70020	−482
40	·27095	−195	·36875	−264	·46187	−329	·54962	−391	·63153	−448
41	·18674	−137	·28647	−209	·38213	−279	·47294	−345	·55839	−407
42	·10118	− 75	·20211	−151	·29960	−224	·39281	−294	·48117	−361
43	·01486	− 11	·11624	− 89	·21483	−164	·30974	−238	·40034	−309
44	− ·07157	55	·02949	− 23	·12842	−100	·22429	−176	·31642	−251
45	−0·15745	123	−0·05749	46	0·04101	− 32	0·13708	−110	0·23000	−187

TABLE XXIII—*continued.*

$se_5(x, \theta)$. $\theta=6$ to $\theta=10$.

x	$\theta=6$ $b_5=25{\cdot}72341$		$\theta=7$ $b_5=25{\cdot}96245$		$\theta=8$ $b_5=26{\cdot}22100$		$\theta=9$ $b_5=26{\cdot}49155$		$\theta=10$ $b_5=26{\cdot}76643$	
°		Δ^2		Δ^2		Δ^2		Δ^2		Δ^2
45	−0·15745	123	−0·05749	46	0·04101	−32	0·13708	−110	0·23000	−187
46	−·24209	193	−·14402	116	−·04673	38	·04876	−40	·14170	−118
47	−·32480	263	−·22938	188	−·13408	112	−·03995	34	·05222	−44
48	−·40489	333	−·31285	261	−·22031	187	−·12832	111	−·03771	33
49	−·48165	402	−·39372	335	−·30467	264	−·21559	191	−·12730	115
50	−·55439	469	−·47123	407	−·38639	341	−·30094	272	−·21574	199
51	−·62244	535	−·54468	479	−·46470	418	−·38358	353	−·30219	285
52	−·68513	597	−·61333	548	−·53882	494	−·46269	435	−·38580	371
53	−·74186	656	−·67650	614	−·60801	567	−·53746	515	−·46569	458
54	−·79203	710	−·73353	676	−·67153	637	−·60707	592	−·54100	543
55	−·83511	758	−·78380	734	−·72867	703	−·67076	667	−·61089	625
56	−·87060	801	−·82673	785	−·77879	764	−·72778	736	−·67452	703
57	−·89808	837	−·86180	830	−·82127	818	−·77744	800	−·73113	777
58	−·91720	865	−·88857	868	−·85556	865	−·81910	857	−·77996	844
59	−·92767	885	−·90666	898	−·88120	905	−·85219	906	−·82036	903
60	−·92928	897	−·91577	919	−·89780	935	−·87621	946	−·85173	953
61	−·92192	900	−·91570	930	−·90504	956	−·89077	977	−·87356	993
62	−·90556	894	−·90632	932	−·90273	966	−·89556	996	−·88547	1023
63	−·88026	878	−·88762	924	−·89075	966	−·89039	1004	−·88715	1040
64	−·84618	853	−·85968	905	−·86912	954	−·87517	1001	−·87843	1045
65	−·80357	818	−·82270	875	−·83794	931	−·84995	984	−·85926	1036
66	−·75279	773	−·77696	835	−·79746	896	−·81488	956	−·82973	1014
67	−·69427	720	−·72287	785	−·74802	850	−·77025	914	−·79007	977
68	−·62856	657	−·66093	725	−·69008	792	−·71649	860	−·74063	927
69	−·55627	586	−·59175	655	−·62421	724	−·65413	793	−·68193	864
70	−·47812	508	−·51601	576	−·55111	645	−·58384	715	−·61458	787
71	−·39490	423	−·43452	489	−·47156	557	−·50639	627	−·53936	698
72	−·30744	331	−·34813	395	−·38643	460	−·42267	528	−·45717	597
73	−·21668	235	−·25780	295	−·29671	356	−·33368	420	−·36901	487
74	−·12356	135	−·16452	189	−·20342	246	−·24048	305	−·27598	367
75	−·02909	32	−·06935	80	−·10766	131	−·14423	185	−·17928	240
76	·06570	−73	·02663	−31	−·01060	13	−·04614	59	−·08017	108
77	·15976	−178	·12229	−144	·08660	−107	·05255	−68	·02001	−27
78	·25204	−281	·21652	−255	·18272	−227	·15056	−197	·11992	−164
79	·34151	−383	·30819	−365	·27657	−340	·24660	−324	·21819	−301
80	·42715	−481	·39622	−472	·36696	−461	·33939	−448	·31345	−435
81	·50798	−574	·47952	−573	·45275	−571	·42771	−568	·40436	−563
82	·58306	−661	·55709	−668	·53283	−675	·51034	−680	·48964	−685
83	·65154	−741	·62798	−756	·60616	−770	·58618	−784	·56806	−798
84	·71261	−812	·69131	−834	·67179	−856	·65418	−878	·63851	−900
85	·76555	−875	·74630	−903	·72886	−931	·71339	−960	·69995	−989
86	·80974	−927	·79226	−960	·77663	−994	·76301	−1029	·75150	−1065
87	·84467	−968	·82862	−1006	·81445	−1044	·80234	−1084	·79240	−1125
88	·86992	−998	·85492	−1039	·84183	−1081	·83084	−1124	·82205	−1168
89	·88519	−1016	·87084	−1059	·85840	−1103	·84809	−1148	·84002	−1195
90	0·89030	−1022	0·87616	−1065	0·86395	−1110	0·85387	−1156	0·84604	−1204

TABLE XXIV.

$ce_6(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta=1$ $a_6=25{\cdot}02085$	Δ^2	$\theta=2$ $a_6=25{\cdot}08378$	Δ^2	$\theta=3$ $a_6=25{\cdot}19029$	Δ^2	$\theta=4$ $a_6=25{\cdot}34376$	Δ^2	$\theta=5$ $a_6=25{\cdot}54997$	Δ^2
° 0	1·02142	−716	1·04428	−670	1·06899	−625	1·09583	−579	1·12481	−533
1	1·01784	−713	1·04093	−668	1·06586	−623	1·09293	−577	1·12214	−532
2	1·00713	−706	1·03090	−662	1·05651	−618	1·08427	−573	1·11417	−528
3	0·98936	−694	1·01424	−652	1·04098	−609	1·06987	−566	1·10090	−523
4	·96464	−677	0·99107	−637	1·01936	−597	1·04980	−557	1·08241	−516
5	·93317	−655	·96152	−619	0·99176	−582	1·02417	−545	1·05876	−506
6	·89514	−629	·92579	−597	·95834	−564	0·99309	−530	1·03004	−495
7	·85083	−598	·88408	−571	·91929	−542	·95671	−512	0·99638	−481
8	·80054	−563	·83667	−541	·87481	−517	·91522	−492	·95791	−465
9	·74462	−524	·78385	−508	·82516	−489	·86880	−469	·91480	−447
10	·68346	−482	·72596	−471	·77061	−459	·81769	−444	·86721	−427
11	·61749	−435	·66334	−432	·71147	−425	·76214	−416	·81536	−404
12	·54716	−386	·59642	−389	·64809	−389	·70243	−386	·75947	−380
13	·47297	−334	·52560	−344	·58081	−350	·63887	−353	·69979	−353
14	·39543	−280	·45134	−296	·51004	−309	·57177	−318	·63657	−324
15	·31509	−223	·37412	−246	·43618	−265	·50149	−281	·57011	−293
16	·23252	−165	·29444	−194	·35966	−220	·42840	−242	·50073	−260
17	·14830	−105	·21281	−141	·28094	−173	·35290	−201	·42874	−225
18	·06302	− 45	·12978	− 86	·20049	−124	·27538	−158	·35450	−188
19	− ·02270	16	·04588	− 31	·11880	− 74	·19627	−114	·27837	−150
20	− ·10826	77	− ·03833	26	·03637	− 23	·11604	− 68	·20075	−109
21	− ·19305	138	− ·12227	82	− ·04629	29	·03512	− 21	·12203	− 67
22	− ·27646	198	− ·20540	139	− ·12865	82	− ·04600	28	·04264	− 24
23	− ·35788	257	− ·28713	195	− ·21019	135	− ·12685	77	− ·03698	21
24	− ·43672	315	− ·36692	250	− ·29039	187	− ·20693	126	− ·11639	67
25	− ·51242	370	− ·44420	304	− ·36872	240	− ·28576	176	− ·19514	114
26	− ·58441	423	− ·51843	357	− ·44465	291	− ·36282	226	− ·27275	161
27	− ·65217	473	− ·58910	408	− ·51767	341	− ·43763	275	− ·34874	209
28	− ·71520	520	− ·65569	456	− ·58728	390	− ·50969	324	− ·42265	257
29	− ·77302	564	− ·71771	502	− ·65298	438	− ·57850	372	− ·49399	305
30	− ·82520	603	− ·77472	544	− ·71430	483	− ·64360	418	− ·56228	352
31	− ·87135	639	− ·82628	584	− ·77080	525	− ·70452	463	− ·62705	398
32	− ·91111	670	− ·87201	619	− ·82205	565	− ·76080	506	− ·68784	443
33	− ·94417	696	− ·91154	651	− ·86765	601	− ·81203	546	− ·74419	487
34	− ·97027	717	− ·94456	678	− ·90724	634	− ·85779	584	− ·79568	528
35	−0·98921	733	− ·97080	701	− ·94049	663	− ·89772	618	− ·84188	567
36	−1·00082	744	−0·99004	719	− ·96712	687	− ·93147	649	− ·88241	603
37	−1·00499	749	−1·00208	732	− ·98688	707	− ·95873	675	− ·91691	636
38	−1·00167	748	−1·00681	739	−0·99956	722	− ·97924	698	− ·94504	666
39	−0·99087	742	−1·00414	741	−1·00503	733	− ·99277	716	− ·96652	691
40	− ·97265	731	−0·99406	738	−1·00316	737	− ·99914	729	− ·98109	711
41	− ·94713	713	− ·97660	729	−0·99392	737	− ·99823	736	− ·98855	727
42	− ·91447	691	− ·95185	715	− ·97732	731	− ·98995	739	− ·98874	738
43	− ·87490	663	− ·91995	695	− ·95340	719	− ·97429	735	− ·98155	743
44	− ·82871	629	− ·88111	669	− ·92229	701	− ·95127	726	− ·96694	742
45	−0·77622	591	−0·83557	638	−0·88418	678	−0·92100	711	−0·94491	735

TABLE XXIV—*continued.*

$ce_6(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta = 1$ $a_6 = 25{\cdot}02085$		$\theta = 2$ $a_6 = 25{\cdot}08378$		$\theta = 3$ $a_6 = 25{\cdot}19029$		$\theta = 4$ $a_6 = 25{\cdot}34376$		$\theta = 5$ $a_6 = 25{\cdot}54997$	
°		Δ^2		Δ^2		Δ^2		Δ^2		Δ^2
45	−0·77622	591	−0·83557	638	−0·88418	678	−0·92100	711	−0·94491	735
46	− ·71782	548	− ·78366	602	− ·83928	649	− ·88362	689	− ·91553	722
47	− ·65394	501	− ·72573	560	− ·78789	614	− ·83935	662	− ·87893	702
48	− ·58505	449	− ·66219	514	− ·73036	574	− ·78847	628	− ·83531	676
49	− ·51166	394	− ·59352	463	− ·66709	528	− ·73130	589	− ·78493	644
50	− ·43434	335	− ·52021	408	− ·59854	478	− ·66824	544	− ·72811	605
51	− ·35366	274	− ·44282	349	− ·52521	423	− ·59974	493	− ·66524	559
52	− ·27024	210	− ·36194	287	− ·44765	363	− ·52632	437	− ·59679	508
53	− ·18473	144	− ·27819	222	− ·36646	299	− ·44852	376	− ·52325	451
54	− ·09778	76	− ·19222	154	− ·28228	232	− ·36697	311	− ·44520	388
55	− ·01006	8	− ·10471	84	− ·19578	162	− ·28231	241	− ·36328	320
56	·07773	− 61	− ·01636	13	− ·10765	90	− ·19523	168	− ·27815	248
57	·16492	−130	·07213	− 59	− ·01863	16	− ·10648	93	− ·19054	172
58	·25080	−198	·16002	−131	·07056	− 60	− ·01679	15	− ·10122	92
59	·33471	−265	·24661	−202	·15914	−136	·07303	− 65	− ·01098	10
60	·41598	−330	·33117	−273	·24636	−211	·16221	−145	·07937	− 74
61	·49395	−392	·41301	−342	·33147	−286	·24994	−225	·16897	−159
62	·56799	−452	·49142	−409	·41372	−360	·33542	−305	·25699	−244
63	·63752	−508	·56574	−473	·49237	−430	·41785	−382	·34256	−328
64	·70196	−561	·63534	−533	·56672	−498	·49647	−457	·42487	−410
65	·76080	−609	·69961	−589	·63608	−562	·57050	−529	·50306	−490
66	·81354	−653	·75800	−641	·69982	−622	·63924	−597	·57637	−566
67	·85975	−691	·80997	−687	·75734	−677	·70201	−660	·64401	−637
68	·89905	−724	·85508	−728	·80809	−726	·75818	−718	·70529	−703
69	·93111	−751	·89291	−763	·85158	−769	·80716	−769	·75953	−762
70	·95565	−772	·92311	−791	·88738	−805	·84846	−813	·80615	−815
71	·97247	−787	·94540	−813	·91514	−833	·88164	−849	·84462	−859
72	·98142	−796	·95957	−827	·93457	−855	·90631	−878	·87450	−895
73	·98241	−798	·96547	−835	·94545	−868	·92222	−898	·89543	−922
74	·97542	−793	·96302	−835	·94765	−873	·92915	−909	·90713	−940
75	·96050	−782	·95222	−827	·94111	−870	·92699	−911	·90943	−947
76	·93775	−765	·93315	−813	·92587	−859	·91572	−903	·90227	−944
77	·90736	−741	·90594	−791	·90204	−840	·89543	−887	·88567	−931
78	·86956	−711	·87084	−762	·86982	−812	·86627	−861	·85976	−908
79	·82466	−675	·82811	−726	·82947	−776	·82850	−826	·82477	−874
80	·77301	−633	·77813	−683	·78136	−733	·78247	−783	·78104	−831
81	·71502	−586	·72131	−634	·72591	−683	·72861	−731	·72900	−778
82	·65118	−534	·65815	−580	·66364	−625	·66744	−671	·66919	−716
83	·58200	−478	·58920	−519	·59512	−562	·59957	−604	·60222	−646
84	·50804	−417	·51505	−455	·52098	−492	·52565	−531	·52878	−569
85	·42991	−353	·43636	−385	·44191	−418	·44643	−451	·44966	−484
86	·34825	−286	·35381	−313	·35867	−340	·36270	−367	·36570	−395
87	·26372	−217	·26813	−237	·27202	−258	·27529	−279	·27779	−300
88	·17703	−146	·18008	−159	·18279	−173	·18509	−188	·18688	−202
89	·08888	− 73	·09044	− 80	·09183	− 87	·09302	− 94	·09395	−102
90	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0

TABLE XXIV—*continued.*

$ce_5(x, \theta)$. $\theta = 6$ to $\theta = 10$.

x	$\theta=6$ $a_5=25·81727$	Δ^2	$\theta=7$ $a_5=26·15612$	Δ^2	$\theta=8$ $a_5=26·57775$	Δ^2	$\theta=9$ $a_5=27·09187$	Δ^2	$\theta=10$ $a_5=27·70377$	Δ^2
0°	1·15541	−486	1·18640	−439	1·21568	−392	1·24052	−344	1·25802	−295
1	1·15298	−485	1·18420	−439	1·21372	−391	1·23880	−343	1·25654	−295
2	1·14570	−483	1·17762	−437	1·20785	−391	1·23365	−343	1·25211	−296
3	1·13358	−479	1·16666	−435	1·19807	−389	1·22506	−343	1·24473	−296
4	1·11667	−474	1·15136	−431	1·18440	−387	1·21305	−342	1·23438	−297
5	1·09502	−467	1·13175	−426	1·16686	−385	1·19761	−342	1·22106	−298
6	1·06871	−458	1·10787	−420	1·14547	−381	1·17875	−341	1·20476	−299
7	1·03781	−448	1·07979	−413	1·12027	−377	1·15649	−339	1·18547	−300
8	1·00243	−436	1·04758	−405	1·09130	−372	1·13084	−337	1·16319	−300
9	0·96270	−422	1·01131	−395	1·05861	−366	1·10181	−335	1·13791	−301
10	·91874	−407	0·97109	−384	1·02226	−359	1·06944	−331	1·10961	−301
11	·87071	−390	·92703	−372	0·98231	−351	1·03376	−327	1·07830	−301
12	·81879	−370	·87924	−358	·93885	−342	0·99480	−323	1·04399	−300
13	·76316	−349	·82788	−342	·89196	−331	·95261	−317	1·00667	−298
14	·70404	−326	·77310	−325	·84177	−319	·90726	−309	0·96638	−296
15	·64166	−301	·71507	−305	·78839	−306	·85881	−301	·92313	−292
16	·57626	−274	·65398	−284	·73195	−290	·80736	−291	·87695	−287
17	·50813	−245	·59005	−261	·67261	−273	·75300	−279	·82792	−280
18	·43753	−214	·52351	−236	·61054	−253	·69585	−265	·77607	−272
19	·36479	−182	·45460	−209	·54595	−232	·63604	−250	·72151	−262
20	·29024	−147	·38360	−180	·47903	−209	·57374	−232	·66432	−250
21	·21421	−110	·31080	−149	·41002	−183	·50911	−213	·60462	−236
22	·13709	−72	·23651	−116	·33918	−155	·44236	−190	·54257	−220
23	·05925	−31	·16106	−80	·26679	−125	·37370	−166	·47831	−201
24	−·01891	10	·08481	−43	·19314	−93	·30339	−139	·41204	−180
25	−·09696	54	·00812	−4	·11855	−59	·23168	−109	·34398	−155
26	−·17448	98	−·06860	37	·04339	−22	·15889	−77	·27437	−128
27	−·25101	144	−·14496	79	−·03200	17	·08532	−43	·20347	−99
28	−·32611	190	−·22052	123	−·10722	58	·01132	−6	·13159	−66
29	−·39931	237	−·29486	168	−·18186	100	−·06273	34	·05904	−31
30	−·47015	284	−·36750	215	−·25550	145	−·13645	75	−·01381	8
31	−·53814	331	−·43801	261	−·32769	190	−·20941	119	−·08658	49
32	−·60283	377	−·50590	309	−·39797	237	−·28118	165	−·15887	92
33	−·66375	423	−·57070	356	−·46589	285	−·35131	212	−·23024	137
34	−·72043	468	−·63195	402	−·53095	333	−·41932	260	−·30023	185
35	−·77243	511	−·68918	448	−·59269	381	−·48473	309	−·36838	234
36	−·81933	552	−·74191	493	−·65061	429	−·54704	359	−·43418	285
37	−·86071	590	−·78972	536	−·70425	475	−·60577	408	−·49714	336
38	−·89619	625	−·83217	577	−·75313	521	−·66042	457	−·55673	388
39	−·92542	657	−·86884	615	−·79681	564	−·71049	505	−·61245	439
40	−·94808	685	−·89937	650	−·83485	605	−·75552	551	−·66378	490
41	−·96389	709	−·92340	681	−·86683	643	−·79502	595	−·71020	539
42	−·97261	727	−·94062	707	−·89239	677	−·82858	636	−·75124	586
43	−·97406	741	−·95077	729	−·91118	706	−·85577	673	−·78642	630
44	−·96810	748	−·95364	745	−·92291	731	−·87624	706	−·81530	670
45	−0·95465	750	−0·94905	756	−0·92733	750	−0·88964	734	−0·83748	706

TABLE XXIV—*continued.*

$ce_8(x, \theta)$. $\theta = 6$ to $\theta = 10$.

x	$\theta=6$ $a_8=25{\cdot}81727$	Δ^2	$\theta=7$ $a_8=26{\cdot}15612$	Δ^2	$\theta=8$ $a_8=26{\cdot}57775$	Δ^2	$\theta=9$ $a_8=27{\cdot}09187$	Δ^2	$\theta=10$ $a_8=27{\cdot}70377$	Δ^2
° 45	−0·95465	750	−0·94905	756	−0·92733	750	−0·88964	734	−0·83748	706
46	− ·93371	746	− ·93691	760	− ·92424	763	− ·89571	756	− ·85259	737
47	− ·90530	735	− ·91716	757	− ·91353	770	− ·89422	772	− ·86033	762
48	− ·86955	716	− ·88985	748	− ·89511	770	− ·88501	780	− ·86045	780
49	− ·82664	692	− ·85505	731	− ·86899	762	− ·86800	782	− ·85277	791
50	− ·77681	660	− ·81294	707	− ·83526	746	− ·84317	775	− ·83717	794
51	− ·72039	621	− ·76375	676	− ·79406	723	− ·81059	761	− ·81364	789
52	− ·65776	575	− ·70781	636	− ·74564	691	− ·77040	737	− ·78221	775
53	− ·58937	522	− ·64551	590	− ·69030	651	− ·72283	705	− ·74303	751
54	− ·51577	463	− ·57731	535	− ·62846	603	− ·66822	664	− ·69635	718
55	− ·43753	398	− ·50375	474	− ·56059	547	− ·60697	614	− ·64248	675
56	− ·35531	328	− ·42546	406	− ·48726	483	− ·53957	555	− ·58187	623
57	− ·26981	252	− ·34310	332	− ·40909	412	− ·46663	488	− ·51502	561
58	− ·18179	172	− ·25741	253	− ·32681	334	− ·38879	414	− ·44256	491
59	− ·09206	88	− ·16920	168	− ·24119	250	− ·30682	332	− ·36519	412
60	− ·00144	1	− ·07930	80	− ·15307	161	− ·22154	243	− ·28369	325
61	·08918	− 88	·01140	− 12	− ·06334	67	− ·13382	149	− ·19895	232
62	·17894	−177	·10198	− 106	·02707	− 29	− ·04461	50	− ·11189	132
63	·26692	−267	·19150	− 201	·11717	− 120	·04500	− 52	− ·02351	28
64	·35222	−356	·27901	− 295	·20600	− 229	·13428	− 156	·06516	− 80
65	·43397	−443	·36357	− 389	·29254	− 328	·22191	− 261	·15302	− 189
66	·51129	−527	·44424	− 480	·37579	− 427	·30692	− 366	·23900	− 299
67	·58333	−606	·52011	− 568	·45478	− 522	·38828	− 468	·32198	− 408
68	·64932	−681	·59029	− 651	·52855	− 613	·46496	− 567	·40089	− 514
69	·70849	−749	·65396	− 728	·59619	− 698	·53597	− 660	·47466	− 615
70	·76018	−810	·71036	− 797	·65686	− 776	·60038	− 747	·54228	− 710
71	·80376	−863	·75879	− 859	·70976	− 846	·65732	− 826	·60280	− 797
72	·83872	−907	·79862	− 911	·75419	− 907	·70600	− 895	·65535	− 875
73	·86461	−941	·82935	− 953	·78955	− 957	·74573	− 953	·69915	− 942
74	·88108	−965	·85054	− 984	·81534	− 996	·77593	−1000	·73352	− 997
75	·88791	−978	·86190	−1004	·83117	−1023	·79613	−1034	·75793	−1038
76	·88495	−981	·86321	−1012	·83678	−1036	·80598	−1054	·77195	−1065
77	·87218	−972	·85440	−1007	·83202	−1037	·80530	−1060	·77532	−1078
78	·84970	−951	·83552	− 990	·81689	−1024	·79402	−1052	·76791	−1074
79	·81771	−919	·80674	− 961	·79152	− 997	·77221	−1029	·74977	−1055
80	·77652	−877	·76835	− 919	·75617	− 957	·74012	− 991	·72107	−1020
81	·72657	−823	·72077	− 866	·71125	− 904	·69812	− 939	·68217	− 970
82	·66839	−760	·66454	− 801	·65729	− 839	·64673	− 874	·63357	− 905
83	·60261	−687	·60029	− 726	·59494	− 762	·58661	− 795	·57593	− 825
84	·52996	−606	·52879	− 641	·52496	− 674	·51853	− 705	·51004	− 733
85	·45126	−517	·45087	− 548	·44824	− 577	·44340	− 605	·43681	− 630
86	·36739	−421	·36747	− 447	·36574	− 472	·36223	− 495	·35728	− 516
87	·27930	−321	·27960	− 341	·27853	− 360	·27610	− 378	·27259	− 395
88	·18800	−216	·18832	− 230	·18771	− 243	·18620	− 255	·18395	− 267
89	·09455	−109	·09474	− 116	·09447	− 122	·09374	− 129	·09265	− 134
90	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0

TABLE XXV.

$se_6(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta=1$, $b_6=36{\cdot}01429$	Δ^2	$\theta=2$, $b_6=36{\cdot}05721$	Δ^2	$\theta=3$, $b_6=36{\cdot}12887$	Δ^2	$\theta=4$, $b_6=36{\cdot}22941$	Δ^2	$\theta=5$, $b_6=36{\cdot}35887$	Δ^2
0	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0
1	·10300	−107	·10138	−99	·09964	−91	·09780	−84	·09585	−77
2	·20493	−212	·20176	−197	·19838	−182	·19477	−167	·19092	−153
3	·30474	−316	·30017	−293	·29529	−271	·29006	−250	·28446	−229
4	·40140	−416	·39566	−387	·38949	−358	·38285	−330	·37572	−303
5	·49390	−512	·48728	−476	·48011	−442	·47234	−408	·46395	−374
6	·58128	−603	·57413	−562	·56631	−522	·55776	−482	·54843	−444
7	·66263	−687	·65537	−642	·64730	−597	·63836	−553	·62848	−510
8	·73711	−765	·73019	−716	·72231	−668	·71342	−620	·70342	−573
9	·80395	−835	·79785	−783	·79065	−732	·78228	−682	·77264	−632
10	·86243	−896	·85767	−843	·85167	−790	·84433	−738	·83554	−686
11	·91196	−948	·90907	−895	·90479	−842	·89900	−788	·89159	−735
12	·95201	−991	·95151	−938	·94948	−886	·94578	−833	·94028	−779
13	0·98215	−1023	0·98458	−973	0·98532	−922	0·98424	−870	0·98118	−818
14	1·00206	−1045	1·00791	−998	1·01194	−950	1·01400	−900	1·01391	−850
15	1·01152	−1055	1·02126	−1013	1·02906	−969	1·03475	−923	1·03813	−875
16	1·01044	−1055	1·02448	−1019	1·03650	−979	1·04627	−938	1·05361	−894
17	0·99879	−1044	1·01752	−1014	1·03414	−981	1·04842	−945	1·06014	−906
18	·97671	−1022	1·00042	−999	1·02197	−973	1·04112	−943	1·05761	−910
19	·94440	−990	0·97332	−975	1·00008	−956	1·02439	−933	1·04599	−907
20	·90219	−947	·93648	−940	0·96862	−930	0·99833	−915	1·02529	−896
21	·85052	−894	·89023	−896	·92788	−894	·96313	−888	0·99564	−877
22	·78990	−831	·83502	−843	·87818	−850	·91904	−852	·95722	−850
23	·72097	−760	·77137	−781	·81999	−798	·86643	−809	·91031	−815
24	·64445	−680	·69992	−711	·75381	−737	·80574	−757	·85524	−772
25	·56113	−593	·62135	−633	·68027	−668	·73747	−698	·79246	−722
26	·47187	−500	·53645	−548	·60005	−592	·66222	−631	·72245	−664
27	·37762	−400	·44606	−458	·51391	−510	·58066	−557	·64580	−599
28	·27937	−297	·35110	−361	·42266	−422	·49354	−477	·56316	−527
29	·17814	−189	·25253	−261	·32720	−328	·40164	−391	·47525	−449
30	·07503	−80	·15135	−157	·22847	−230	·30583	−300	·38285	−365
31	−·02889	31	·04860	−50	·12742	−129	·20703	−205	·28680	−276
32	−·13249	142	−·05466	57	·02509	−25	·10618	−106	·18798	−183
33	−·23468	251	−·15734	165	−·07750	80	·00427	−4	·08733	−86
34	−·33436	359	−·25837	272	−·17929	185	−·09768	99	−·01417	14
35	−·43044	463	−·35669	377	−·27923	290	−·19864	203	−·11553	116
36	−·52190	562	−·45124	478	−·37628	393	−·29757	306	−·21573	219
37	−·60773	656	−·54101	576	−·46940	493	−·39344	408	−·31374	321
38	−·68701	743	−·62502	668	−·55759	589	−·48524	507	−·40855	422
39	−·75886	822	−·70235	753	−·63990	679	−·57197	602	−·49913	521
40	−·82[illegible]48	893	−·77216	831	−·71541	764	−·65269	692	−·58450	616
41	−·87718	954	−·83365	901	−·78329	841	−·72648	777	−·66371	706
42	−·92233	1005	−·88614	961	−·84274	911	−·79250	854	−·73586	791
43	−·95744	1045	−·92901	1012	−·89310	971	−·84999	923	−·80010	868
44	−·98209	1074	−·96177	1051	−·93374	1021	−·89825	983	−·85565	938
45	−0·99599	1092	−0·98402	1080	−0·96418	1060	−0·93669	1033	−0·90183	998

TABLE XXV—*continued*.

$se_6(x, \theta)$. $\theta = 1$ to $\theta = 5$.

x	$\theta=1$ $b_6=36·01429$		$\theta=2$ $b_6=36·05721$		$\theta=3$ $b_6=36·12887$		$\theta=4$ $b_6=36·22941$		$\theta=5$ $b_6=36·35887$	
°		Δ^2		Δ^2		Δ^2		Δ^2		Δ^2
45	-0·99599	1092	-0·98402	1080	-0·96418	1060	-0·93669	1033	-0·90183	998
46	-·99899	1097	-·99546	1097	-·98402	1088	-·96479	1072	-·93802	1048
47	-·99101	1090	-·99594	1101	-·99297	1104	-·98217	1100	-·96374	1087
48	-·97212	1072	-·98541	1094	-·99088	1108	-·98856	1115	-·97858	1114
49	-·94252	1041	-·96394	1074	-·97771	1100	-·98380	1118	-·98229	1128
50	-·90251	999	-·93173	1042	-·95353	1079	-·96785	1108	-·97471	1130
51	-·85252	945	-·88910	998	-·91857	1045	-·94083	1085	-·95584	1118
52	-·79307	881	-·83648	942	-·87317	999	-·90296	1049	-·92578	1092
53	-·72481	807	-·77444	876	-·81777	940	-·85460	999	-·88479	1053
54	-·64848	723	-·70365	799	-·75298	870	-·79624	938	-·83328	1000
55	-·56493	631	-·62486	712	-·67948	789	-·72851	864	-·77177	934
56	-·47507	531	-·53897	616	-·59809	698	-·65215	778	-·70091	855
57	-·37989	426	-·44691	512	-·50971	598	-·56799	682	-·62150	764
58	-·28046	315	-·34973	402	-·41535	490	-·47702	577	-·53444	662
59	-·17788	200	-·24852	287	-·31610	375	-·38027	463	-·44077	550
60	-·07329	83	-·14445	167	-·21309	254	-·27891	341	-·34158	430
61	·03212	-36	-·03870	45	-·10756	129	-·17412	214	-·23810	302
62	·13716	-155	·06750	-79	-·00073	1	-·06720	83	-·13160	168
63	·24066	-272	·17290	-202	·10610	-128	·04056	-51	-·02343	30
64	·34143	-387	·27629	-324	·21165	-257	·14781	-185	·08505	-110
65	·43833	-498	·37644	-443	·31463	-383	·25320	-319	·19242	-251
66	·53025	-603	·47216	-557	·41379	-506	·35541	-450	·29729	-390
67	·61615	-701	·56232	-665	·50789	-623	·45312	-576	·39826	-525
68	·69503	-792	·64583	-765	·59576	-733	·54507	-696	·49398	-655
69	·76600	-874	·72168	-857	·67630	-835	·63005	-809	·58315	-777
70	·82821	-946	·78897	-939	·74848	-928	·70695	-911	·66456	-890
71	·88097	-1008	·84686	-1010	·81139	-1009	·77473	-1003	·73706	-992
72	·92365	-1058	·89465	-1070	·86421	-1078	·83249	-1082	·79964	-1082
73	·95575	-1096	·93174	-1116	·90626	-1134	·87943	-1147	·85141	-1157
74	·97689	-1121	·95766	-1150	·93696	-1175	·91490	-1197	·89160	-1216
75	·98682	-1134	·97209	-1169	·95592	-1202	·93840	-1232	·91964	-1260
76	·98541	-1133	·97483	-1174	·96285	-1214	·94957	-1251	·93507	-1286
77	·97268	-1119	·96582	-1165	·95765	-1210	·94823	-1253	·93765	-1294
78	·94875	-1093	·94516	-1142	·94035	-1191	·93437	-1238	·92729	-1284
79	·91390	-1053	·91308	-1105	·91114	-1156	·90813	-1206	·90410	-1255
80	·86852	-1002	·86995	-1054	·87037	-1106	·86982	-1158	·86836	-1209
81	·81311	-938	·81628	-990	·81854	-1042	·81994	-1094	·82052	-1145
82	·74833	-864	·75271	-914	·75630	-964	·75913	-1014	·76124	-1065
83	·67491	-780	·67999	-826	·68440	-874	·68817	-921	·69131	-969
84	·59369	-686	·59902	-729	·60378	-772	·60799	-815	·61169	-859
85	·50561	-584	·51075	-622	·51543	-659	·51967	-697	·52348	-736
86	·41169	-476	·41628	-507	·42050	-538	·42437	-570	·42791	-602
87	·31301	-362	·31673	-386	·32018	-410	·32338	-435	·32633	-460
88	·21071	-244	·21332	-260	·21576	-276	·21803	-293	·22014	-310
89	·10597	-123	·10731	-131	·10858	-139	·10975	-148	·11085	-156
90	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0

TABLE XXV—*continued.*

$se_6(x, \theta)$. $\theta=6$ to $\theta=10$.

x	$\theta=6$ $b_6=36{\cdot}51707$	Δ^2	$\theta=7$ $b_6=36{\cdot}70350$	Δ^2	$\theta=8$ $b_6=36{\cdot}91721$	Δ^2	$\theta=9$ $b_6=37{\cdot}15669$	Δ^2	$\theta=10$ $b_6=37{\cdot}41986$	Δ^2
0°	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0
1	·09376	− 70	·09153	− 63	·08916	− 57	·08665	− 51	·08400	− 45
2	·18681	−140	·18243	−126	·17776	−113	·17280	−101	·16756	− 89
3	·27847	−208	·27206	−189	·26522	−170	·25794	−151	·25022	−134
4	·36805	−276	·35981	−250	·35099	−225	·34156	−201	·33155	−178
5	·45486	−342	·44506	−311	·43450	−280	·42318	−250	·41110	−222
6	·53826	−406	·52720	−369	·51521	−334	·50228	−299	·48843	−266
7	·61759	−468	·60564	−426	·59259	−386	·57840	−347	·56310	−309
8	·69225	−527	·67983	−481	·66610	−437	·65105	−394	·63469	−352
9	·76164	−582	·74920	−534	·73525	−486	·71977	−439	·70276	−394
10	·82521	−634	·81323	−583	·79954	−533	·78409	−483	·76689	−435
11	·88244	−682	·87144	−629	·85850	−577	·84358	−526	·82666	−475
12	·93285	−726	·92335	−672	·91169	−619	·89781	−566	·88169	−514
13	0·97600	−765	0·96854	−711	·95869	−658	·94637	−605	·93158	−552
14	1·01150	−798	1·00661	−746	0·99911	−693	0·98890	−640	0·97594	−587
15	1·03902	−826	1·03722	−776	1·03259	−725	1·02502	−673	1·01444	−621
16	1·05828	−849	1·06008	−801	1·05883	−753	1·05440	−703	1·04673	−652
17	1·06905	−865	1·07492	−821	1·07754	−776	1·07676	−729	1·07249	−681
18	1·07117	−874	1·08154	−836	1·08849	−794	1·09183	−751	1·09145	−706
19	1·06456	−877	1·07981	−844	1·09149	−808	1·09939	−769	1·10335	−728
20	1·04917	−873	1·06964	−846	1·08642	−816	1·09926	−782	1·10798	−745
21	1·02505	−861	1·05101	−841	1·07319	−818	1·09130	−790	1·10515	−759
22	0·99233	−842	1·02397	−830	1·05178	−814	1·07545	−793	1·09473	−768
23	·95118	−816	0·98863	−812	1·02224	−803	1·05167	−789	1·07663	−771
24	·90187	−782	·94516	−786	0·98467	−786	1·02000	−780	1·05083	−769
25	·84475	−741	·89384	−754	·93925	−761	0·98053	−764	1·01733	−761
26	·78021	−692	·83497	−714	·88621	−730	·93343	−741	0·97622	−746
27	·70876	−636	·76897	−666	·82587	−692	·87892	−711	·92766	−725
28	·63096	−572	·69631	−612	·75861	−646	·81729	−674	·87185	−696
29	·54743	−502	·61752	−550	·68490	−593	·74893	−629	·80908	−660
30	·45888	−426	·53324	−482	·60526	−533	·67428	−578	·73970	−617
31	·36606	−344	·44413	−407	·52029	−465	·59384	−519	·66415	−566
32	·26981	−257	·35095	−326	·43067	−392	·50822	−453	·58294	−508
33	·17099	−164	·25451	−240	·33713	−312	·41808	−379	·49664	−442
34	·07053	− 69	·15567	−149	·24047	−226	·32414	−300	·40593	−369
35	− ·03062	30	·05534	− 54	·14155	−135	·22720	−214	·31151	−290
36	− ·13146	131	− ·04552	45	·04128	− 40	·12813	−123	·21420	−203
37	− ·23099	234	− ·14594	146	− ·05939	59	·02782	− 27	·11486	−111
38	− ·32818	336	− ·24489	249	− ·15947	161	− ·07276	73	·01440	− 14
39	− ·42201	437	− ·34135	351	− ·25794	264	− ·17260	176	− ·08619	88
40	− ·51148	536	− ·43431	453	− ·35377	368	− ·27069	281	− ·18591	192
41	− ·59557	632	− ·52273	553	− ·44593	471	− ·36598	386	− ·28371	299
42	− ·67335	723	− ·60562	650	− ·53337	572	− ·45739	491	− ·37851	407
43	− ·74391	808	− ·68201	742	− ·61509	670	− ·54390	595	− ·46924	515
44	− ·80638	886	− ·75098	828	− ·69011	764	− ·62446	694	− ·55482	620
45	−0·85999	956	−0·81168	907	−0·75748	851	−0·69808	790	−0·63419	722

TABLE XXV—*continued.*

$se_6(x, \theta)$. $\theta=6$ to $\theta=10$.

x	$\theta=6$ $b_6=36{\cdot}51707$		$\theta=7$ $b_6=36{\cdot}70350$		$\theta=8$ $b_6=36{\cdot}91721$		$\theta=9$ $b_6=37{\cdot}15669$		$\theta=10$ $b_6=37{\cdot}41986$	
°		Δ^2		Δ^2		Δ^2		Δ^2		Δ^2
45	−0·85999	956	−0·81168	907	−0·75748	851	−0·69808	790	−0·63419	722
46	− ·90405	1016	− ·86331	977	− ·81635	931	− ·76380	878	− ·70635	820
47	− ·93794	1066	− ·90516	1038	− ·86590	1002	− ·82073	960	− ·77030	910
48	− ·96117	1105	− ·93664	1088	− ·90543	1063	− ·86807	1031	− ·82516	992
49	− ·97335	1131	− ·95724	1126	− ·93433	1113	− ·90510	1093	− ·87009	1065
50	− ·97421	1144	− ·96657	1151	− ·95209	1150	− ·93120	1142	− ·90437	1125
51	− ·96364	1144	− ·96440	1163	− ·95836	1174	− ·94588	1177	− ·92740	1173
52	− ·94162	1130	− ·95059	1160	− ·95289	1183	− ·94880	1198	− ·93870	1207
53	− ·90831	1101	− ·92520	1142	− ·93559	1177	− ·93972	1204	− ·93793	1225
54	− ·86399	1057	− ·88838	1109	− ·90652	1155	− ·91861	1194	− ·92490	1227
55	− ·80910	1000	− ·84047	1061	− ·86591	1117	− ·88555	1167	− ·89961	1211
56	− ·74421	929	− ·78195	998	− ·81414	1063	− ·84083	1123	− ·86220	1178
57	− ·67003	844	− ·71346	920	− ·75173	993	− ·78487	1062	− ·81301	1127
58	− ·58741	747	− ·63576	829	− ·67939	908	− ·71830	984	− ·75255	1057
59	− ·49733	638	− ·54977	724	− ·59797	808	− ·64188	890	− ·68152	970
60	− ·40087	518	− ·45655	607	− ·50848	695	− ·55656	781	− ·60078	867
61	− ·29922	390	− ·35726	479	− ·41203	569	− ·46342	658	− ·51138	747
62	− ·19367	255	− ·25317	343	− ·30990	432	− ·36371	522	− ·41452	613
63	− ·08558	113	− ·14566	199	− ·20344	286	− ·25877	376	− ·31152	466
64	·02364	− 32	− ·03615	50	− ·09412	134	− ·15007	220	− ·20387	308
65	·13255	− 179	·07385	− 103	·01654	− 24	− ·03918	58	− ·09314	142
66	·23967	− 325	·18282	− 256	·12696	− 184	·07230	− 108	·01901	− 30
67	·34354	− 469	·28923	− 409	·23553	− 344	·18269	− 276	·13087	− 205
68	·44273	− 608	·39154	− 557	·34066	− 502	·29031	− 443	·24068	− 380
69	·53583	− 741	·48829	− 700	·44078	− 655	·39351	− 605	·34670	− 552
70	·62152	− 864	·57803	− 834	·53434	− 800	·49066	− 761	·44720	− 718
71	·69856	− 977	·65944	− 958	·61991	− 934	·58020	− 906	·54052	− 875
72	·76583	−1077	·73126	−1069	·69614	−1056	·66067	−1040	·62510	−1019
73	·82234	−1163	·79240	−1165	·76180	−1163	·73075	−1158	·69948	−1149
74	·86721	−1232	·84189	−1244	·81584	−1253	·78926	−1259	·76237	−1261
75	·89977	−1284	·87894	−1306	·85734	−1324	·83517	−1340	·81265	−1353
76	·91948	−1318	·90293	−1348	·88560	−1375	·86769	−1400	·84940	−1423
77	·92601	−1333	·91344	−1370	·90011	−1405	·88620	−1438	·87191	−1469
78	·91922	−1328	·91026	−1371	·90057	−1412	·89033	−1452	·87974	−1490
79	·89915	−1303	·89337	−1350	·88692	−1396	·87995	−1441	·87266	−1485
80	·86604	−1259	·86298	−1309	·85930	−1358	·85515	−1407	·85072	−1455
81	·82034	−1196	·81950	−1247	·81810	−1297	·81629	−1348	·81424	−1398
82	·76268	−1115	·76355	−1165	·76392	−1215	·76395	−1265	·76378	−1316
83	·69388	−1017	·69594	−1065	·69760	−1113	·69896	−1161	·70017	−1210
84	·61490	− 903	·61769	− 947	·62014	− 992	·62235	−1037	·62445	−1082
85	·52690	− 775	·52997	− 814	·53277	− 854	·53538	− 894	·53791	− 934
86	·43115	− 635	·43411	− 668	·43686	− 701	·43947	− 735	·44202	− 769
87	·32905	− 485	·33158	− 511	·33395	− 537	·33621	− 563	·33844	− 590
88	·22210	− 328	·22393	− 345	·22566	− 363	·22732	− 381	·22896	− 400
89	·11188	− 165	·11284	− 174	·11375	− 183	·11462	− 192	·11549	− 202
90	0·00000	0	0·00000	0	0·00000	0	0·00000	0	0·00000	0

XXIII.—Zeros and Turning Points of the Elliptic-cylinder Functions.

By E. L. Ince, M.A. (Cantab.), D.Sc. (Edin.).

(MS. received January 3, 1931. Revised MS. received November 24, 1931. Read February 2, 1931.)

1. Introduction.

In any discussion of the zeros and turning points of the elliptic-cylinder function it is not necessary to define them more precisely than as functions of period π or 2π which satisfy the Mathieu equation

$$\frac{d^2y}{dx^2} + (a - 2\theta \cos 2x)y = 0$$

for the corresponding *characteristic* values of a. To simplify the following discussion, it is convenient merely to impose the condition that $y(0) > 0$ if $y(x)$ is even, and $y'(0) > 0$ if $y(x)$ is odd.

The functions considered may then be regarded as any positive multiples of the functions $ce_{2n}(x, \theta)$, $se_{2n+1}(x, \theta)$, $ce_{2n+1}(x, \theta)$, and $se_{2n+2}(x, \theta)$ specifically defined and in part tabulated in the preceding paper. The number of zeros of each function in the open interval $0 < x < \frac{1}{2}\pi$ is independent of θ and is indicated by the n in the suffix.*

One remarkable feature of these functions, revealed by the tables, is that when the definitions of $ce_{2n}(x, \theta)$ and $se_{2n+1}(x, \theta)$ are unified either by the condition $ce_{2n}(\frac{1}{2}\pi, \theta) = se_{2n+1}(\frac{1}{2}\pi, \theta)$ or by the condition

$$\int_0^{\frac{1}{2}\pi} \{ce_{2n}(x, \theta)\}^2 dx = \int_0^{\frac{1}{2}\pi} \{se_{2n+1}(x, \theta)\}^2 dx,$$

then these two functions approach one another asymptotically over any interval $0 < a < x \leqslant \frac{1}{2}\pi$, for large positive θ. Likewise, if $ce_{2n+1}(x, \theta)$ and $se_{2n+2}(x, \theta)$ are related by the condition $ce'_{2n+1}(x, \theta) = se'_{2n+2}(x, \theta)$ for $x = \frac{1}{2}\pi$, or by an integral condition similar to the above, they also approach asymptotically.

Thus for every m, the zeros and turning points of $se_{m+1}(x, \theta)$ in the open interval $(0, \frac{1}{2}\pi)$ approach those of $ce_m(x, \theta)$ with positive increasing θ.

2. Zeros.

It is a consequence of the Sturmian theory that, as θ increases, the zeros in the interval $0 < x < \frac{1}{2}\pi$ ultimately move towards the right-hand end-point.† Table I, which gives these zeros for the four functions $n = 1$

* Ince, *Journ. London Math. Soc.*, ii, 1927, 47.

† Confirmation of this fact is given in § 5.

and the four functions $n=2$, seems to indicate that this property is general and not merely ultimate.

For the ten lowest values of θ the zeros were first calculated from the existing tables of the functions by inverse interpolation. The values of the function for the nearest minute above and below the zero were calculated, and the zero determined more precisely by interpolation. For the ten higher values of θ tables do not exist, and therefore the zeros had first to be estimated by extrapolation from the lower values and then calculated in the same way.

3. Turning Points.

If $a>2\theta$, an elliptic-cylinder function having n zeros in the open interval $(0, \frac{1}{2}\pi)$ has also n inflexions coincident with the zeros, and no more. The number of turning points in the open interval is therefore $n-1$, n, or $n+1$, according to the behaviour of the functions at the end-points. In particular,

$ce_{2n}(x, \theta)$, for $n\geqslant 1$, has $n-1$ turning points in the open interval,
$se_{2n+1}(x, \theta)$ and $ce_{2n+1}(x, \theta)$ have n ,, ,, ,, ,, ,, ,
$se_{2n+2}(x, \theta)$ has $n+1$,, ,, ,, ,, ,, .

If, however, $a<2\theta$, which is always the case for a sufficiently large θ, an additional inflexion exists in the interval, namely, at the point where $2\theta\cos 2x=a$. Now it is important to note that this inflexion must lie between 0 and ζ, where ζ is the lowest zero in the open interval. For if this were not the case, the identity

$$y'(\zeta)-y'(0)=\int_0^{\zeta}(2\theta\cos 2x-a)y\,dx$$

would be definitely negative on the left and definitely positive on the right.

Thus if $x=\xi$ is the root of $2\theta\cos 2x=a$ in the interval $(0, \frac{1}{2}\pi)$, there is no zero in the interval $0<x\leqslant\xi$.

Moreover, at $x=\xi$ the sign of y'' changes from $+$ to $-$. As the turning point in the interval $(0, \zeta)$ must correspond to a maximum, it must lie to the right of ξ; that is, there are no turning points in the interval $0<x<\xi$.

In the case of the odd functions there is one, and only one, turning point between 0 and ζ whether there is an inflexion in that interval or not. In the case of the even functions, however, the inflexion involves the existence of a maximum in (ξ, ζ). Hence, if $a<2\theta$,

$ce_{2n}(x, \theta)$ and $se_{2n+1}(x, \theta)$ have n turning points in the open interval $(0, \frac{1}{2}\pi)$,
$ce_{2n+1}(x, \theta)$ and $se_{2n+2}(x, \theta)$,, $n+1$,, ,, ,, ,, ,, .

The table of turning points (Table II) was calculated on much the same lines as the table of zeros.

4. Transition Values.

The values of θ at which a change occurs in the number of turning values of the even functions are calculated from the familiar continued-fraction relations between a and θ. When $a=2\theta$ the relation appropriate to the even functions of even order becomes

$$1=-\frac{\theta/4}{1-\frac{1}{2}\theta}-\frac{\theta^2/4\cdot16}{1-\frac{1}{8}\theta}-\frac{\theta^2/16\cdot36}{1-\frac{1}{18}\theta}-\frac{\theta^2/36\cdot64}{1-\frac{1}{32}\theta}-\quad . \quad . \quad .,$$

and that appropriate to the even functions of odd order becomes

$$\theta-1=-\frac{\theta^2/9}{1-\frac{2}{9}\theta}-\frac{\theta^2/9\cdot25}{1-\frac{2}{25}\theta}-\frac{\theta^2/25\cdot49}{1-\frac{2}{49}\theta}-\quad . \quad . \quad . .$$

The actual values to six decimal places were found to be *:

for $ce_1(x,\theta)$, $\theta=0{\cdot}889820\ldots$,
,, $ce_2(x,\theta)$, $\theta=3{\cdot}039074\ldots$,
,, $ce_3(x,\theta)$, $\theta=6{\cdot}425863\ldots$,
,, $ce_4(x,\theta)$, $\theta=11{\cdot}047992\ldots$,
,, $ce_5(x,\theta)$, $\theta=16{\cdot}904741\ldots$.

5. Asymptotic Approximations.

For large θ, write $h^2=4\theta$; then,† if $h>0$,

$$a+2\theta=(2m+1)h+O(1),$$

where

$$m=2n \text{ if } a=a_{2n} \text{ or } b_{2n+1}\,;\ m=2n+1 \text{ if } a=a_{2n+1} \text{ or } b_{2n+2}.$$

Hence

$$\cos^2\xi=\frac{a+2\theta}{4\theta}=\frac{2m+1}{h}+O(h^{-2}).$$

Thus, as θ increases, ξ ultimately approaches the right-hand end-point of the interval $(0, \frac{1}{2}\pi)$, and as it has already been seen that the zeros and turning points of the open interval lie between ξ and $\frac{1}{2}\pi$, it follows that these points tend to accumulate in an interval of length $\surd\{(2m+1)h^{-1}\}$ abutting on the end-point.

The Mathieu equation may be formally satisfied by ‡

$$y=y_1=e^{h\sin x}u_1,$$

where, if $t=\frac{1}{2}\pi-x$,

$$u_1=\frac{\sin^m\frac{1}{2}t}{\cos^{m+1}\frac{1}{2}t}\left[1+\left\{\frac{(m+1)(m+2)}{\cos^2\frac{1}{2}t}-\frac{m(m-1)}{\sin^2\frac{1}{2}t}\right\}\frac{h^{-1}}{16}\right.$$
$$+\left\{\frac{(m+1)\ldots(m+4)}{\cos^4\frac{1}{2}t}-\frac{2(m+1)(m+2)(m^2-5m-2)}{\cos^2\frac{1}{2}t}\right.$$
$$\left.\left.-\frac{2m(m-1)(m^2+7m+4)}{\sin^2\frac{1}{2}t}+\frac{m\ldots(m-3)}{\sin^4\frac{1}{2}t}\right\}\frac{h^{-2}}{512}+\ldots\right].$$

* Cf. Goldstein, *Proc. London Math. Soc.*, (2) xxviii, 1928, 94.
† Ince, *Proc. Roy. Soc. Edin.*, xlvi, 1926, 318. ‡ Ince, *loc. cit.*

It is also satisfied by

$$y = y_2 = e^{-h \sin x} u_2,$$

where u_2 is derived from u_1 by changing h into $-h$ and t into $\pi - t$.

The present author pointed out that these are asymptotic representations of Mathieu functions of period 4π, and Goldstein has shown * that $y_1 + y_2$ is an approximation to a multiple of the even function which may be denoted by $ce_{m+\frac{1}{2}}(x, \theta)$ and $y_1 - y_2$ to a multiple of the corresponding odd function $se_{m+\frac{1}{2}}(x, \theta)$.

So far as an interval $0 < a < x < \frac{1}{2}\pi$ is concerned, for large positive h,

$$ce_{m+\frac{1}{2}}(x, \theta) \sim y_1 \sim se_{m+\frac{1}{2}}(x, \theta).$$

Since $ce_m(x, \theta)$ and $se_{m+1}(x, \theta)$ have, asymptotically, the same characteristic numbers as $ce_{m+\frac{1}{2}}(x, \theta)$ and $se_{m+\frac{1}{2}}(x, \theta)$, it is reasonable to conjecture that

$$ce_m(x, \theta) \sim y_1 \sim se_{m+1}(x, \theta).$$

It is not our purpose here to attempt to justify this relation, but to show by actual examples that for moderately large positive h, the zeros of y_1 do in fact approach closely to the zeros of the elliptic-cylinder functions. The same also may be said of the turning points.

Take first of all the cases $m = 2$ and $m = 3$. It is not difficult to show that y_1 has one zero given by

$$\sin^2 \tfrac{1}{2}t = \frac{m(m-1)}{16h}\left\{1 + \frac{2m+1}{8h} + O(h^{-2})\right\}$$

or, approximately,

$$\sin x = 1 - \frac{m(m-1)}{8h}\left(1 + \frac{2m+1}{8h}\right).$$

That this formula does give a fair approximation will be seen by the following comparison ($\theta = 36$, $h = 12$):

Zero of ce_2 and se_3 $(m=2)$: approx. $77^\circ\ 58'{\cdot}9$, true $77^\circ\ 56'{\cdot}5$,
" " ce_3 and se_4 $(m=3)$: " $68^\circ\ 53'{\cdot}9$, " $68^\circ\ 44'{\cdot}6$.

In the cases $m = 4$ and $m = 5$, y_1 has two zeros given by

$$0 = 1 - \frac{m(m-1)}{16hs^2} + \frac{m(m-1)(m-2)(m-3)}{512h^2s^4} + \frac{(m+1)(m+2)}{16h} - \frac{m(m-1)(m^2+7m+4)}{256h^2s^2} + \frac{m(m-1)(m-2)(m-3)(m^2+11m+6)}{8192h^3s^4} + O(h^{-2}),$$

where $s = \sin \frac{1}{2}t$, or approximately

$$\sin x = 1 - \frac{m(m-1)}{16h}\left(1 + \frac{2m+1}{8h}\right)\left(1 \pm \sqrt{1 - \frac{2(m-2)(m-3)}{m(m-1)}}\right).$$

---

* *Proc. Roy. Soc. Edin.*, **xlix**, 1929, 215.

For $\theta=36$ this approximate formula gives

Zeros of ce_4 and se_5 $(m=4)$: approx. 61° 9′, true 60° 45′,
„ 80° 55′, „ 80° 49′.
„ „ ce_5 and se_6 $(m=5)$: „ 54° 8′, „ 53° 16′,
„ 73° 12′, „ 72° 52′.

In a similar way approximation to the turning points may be found. In the cases $m=1$ and $m=2$ the second order approximation is

$$\sin x = 1 - \frac{m(m+3)}{8h} - \frac{m(2m^2+7m+7)}{64h^2}.$$

This gives, for $\theta=36$:

Turning point of ce_1 and se_2 $(m=1)$: approx. 73° 3′·4, true 73° 1′·2,
„ „ „ ce_2 and se_3 $(m=2)$: „ 62° 48′·9, „ 62° 40′·1.

In the cases $m=3$ and $m=4$ the second order approximation is too complicated to be of practical value. The first order approximation is

$$\sin x = 1 - \frac{m(m+3)}{16h}\left(1 \pm \sqrt{1 - \frac{2(m-1)(m-2)(m+5)}{m(m+3)^2}}\right)$$

and gives, for $\theta=36$:

Turning points of ce_3 and se_4 $(m=3)$: approx. 55° 51′, true 54° 4′,
„ 80° 1′, „ 79° 35′.
„ „ „ ce_4 and se_5 $(m=4)$: „ 49° 9′, „ 46° 11′,
„ 71° 53′, „ 71° 4′.

6. Approximations for Small θ.

It is known * that if an elliptic-cylinder function is developed as a power series in θ (with coefficients periodic in x), the development will be convergent for sufficiently small values of $|\theta|$. From the series (and the derived series) it is possible to obtain expressions for the zeros and turning points which are of some practical utility in computation with values of θ small in comparison with unity.

The expressions for the zeros, when $\theta>0$, are as follows:

in $ce_2(x, \theta)$, $\cos 2x = -\frac{1}{3}\theta + \frac{7}{180}\theta^3 - \frac{39217}{4354560}\theta^5 + \ldots;$

in $se_3(x, \theta)$, $\cos 2x = -\frac{1}{2} - \frac{3}{32}\theta + \frac{33}{2560}\theta^2 - \frac{21}{40960}\theta^3 + \ldots;$

in $ce_3(x, \theta)$, $\cos 2x = \frac{1}{2} - \frac{3}{32}\theta - \frac{33}{2560}\theta^2 - \frac{21}{40960}\theta^3 + \ldots;$

in $se_4(x, \theta)$, $\cos 2x = -\frac{1}{15}\theta + \frac{17}{47250}\theta^3 + \ldots;$

* Whittaker and Watson, *Modern Analysis*, § 19.61.

in $ce_4(x, \theta)$, $\cos 2x = \frac{1}{\sqrt{2}} - \frac{1}{30}\theta - \frac{31\sqrt{2}}{14400}\theta^2 + \ldots,$

and $\cos 2x = -\frac{1}{\sqrt{2}} - \frac{1}{30}\theta + \frac{31\sqrt{2}}{14400}\theta^2 + \ldots;$

in $se_5(x, \theta)$, $\cos 2x = \frac{\sqrt{5}-1}{4} - \frac{5+\sqrt{5}}{192}\theta - \frac{13\sqrt{5}}{32256}\theta^2 + \ldots,$

and $\cos 2x = -\frac{\sqrt{5}+1}{4} - \frac{5-\sqrt{5}}{192}\theta + \frac{13\sqrt{5}}{32256}\theta^2 + \ldots;$

in $ce_5(x, \theta)$, $\cos 2x = \frac{\sqrt{5}+1}{4} - \frac{5-\sqrt{5}}{192}\theta - \frac{13\sqrt{5}}{32256}\theta^2 + \ldots,$

and $\cos 2x = -\frac{\sqrt{5}-1}{4} - \frac{5+\sqrt{5}}{192}\theta + \frac{13\sqrt{5}}{32256}\theta^2 + \ldots;$

in $se_6(x, \theta)$, $\cos 2x = \frac{1}{2} - \frac{3}{140}\theta - \frac{699}{1254400}\theta^2 + \ldots,$

and $\cos 2x = -\frac{1}{2} - \frac{3}{140}\theta + \frac{699}{1254400}\theta^2 + \ldots.$

The expressions for the turning points are:

in $se_2(x, \theta)$, $\cos 2x = -\frac{1}{6}\theta + \frac{1}{160}\theta^3 + \ldots;$

in $se_3(x, \theta)$, $\cos 2x = \frac{1}{2} - \frac{7}{96}\theta - \frac{73}{23040}\theta^2 + \ldots;$

in $ce_3(x, \theta)$, $\cos 2x = -\frac{1}{2} - \frac{7}{96}\theta + \frac{73}{23040}\theta^2 + \ldots;$

in $se_4(x, \theta)$, $\cos 2x = \frac{1}{\sqrt{2}} - \frac{7}{240}\theta - \frac{143\sqrt{2}}{115200}\theta^2 + \ldots,$

and $\cos 2x = -\frac{1}{\sqrt{2}} - \frac{7}{240}\theta + \frac{143\sqrt{2}}{115200}\theta^2 + \ldots;$

in $ce_4(x, \theta)$, $\cos 2x = -\frac{7}{120}\theta - \frac{67}{3024000}\theta^3 + \ldots;$

in $se_5(x, \theta)$, $\cos 2x = \frac{\sqrt{5}+1}{4} - \frac{23(5-\sqrt{5})}{4800}\theta - \frac{5917\sqrt{5}}{20160000}\theta^2 + \ldots,$

and $\cos 2x = -\frac{\sqrt{5}-1}{4} - \frac{23(5+\sqrt{5})}{4800}\theta + \frac{5917\sqrt{5}}{20160000}\theta^2 + \ldots;$

in $ce_5(x, \theta)$, $\cos 2x = \frac{\sqrt{5}-1}{4} - \frac{23(5+\sqrt{5})}{4800}\theta - \frac{5917\sqrt{5}}{20160000}\theta^2 + \ldots,$

and $\cos 2x = -\frac{\sqrt{5}+1}{4} - \frac{23(5-\sqrt{5})}{4800}\theta + \frac{5917\sqrt{5}}{20160000}\theta^2 + \ldots;$

in $se_6(x, \theta)$, $\cos 2x = \frac{\sqrt{3}}{2} - \frac{17}{2520}\theta - \frac{5101\sqrt{3}}{33868800}\theta^2 + \ldots,$

$\cos 2x = -\frac{17}{630}\theta + \ldots,$

and $\cos 2x = -\frac{\sqrt{3}}{2} - \frac{17}{2520}\theta + \frac{5101\sqrt{3}}{33868800}\theta^2 + \ldots.$

There appears to be better agreement with true values in the case of functions of low order than in those of high order.

7. The Tables.

The zeros and turning points of the twelve elliptic-cylinder functions of lowest order (when they exist in the interval $0<x<\frac{1}{2}\pi$) are given in Tables I and II respectively for a set of positive values of θ. The entries represent degrees and decimals. Tables for negative θ would be obtained by writing down complementary angles and interchanging se_r with ce_r for every odd r.

Interpolation to the degree of precision required by ordinary physical applications is usually possible, except at the beginning of the entries in the first column of turning points of the even functions. The graphs of these particular turning points are orthogonal to the θ-axis.

The computation of tables of the zeros and turning points for the hyperbolic-cylinder functions

$$ch_m(x,\theta)=ce_m(ix,\theta)\quad ; \quad sh_m(x,\theta)=-i\,se_m(ix,\theta)$$

is a task which is awaiting any worker who is prepared to undertake the necessary labour. The physical problems in which the hyperbolic-cylinder functions arise require the tabulation of θ against x (zero or turning point), rather than of x against θ; thus the tables would be set out in a form different from those given here.

I have to thank the Carnegie Trust for the Universities of Scotland for a grant in aid of the publication of the Tables.

TABLE I.—ZEROS.

θ	ce_2	se_3	ce_3	se_4	ce_4		se_5		ce_5		se_6	
0	45·0000	60·0000	30·0000	45·0000	22·5000	67·5000	36·0000	72·0000	18·0000	54·0000	30·0000	60·0000
1	53·7730	62·7806	33·4235	46·9011	23·9480	68·7650	37·1539	72·6710	18·7353	55·1145	30·7222	60·6955
2	59·8189	64·9863	37·3678	48·7512	25·6137	69·8825	38·3396	73·2861	19·5448	56·1847	31·4706	61·3643
3	63·7150	66·7387	41·4527	50·5086	27·5217	70·8841	39·5468	73·8521	20·4400	57·2087	32·2444	62·0069
4	66·3489	68·1479	45·2485	52·1450	29·6713	71·7954	40·7633	74·3747	21·4329	58·1868	33·0418	62·6238
5	68·2220	69·2984	48·5290	53·6468	32·0206	72·6341	41·9760	74·8584	22·5356	59·1214	33·8607	63·2158
6	69·6082	70·2524	51·2703	55·0116	34·4824	73·4095	43·1718	75·3067	23·7581	60·0159	34·6974	63·7838
7	70·6693	71·0549	53·5385	56·2450	36·9453	74·1244	44·3389	75·7227	25·1058	60·8744	35·5487	64·3285
8	71·5073	71·7392	55·4170	57·3565	39·3085	74·7786	45·4674	76·1089	26·5755	61·7007	36·4100	64·8508
9	72·1893	72·3296	56·9805	58·3575	41·5048	75·3725	46·5497	76·4674	28·1528	62·4973	37·2764	65·3513
10	72·7591	72·8449	58·2909	59·2600	43·5031	75·9084	47·5807	76·8005	29·8106	63·2649	38·1430	65·8307
12	73·6700	73·7032	60·3415	60·8136	46·8992	76·8219	49·4790	77·3973	33·2167	64·7080	39·8565	66·7286
14	74·3798	74·3933	61·8711	62·0963	49·5863	77·5560	51·1524	77·9133	36·4933	66·0122	41·5135	67·5490
16	74·9583	74·9639	63·0621	63·1703	51·7074	78·1469	52·6349	78·3610	39·4423	67·1648	43·0856	68·2961
18	75·4443	75·4468	64·0301	64·0828	53·3966	78·6266	53·9293	78·7553	41·9970	68·1672	44·5543	68·9749
20	75·8620	75·8631	64·8429	64·8690	54·7657	79·0222	55·0666	79·0932	44·1685	69·0309	45·9112	69·5906
24	76·5499	76·5501	66·1542	66·1610	56·8661	79·6374	56·9609	79·6633	47·5426	70·4075	48·2909	70·6545
28	77·1001	77·1002	67·1858	67·1876	58·4412	80·1122	58·4715	80·1196	49·9696	71·4316	50·2678	71·5320
32	77·5554	77·5554	68·0308	68·0314	59·6979	80·4922	59·7079	80·4946	51·7983	72·2236	51·9136	72·2629
36	77·9415	77·9415	68·7427	68·7429	60·7406	80·8094	60·7440	80·8103	53·2534	72·8646	53·2980	72·8799
40	78·2753	78·2753	69·3550	69·3551	61·6289	81·0808	61·6301	81·0810	54·4613	73·4024	54·4788	73·4085

TABLE II.—TURNING POINTS.

θ	ce_1	se_2	ce_2	se_3	ce_3		se_4	
0	..	45·0000	..	30·0000	..	60·0000	22·5000	67·5000
1	18·8163	49·6279	..	32·4561	..	62·3639	23·7277	68·6344
2	46·0988	53·5569	..	34·9812	..	64·6111	25·0445	69·6728
3	53·8620	56·6181	..	37·5143	..	66·6768	26·4443	70·6152
4	57·8130	58·9641	27·1987	39·9648	..	68·4870	27·9180	71·4644
5	60·2673	60·7848	35·6511	42·2486	..	70·0158	29·4515	72·2260
6	61·9875	62·2344	40·6876	44·3150	..	71·2856	31·0239	72·9070
7	63·2926	63·4162	44·0950	46·1506	15·1015	72·3374	32·6084	73·5157
8	64·3366	64·4010	46·5780	47·7669	23·7950	73·2118	34·1752	74·0600
9	65·2028	65·2376	48·4873	49·1872	29·0581	73·9431	35·6963	74·5478
10	65·9405	65·9598	50·0194	50·4383	32·7955	74·5589	37·1493	74·9861
12	67·1458	67·1522	52·3745	52·5316	37·8654	75·5313	39·7987	75·7385
14	68·1032	68·1054	54·1494	54·2119	41·2074	76·2585	42·0838	76·3588
16	68·8921	68·8929	55·5686	55·5947	43·6259	76·8297	44·0325	76·8781
18	69·5592	69·5596	56·7469	56·7583	45·5038	77·2964	45·6972	77·3195
20	70·1348	70·1349	57·7509	57·7560	47·0360	77·6883	47·1297	77·6999
24	71·0863	71·0863	59·3910	59·3921	49·4470	78·3233	49·4708	78·3263
28	71·8498	71·8498	60·6926	60·6928	51·3074	78·8239	51·3138	78·8247
32	72·4829	72·4829	61·7634	61·7635	52·8144	79·2346	52·8163	79·2348
36	73·0206	73·0206	62·6678	62·6678	54·0736	79·5809	54·0750	79·5809
40	73·4860	73·4860	63·4470	63·4470	55·1520	79·8790	55·1530	79·8790

TABLE II.—TURNING POINTS—*continued.*

θ	ce_4		se_5		ce_5			se_6		
0	..	45·0000	18·0000	54·0000	..	36·0000	72·0000	15·0000	45·0000	75·0000
1	..	46·6714	18·6679	55·0303	..	37·0576	72·6240	15·3972	45·7728	75·3763
2	..	48·3445	19·3820	56·0312	..	38·1405	73·2069	15·8166	46·5438	75·7331
3	..	50·0195	20·1441	56·9995	..	39·2469	73·7521	16·2597	47·3115	76·0718
4	..	51·6921	20·9553	57·9324	..	40·3757	74·2632	16·7278	48·0741	76·3936
5	..	53·3482	21·8160	58·8259	..	41·5263	74·7441	17·2223	48·8298	76·6996
6	..	54·9626	22·7256	59·6810	..	42·6974	75·1985	17·7441	49·5766	76·9909
7	..	56·5023	23·6826	60·4923	..	43·8874	75·6300	18·2945	50·3126	77·2684
8	..	57·9416	24·6838	61·2598	..	45·0924	76·0417	18·8742	51·0360	77·5329
9	..	59·2604	25·7244	61·9832	..	46·3056	76·4358	19·4839	51·7448	77·7852
10	..	60·4543	26·7972	62·6629	..	47·5167	76·8134	20·1237	52·4372	78·0258
12	14·5801	62·4859	29·0007	63·8961	..	49·8772	77·5194	21·4934	53·7658	78·4742
14	24·1919	64·1062	31·2053	64·9737	..	52·0659	78·1548	22·9763	55·0109	78·8820
16	29·6266	65·3982	33·3225	65·9138	..	54·0142	78·7158	24·5538	56·1656	79·2520
18	33·3104	66·4373	35·2898	66·7353	12·5599	55·7055	79·2039	26·1945	57·2276	79·5874
20	36·0115	67·2870	37·0786	67·4563	20·2695	57·1524	79·6250	27·8569	58·1986	79·8911
24	39·7951	68·6033	40·1220	68·6570	28·3564	59·4321	80·2977	31·0767	59·8874	80·4155
28	42·4637	69·5989	42·5618	69·6162	32·9671	61·1009	80·7999	33·9599	61·2842	80·8478
32	44·5154	70·3971	44·5472	70·4029	36·0670	62·3756	81·1892	36·4317	62·4469	81·2080
36	46·1872	71·0615	46·1980	71·0635	38·3862	63·3990	81·5049	38·5219	63·4267	81·5122
40	47·5952	71·6287	47·5990	71·6294	40·2458	64·2533	81·7701	40·2981	64·2642	81·7758

(*Issued separately October* 18, 1932.)

XXIV.—Adrenaline and the Œstrous Cycle in the Mouse. By J. M. Robson, M.D., B.Sc. (From the Macaulay Laboratory, Institute of Animal Genetics, University of Edinburgh.)

(MS. received June 9, 1932. Read July 4, 1932.)

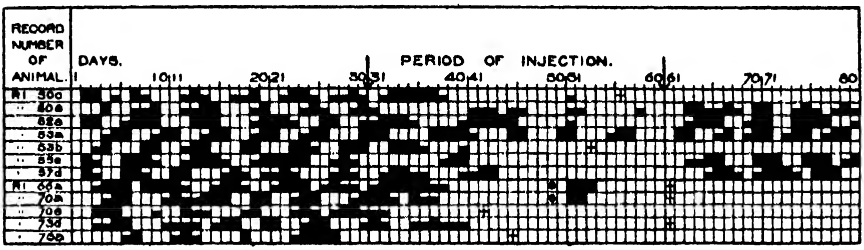
INHIBITION of the œstrous cycle has been brought about by a variety of experimental procedures in an attempt to investigate the factors which normally control the maintenance of the cyclic activity of the sex organs. It is not intended here to enter into a discussion of all the methods which have been used experimentally to affect the periodicity of œstrus. Various preparations of the corpus luteum have been shown to interrupt the cycle (Parkes and Bellerby, 1927; Patel, 1930), and it has been suggested that the action is specific and corresponds to the physiological interruption obtained during pregnancy and pseudopregnancy. A number of chemical substances have been used to obtain the same result. Buschke, Zondek, and Berman showed that the feeding of thallium interrupts œstrus; lead (Buschke and Berman, 1927) and iodine (Braude and Schwarzmann, 1929) give the same effect.

In an attempt to determine the part played by the sympathetic and parasympathetic innervation in the activity of the ovary, Kraul (1929) investigated the effect of pilocarpine and adrenaline on the œstrous cycle. He was able to cause interruption of œstrus, and related the effect to the action on the nerve supply of the ovary. The site and method of action of the drugs were, however, not determined. Zondek (1931) could only partly reproduce this effect and was unable to prevent the onset of œstrus in immature animals treated with adrenaline. In the experiments to be described an attempt has been made to analyse the action of adrenaline on the sex-cycle in the mouse.

A number of conventional signs have been used in the tables and they will be described here to avoid repetition. A full block shows that the smear contains only cornified cells. Partial filling in of the block shows that the smear contains other cells in addition to cornified cells, and an attempt has been made to represent the percentage of cornified cells present in the smears by the degree of blackening of the squares. The + sign means animal killed. M.U. refers to mouse units. Other signs used are explained in the separate tables.

A preliminary series of experiments were performed in the first instance in order to determine the action of adrenaline on the œstrous cycle of mice. A number of animals were smeared daily for several weeks, and a group in which œstrus occurred at regular intervals was chosen for the experiments. These were then injected twice daily with 0·05 c.c. of a 1 per cent. solution of adrenaline hydrochloride (Parke Davis). It was found that the injections had a marked effect upon the occurrence of œstrus; in the majority of animals œstrus was completely inhibited for varying periods up to several weeks. In some animals œstrus still occurred, but at much longer intervals than that obtaining during the non-injected period. When the injections were discontinued regular œstrous cycles occurred again, and the first œstrous period was observed within one or two days after the cessation of adrenaline injections (see Table I, 5 animals). This finding

Table I.—Showing the Effect of Adrenaline Injections on the Œstrous Cycle.



Animals RI 50*c*–57*d* received 0·05 c.c. adrenaline (P.D.) twice a day between the thirtieth to the sixtieth days.

Animals RI 66*a*–75*a* received 0·07 c.c. adrenaline solution (1 : 1000) twice a day between the thirtieth to the sixtieth days.

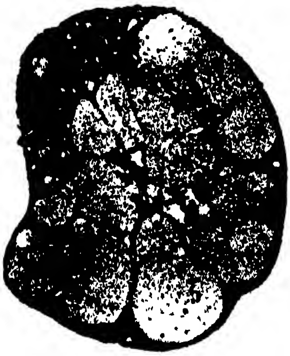
● = 1 M.U. of α-hormone (œstrin).

strongly suggests that the inhibitory action of the drug is not due to a toxic effect.

The preparation of adrenaline (P.D.) used in the preliminary experiments contained a certain amount of chloretone (0·5 per cent.), which might to some extent have been responsible for the effects observed. Control experiments were therefore performed in which 0·05 c.c. of saturated chloretone solution in saline was injected twice daily as above. There was no effect on the œstrous cycle. Further, a pure solution of adrenaline, free from antiseptic, was injected into another group of animals and produced the typical results obtained with adrenaline, which are also recorded in Table I.

Section of the genital tract of a number of animals injected with

adrenaline and in which œstrus had been inhibited was performed. The vaginal epithelium showed in some cases some degree of proliferation but no cornification. In other animals mucification was observed in the sections of the vagina. These findings suggest that the inhibition of a production was, in these experiments, only partial, and that sufficient a was being secreted to bring about mucification or some proliferation, but not cornification. In other animals, again, the vagina showed neither proliferation nor mucification, and it is likely that a secretion had, in these cases, been completely or almost completely inhibited. The ovaries, as a rule, showed a paucity of large follicles. They contained a large number of corpora lutea which were, however, not normal but contained an excess of fibrous tissue. The blood-supply of these corpora was poor. On the other hand, the blood-supply to the remainder of the ovary was, as a rule, very good and the ovaries were, in fact, occasionally markedly hyperæmic. The general histological appearance of the ovaries suggest that adrenaline causes inhibition of follicular maturation, a conclusion similar to that arrived at by Kraul (1929).



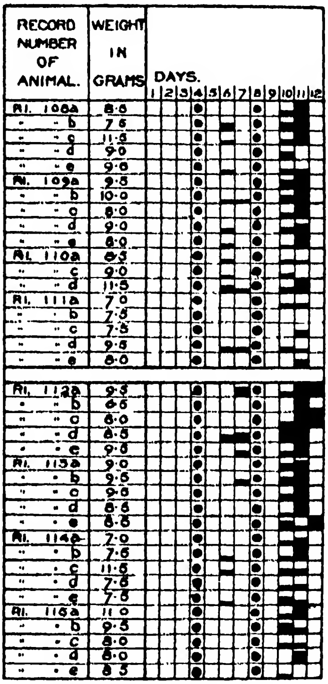
Section of ovary of animal (RI 75a) injected with adrenaline for 15 days. × 20.

Further experiments were now performed in order to determine the site of action of adrenaline in interrupting the œstrous cycle. It appeared possible that the drug might act (1) on the anterior lobe of the pituitary, or (2) on the ovary, or (3) on the vagina and uterus. In the first case, it would prevent or interfere with the secretion of the specific pituitary hormones responsible for stimulation of the ovary; in the second case, it would interfere with the reactivity of the ovary to the pituitary factor or with the secretion of the ovary; while, thirdly, the action of the ovarian secretion (a-hormone, œstrin) on the vagina and uterus might be prevented. Lastly, a combination of any two or all three of these effects may have occurred.

A number of experiments were, in the first instance, performed to determine whether adrenaline interfered with the action of a on the uterus and vagina. Twenty immature animals (of which 2 died during the experiment and are not shown in the table) were injected with 0·015 c.c. of adrenaline hydrochloride twice a day, and 20 animals acted as controls. Of the first group 10 animals received a single injection of 0·08 c.c. of a solution

of a in maize oil, and the other 10 animals half that quantity of a, also in a single injection. The control group was similarly subdivided into two groups of 10 receiving doses of a. The results are figured in Table II. It will be seen that the administration of adrenaline had no appreciable effect on the action of a. The action of a was also investigated in animals injected with adrenaline for prolonged periods and in which the œstrous cycle was

Table II.—Showing the Effect of Combined Injections of Adrenaline and a-hormone (Œstrin) in Immature Animals.



Animals RI 108a–111e received 0·015 c.c. adrenaline (P.D.) twice a day from the first to the twelfth days.

On the fourth day all animals received 0·5 M.U. of a.

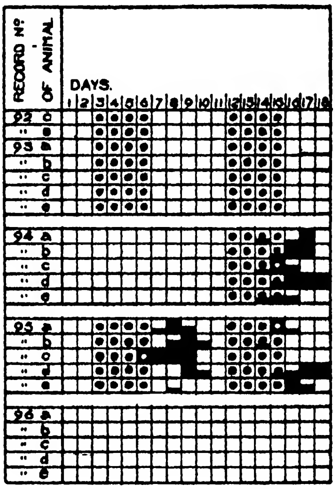
On the eighth day RI 108a–109e and 112a–113e received 1 M.U. a; 110a–111e and 114a–115e received 0·5 M.U. a.

being inhibited (Animals RI 66a and 70a). The injection of one M.U. of a in these animals was sufficient to bring on one œstrous cycle. Here again adrenaline did not interfere with the action of a.

The effect of adrenaline on the reactivity and secretion of the ovary was investigated by the injection of an œstrin-free gonadotropic preparation from pregnancy urine (ρ-factors) into immature animals. This was prepared according to the method of Wiesner and Marshall, 1931.

A number of immature animals weighing between 6 and 10 grm. were divided into four groups. The first group (7 mice) was injected with adrenaline and ρ-factors. They received 0·015 c.c. of adrenaline hydrochloride twice a day until the end of the experiment. On the third day of the experiment the injections of ρ-factors were begun and the animals were given six injections of ρ-factors (0·2 c.c. per injection) over a period of three days. (The first injection was given on the evening of the third day, and the last injection on the morning of the sixth day.) The third group was

Table III.—Showing the Effect of Combined Injections of Adrenaline and gonadotropic hormone in Immature Animals.



Animals 92c-93e received 0·015 c.c. adrenaline (P.D.) twice a day from the first to the eighteenth days.

The dots represent the 72 hours periods over which the injections of gonadotropic hormone were spread.

given ρ-factors, as in the first group, and no adrenaline (5 mice). From the first to the eighth day of the experiment the second group received adrenaline alone (5 mice). The fourth group (5 mice) was untreated.

The animals were smeared daily; of the first, second, and fourth groups no animals showed any vaginal cornification, while all the animals in the third group became cornified. A few days were allowed to elapse and the experiments were again repeated on the same animals with similar results.* The data are collected in Table III.

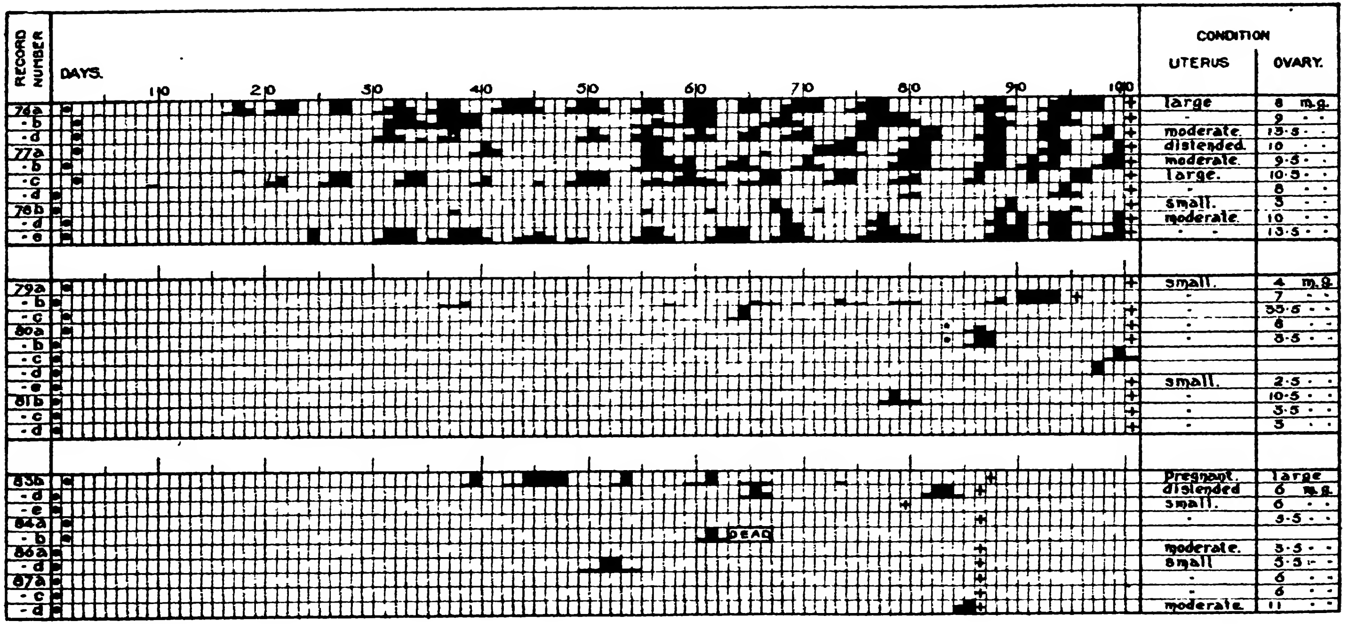
The experiment was performed on thirty other animals with similar results; it could be definitely concluded that the administration of adrenaline

* It will be observed that the second group, in which the injection of adrenaline had been discontinued, showed a positive reaction to the ρ-factors.

prevented, or interfered with, the production of cornification, although ρ-factors were administered. Post-mortems performed on the animals showed that the uterus also had been interfered with. The morphological changes in the ovaries (formation of follicles and of blood-spots) were not markedly different in the groups injected or not injected with adrenaline. The vaginæ of the experimental animals were sectioned. The animals injected with ρ-factors alone showed complete cornification, while in the animals injected both with adrenaline and ρ-factors there was little or no effect in the majority of cases. In a minority of these animals the vaginal mucosa showed mucification, which was sometimes very marked. As it has been previously shown (Robson and Wiesner, 1931; Mirskaia and Wiesner, 1931) that vaginal mucification is due to the action of doses of α smaller than those necessary to bring about cornification, and that the first (puberal) cycle is preceded by mucification, it would appear that in these animals the adrenaline had only partly interfered with the secretory activity of the ovaries. These experiments therefore definitely showed that adrenaline interfered with the secretion of the α-hormone by the ovary.

On the basis of the results so far obtained it appeared likely that the administration of adrenaline in immature animals would prevent the onset of maturity in the genital organs, and experiments were performed to determine whether this was the case. Doses of adrenaline were injected into a group of animals (79*a*–81*d*) for prolonged periods, a second group (76*a*–78*e*) of mice serving as controls. At the beginning of the experiment 0·02 c.c. of a 1 : 1000 solution adrenaline hydrochloride was injected twice a day. This dose was gradually increased until the animals at the end of the experiment were receiving 0·05 c.c. twice a day. The injections appeared to have no appreciable effect on the health of the animals. In practically all the injected mice œstrus was completely inhibited for the whole period of injections, which extended over 100 days. All the control animals came into œstrus. The results of the experiment are given in Table IV. A similar experiment (83*b*–87*d*) was spread over about seven weeks and gave similar results. In two of the injected animals (animals 80*a* and *b*), as is shown in the table, one M.U. of an oily solution of œstrin was injected subcutaneously and produced vaginal cornification, thus further showing that adrenaline did not interfere with the action of the α-hormone. The animals used in these experiments were weighed at fortnightly intervals. The weights are collected in Table V. It will be seen that adrenaline did not have any marked effect on the growth of the animals, although in both experiments the injected animals lagged slightly behind the control groups. Necropsy was performed on all the animals and the sex organs were

TABLE IV.—SHOWING THE EFFECT OF ADRENALINE INJECTIONS IN IMMATURE ANIMALS.



The dots shown at the beginning of the experiment represent the time of opening of the vagina.
Animals *79a–87d* received 0·015–0·05 c.c. adrenaline (P.D.) twice a day for the whole period of the experiment.
Animals *76a–78e* received no injections.
Animals 80*c*, 80*d*, 83*b*, and 87*d* mated.

● = 1 M.U. *a* injected subcutaneously.

subjected to histological examination. In addition, the ovaries were also weighed at the end of the period of injection. The ovaries of the animals injected with adrenaline weighed appreciably less than those of control non-injected animals. The average weight in the injected animals was 5·5 mg. as compared with an average weight of 9·5 mg. for the control group. Histological sections showed that the degree of development of

TABLE V.

SHOWING THE EFFECT OF ADRENALINE INJECTIONS ON THE GROWTH OF MICE.

Age of Animals.	Average Weight of Animals.		
	Control Group.	Injected with Adrenaline.	
Days.		Group I.	Group II.
	grm.	grm.	grm.
21	8·2	7·8	9·0
35	13·4	13·3	13·2
49	17·8	16·2	15·5
63	20·2	17·8	17·5
77	20·7	18·5	19·4
91	22·7	19·5	19·6
105	23·1	19·4	22·8
119	25·2	20·1	

Control Group = 10 animals.
Group I. = 11 ,,
Group II. = 10 ,,

the ovaries in the injected animals was less than that of the control animals. They did not contain any luteal tissue. The degree of development of the follicles was, on the whole, less than in the control group. There was, in some cases, a paucity of mature follicles with fully developed antrum. In other animals, however, the morphological appearance was similar in type to that of non-injected animals in so far as follicular development was concerned, although the ovaries were appreciably smaller. The effect on the histological appearance of the ovaries was, on the whole, less than in the mature animals. The uterus and vagina of the animals treated with adrenaline were considerably less developed than those of the control mice.

The immature animals which had been injected with adrenaline for long periods (more than six weeks, the injections being started when the animals were three weeks old), and in which maturity had been

prevented, were mated with fertile males, the fertility being tested before and after the matings. Mating was allowed on several successive days and at different periods. Out of twenty animals on which these experiments were performed, only four animals mated (once each). These matings occurred towards the end of the period of injection, and it appears likely that the adrenaline was then no longer capable of completely inhibiting the action of the ovary in all cases. This view is supported by the fact that, as will be seen in Table IV, isolated periods of cornification were already being observed.

Discussion.

The experiments described in this paper show that adrenaline is capable of interrupting the œstrous cycle when administered to mature female mice, and of interfering with the onset of maturity in immature animals. Interruption of the œstrous cycle may be complete and prolonged for several weeks, or it may be of a shorter and more intermittent character. In the same way the inhibitory action in immature animals may also be partial, for in a few animals œstrus did occur towards the end of the experimental period of injection, and mating actually took place in four out of twenty animals. Examination of the sexual organs at the conclusion of the period of injection showed that the growth and development of the ovaries, uterus, and vagina had been interfered with; but nevertheless these organs were appreciably larger and better developed than those of immature animals. These results confirm the findings of Kraul (1924) and do not support the results described by Zondek (1931), especially on the action of adrenaline in immature animals. The fact that two injections a day were given in the experiments here described may, to some extent, account for the different results obtained.

It appears unlikely that the action of the drug is due to a general effect on growth and metabolism, for the growth of the injected animals was not appreciably interfered with and their general appearance was not different from that of non-injected animals. It seems unlikely that the prolonged injections caused any toxic effects, even on the ovaries, as œstrus returned very rapidly when the injections were stopped. It appears more likely that the drug had a specific site of action, and general consideration suggested that adrenaline might act (1) on the pituitary, preventing the secretion of the specific gonadotropic substance; or (2) on the ovary, interfering with the secretion of the α-hormone; or lastly (3) on the uterus and vagina, preventing the action of the α-hormone on these organs.

The experimental data ruled out the third possibility, but showed that adrenaline interfered with the action of gonadotropic hormone on the ovary, and with the stimulation of the specific ovarian secretion. There is, however, no evidence to show whether or not adrenaline has, in addition, any action on the activity of the pituitary.

It must be pointed out that the drug was injected only twice a day. It is well known that adrenaline is rapidly absorbed and destroyed in the body, and it appears, therefore, highly probable that, under the given experimental conditions, the action of the drug was only extended over a comparatively short fraction of twenty-four hours. This factor probably explains the partial nature of the inhibitory action observed in some of the experiments, and it seems likely that continued action of adrenaline would in every case completely interfere with the development and secretion of the ovary.

It is of some interest to compare the results here obtained with the interruption of œstrus brought about by the administration of thallium. Buschke, Zondek, and Berman (1927) found that the administration of *a* in these animals brought about, as a rule, a single œstrous period. Moreover, Bickel and Buschke (1932) have recently shown that the implantation of anterior pituitary lobe into mice fed on thallium is, as a rule, followed by œstrus and that occasionally irregular œstrous cycles ensue. These results are apparently different from those obtained with adrenaline, although the experiments were not carried out quantitatively, and there is no evidence that thallium exerted no inhibitory action whatever on the ovarian response to anterior pituitary lobe hormone. Another difference was observed with the thallium-treated animals, in that no interference with the morphological changes in the ovaries was obtained, whereas the adrenaline-injected animals showed inhibition of follicular maturation.

It seems, therefore, likely that the œstrous cycle can be inhibited by an action on various organs and at various points in the chain of reactions. Experiments with a number of other substances are at present in progress and it seems advisable to postpone any further discussion on the mechanism of œstrus inhibition. Similarly, it would be premature to offer any opinion as to whether the inhibition by adrenaline plays any part in the regulation of the ovarian activity under physiological conditions.

Summary.

The injection of adrenaline hydrochloride into mature female mice is able to inhibit the occurrence of œstrus.

The injection of adrenaline hydrochloride into immature female mice prevents the onset of sexual maturity.

Adrenaline does not interfere with the action of a-hormone (œstrin) on the uterus and vagina; it has an inhibitory effect on the action of gonadotropic hormones (prepared from pregnancy urine) on the ovary.

I should like to thank Mr P. G. Marshall for the supply of gonadotropic preparation from pregnancy urine used in these experiments.

BIBLIOGRAPHY.

Bickel and Buschke, 1932, *Klin. Wschr.*, p. 679.
Braude and Schwarzmann, 1929, *Arch. Gynäcol.*, **138**, 782.
Buschke and Berman, 1927, *Münch. med. Wschr.*, p. 969.
Buschke, Zondek, and Berman, 1927, *Klin. Wschr.*, p. 683.
Kraul, 1927, *Arch. Gynäcol.*, **131**, 600.
Mirskaia and Wiesner, 1931, *Proc. Second Internat. Congress for Sex Research.*
Parkes and Bellerby, 1927, *J. Physiol.*, p. 64.
Patel, 1930, *Quart. J. Exp. Physiol.*, vol. xx, p. 245.
Robson and Wiesner, 1931, *ibid.*, vol. xxi, p. 217.
Wiesner and Marshall, 1931, *ibid.*, p. 147.
Zondek, 1931, *Die Hormone des Ovariums und des Hypophysenvorderlappens*, Berlin.

(*Issued separately October 24, 1932.*)

OBITUARY NOTICES.

Alexander G. Burgess, M.A., D.Sc.

Dr Alexander G. Burgess, Rector of Rothesay Academy, died on 29th March, aged fifty-nine. He was born in Wishaw, where his father was a highly respected banker. His early education was received at Wishaw Academy, whence he proceeded to George Watson's College, Edinburgh. Here he distinguished himself in Mathematics and gained many honours, including the Welsh Mathematical Bursary for entrance to the University of Edinburgh. At the University he had a brilliant career, and in 1894 graduated with 1st Class Honours in Mathematics and Natural Philosophy. On leaving the University he entered the teaching profession and gave valuable service in Merchiston Castle School, the Edinburgh Ladies' College, and finally as Rector of Rothesay Academy since 1917. A man of untiring energy, he gained the respect and admiration of his pupils and of his colleagues. His success as a teacher was shown by the high honours obtained by his pupils on entering the Universities. He was also a well-known and successful tutor for the Army and Higher Civil Service Examinations. In 1902 he was elected a Fellow of the Royal Society of Edinburgh. He took an active part in the proceedings of the Edinburgh Mathematical Society, of which he was Secretary for three years. To its *Proceedings* he contributed at least thirteen articles. Later he was elected President, and it was during his presidency that the first Mathematical Colloquium in 1913 was so successfully carried out in Edinburgh. In 1914 his fine powers of organisation and administration shown during the Colloquium pointed him out as the right man to take charge of the arrangements for the Napier Tercentenary Celebrations in July 1914. His genial tact and enthusiasm again ensured success. In March 1924 his Alma Mater recognised his abilities by conferring on him the degree of D.Sc., for a thesis on Tripolar Co-ordinates. His zeal and untiring energy led him to take part in many other spheres of life. At all times he was a strong supporter of the Educational Institute of Scotland, and in 1924 was elected President of the Renfrew and Bute Branch, while for four years he was

Vice-President of the Bute Natural History Society. He was also a keen Church worker. A man of sound judgment and great experience, he did all he had to do in a quiet unostentatious manner that endeared him to all with whom he came in contact. The sudden death of his charming and helpful wife in the autumn of last year was a heavy loss which he suffered with great resignation. He is survived by a son and daughter.

S. G.

James Edward Crombie, M.A., LL.D.

James Edward Crombie was born in the parish of Old Machar in 1862. He was the son of John Crombie, an *alumnus* of Marischal College, 1833–1834, and the grandson of James Crombie, who founded a woollen manufacturing business at Cothal, near Aberdeen, towards the end of the eighteenth century. James Edward Crombie was educated in Old Aberdeen, became a student at King's College, and graduated Master of Arts in 1882. Thereafter he studied for a time in Germany, and on his return he entered the Grandholm Woollen Manufacturing Works, where, after a short time, he took an active part in the business. He was deeply interested in organisation and in craftsmanship, and was responsible in a large measure for the subsequent development of the business, of which he was a director for over thirty years.

Despite the claims of his business he found time to devote to public work and to other pursuits. He was deeply interested in folk-lore, and to further his studies of the subject acquired a good knowledge of French, Spanish, and German, as well as a serviceable knowledge of Italian and Russian. In pursuit of this interest he travelled widely in the more remote parts of Europe. He studied more particularly the ideas of primitive peoples in relation to death and burial; he also explored some aspects of their recreations and amusements, and had a considerable number of manuscripts dealing with these subjects.

He gave considerable attention to scientific subjects, more especially those connected with meteorology and seismology, and for many years had meteorological instruments and a seismograph installed near to his house, which he tended himself, taking continuous records. He was an active member of the Seismology Committee of the British Association for the Advancement of Science, and did a great deal for the promotion of the study of the subject in this country. For some years he contributed generously to the financial support of the work carried out for the Committee at the University of Oxford.

In addition to all these activities he took a prominent part in public affairs, both local and national. For many years he served on the boards of the Sick Children's Hospital and of the Newhills Home, as well as on other kindred bodies.

The University of Aberdeen is indebted to him for the services he rendered it, first as Rector's Assessor and later for a long period as Chancellor's Assessor, an office which he resigned this year, to the great regret of his colleagues. Only those who had the privilege of working with him can appreciate properly the value of his counsel, his unsparing devotion to every duty he undertook, and his ready and generous response with financial help to any object he thought deserving of his support, for the most part given anonymously.

He took an active part in politics, and it was his exertions during the autumn of 1931, in connection with the then impending general election, which were the immediate cause of his breakdown in health. Acting under medical advice he gave up most of his public work, and after a complete rest of some months he recovered his health. His friends were encouraged by this recovery to hope that he had many years of usefulness before him, but a recurrence of his trouble in the late summer of this year rendered these hopes vain. He died on the 6th of August 1932.

He was elected a Fellow of the Society in 1915.

H. M. M.

Robert W. Dron, M.A., M.Inst.C.E., M.Inst.M.E.

PROFESSOR DRON, who attained distinction both as a mining engineer and as a teacher, died at his home at Bearsden on 16th April 1932, at the age of sixty-four. Trained as a mining engineer and surveyor, he entered practice as a consultant in 1901, and founded the firm of R. W. Dron & Son, Civil and Mining Engineers, Glasgow. Though most of his professional work was undertaken in Scotland, he was engaged from time to time to examine mining properties abroad, and visited places as far afield as the Caucasus, Spain, Nova Scotia, California, and Spitzbergen.

His teaching experience dated from the late 'eighties, when he took charge of classes in the West of Scotland attended by mine officials and miners. Most of these students were preparing for the Colliery Manager's Certificate examinations; many of them must have been older than their instructor, and some reached important positions in the industry; but all had reason to be grateful to Dron for his guidance and help. His first book, *A Text Book of Mining Formulæ,* published when he was twenty-three years of age, was intended for the assistance of students of that period and type. All through his life he was intensely interested in geology; many of his papers deal with problems of coal-field geology, and since 1902, when his best-known work, *The Coal Fields of Scotland,* was published, he has been a recognised authority on the subject of which it treats. A second and revised edition was issued in 1921.

His last book appeared in 1928, under the title of *The Economics of Coal Mining.* In writing it he had again in view the difficulties and aspirations of the younger generation; he essayed the formidable task of expounding the financial aspects of mining operations, and succeeded in producing a volume that is a model of skilful selection, arrangement, and style. In 1922 he accepted the invitation of the University of Glasgow to the then vacant Dixon Chair of Mining, and at his induction the Senatus conferred on him the honorary degree of Master of Arts.

Dron joined the Mining Institute of Scotland in 1888, and soon became an active member and one of its representatives on the Council of the Institution of Mining Engineers. He was president of the Scottish Institute from 1923 to 1925, and Chairman of the Finance and Publications Committee of the Institution from 1916 to 1931. A year ago he was elected by unanimous vote to the Presidency of the Institution, the highest

professional distinction open to a British mining engineer. He was, however, never to enjoy the honour he had so fully earned. The day after his election he was ordered to bed, and there he stayed for many weary months until he died.

Outside his engineering and University activities, Dron found time to devote to educational interests. In 1910 he was elected to the New Kilpatrick School Board; later becoming Chairman of the Dumbartonshire Education Authority, and, more recently, Chairman of the Education Committee of the County Council.

His touch was sure and skilful, whether in practical matters or in the lecture room, and his wide experience of men and affairs commanded respect. He was a keen and fearless critic in matters concerning those branches of technology of which he was master; yet a man of charm and humour, a delightful companion, and the staunchest of friends.

He was elected a Fellow of the Society in 1917.

H. B.

Giulio Fano, 1856–1932.

GIULIO FANO was born in Mantua in 1856 and died on 27th September 1932. He studied in Padua, Bologna, and Turin. He was a pupil of Bizzozero. He graduated in 1879, and after graduation worked in Ludwig's laboratory in Leipzig for a year, then under Angelo Mosso in Turin, later with Luciani in Florence, and in 1884 transferred to Genoa, where he became Professor of Physiology. In 1894 he succeeded Luciani in Florence, and in 1916 was appointed Professor of General Physiology in Rome, a post which he occupied until his death. He was a member of the Accademia dei Lincei, was LL.D. of St Andrews, an honorary member of many societies, Italian and foreign. His contributions to science were numerous. The first, on the behaviour of peptone and tryptone in blood and lymph, was founded on work done in Ludwig's laboratory. This was succeeded by publications on muscular work, on respiration, on the mechanism of voluntary movements, on the heart, etc., extending over many years. He described the oscillations of tone exhibited by the auricles of *Emys europæa*, a fact of great interest in connection with the function of cardiac and plain muscle tissue and the tone of muscle in general. His work on the chemistry of respiration extended over many years. He also carried out important researches on osmotic pressure, surface tension, and viscosity of the blood and similar fluids. For his work on spinal reflexes he was awarded by the Accademia dei Lincei the royal prize for biology. Mention must also be made of his researches on the thyroid and on the labyrinth. Apart from his scientific work he made, especially of late years, important contributions of a more general character. Among these may be enumerated *Fisiologia e Civiltà*, *Un Fisiologo intorno al mondo*, *Inhibition et Volonté*, *Cervello e Cuore* (also published in French and in English), and a memorial tribute to Louis Pasteur. Accompanied by his accomplished wife, he was a frequent attendant at the triennial International Congresses of Physiology and was well known to British physiologists. It was a great regret to us that he was unable to be present at the International Congress held in Edinburgh in 1923. A list of his published works will be found in the *Archivio di Fisiologia*, vol. xxiv, p. viii, a journal which he himself founded in 1904 and edited until his death.

He was elected a Foreign Honorary Fellow of the Society in 1930.

E. A. S.-S.

Sir Patrick Geddes.

By the death of Sir Patrick Geddes (b. 1854) at Montpellier on 17th April 1932 the Royal Society has lost one of its most distinctive Fellows. Whether as botanist, sociologist, reviver of the old-time intellectual associations between Scotland and France, or as the exponent of ideas upon town-planning, which he was sometimes enabled to realise, he was always the pioneer and a unique source of inspiration to those with whom he came into contact. Recognition of his unusual gifts came slowly; it may be that a generation must pass before the full tribute will be paid.

In collaboration with his lifelong friend, Sir John Arthur Thomson, he began and finished his principal general contributions to biology in the two works, *The Evolution of Sex* (1889), and *Life: Outlines of General Biology* (1931). There was also the little volume on *Evolution* in the Home University Library in the course of this memorable association.

His connection with the Royal Society of Edinburgh began in 1880, and was immediately followed by the contribution of a series of notable papers. Their titles and dates are as follows:—

Transactions.

"On the Phenomena of Variegation and Cell-multiplication in a Species of Enteromorpha." *Trans. R.S.E.*, vol. xxix, 1880, pp. 555–559. By Patrick Geddes.

"Sur l'Histologie des Pédicellaires et des Muscles de l'oursin (*Echinus sphæra*, Forbes)." *Trans. R.S.E.*, vol. xxx, 1883, pp. 383–395. By Patrick Geddes and Frank E. Beddard.

Proceedings.

"On the Classification of Statistics and its Results." *Proc. R.S.E.*, vol. xi, 1882, pp. 295–322. By Patrick Geddes.

"On the Nature and Functions of the 'Yellow Cells' of Radiolarians and Cœlenterates." *Proc. R.S.E.*, vol. xi, 1882, pp. 377–396. By Patrick Geddes.

"A Re-statement of the Cell-theory, with Applications to the Morphology, Classification, and Physiology of Protists, Plants, and Animals.

Together with an Hypothesis of Cell-structure, and an Hypothesis of Contractility." *Proc. R.S.E.*, vol. xii, 1884, pp. 266–292. By Patrick Geddes.

"A Synthetic Outline of the History of Biology." *Proc. R.S.E.*, vol. xiii, 1886, pp. 904–911. By Patrick Geddes.

"Theory of Growth, Reproduction, Sex, and Heredity." *Proc. R.S.E.*, vol. xiii, 1886, pp. 911–931. By Patrick Geddes.

"History and Theory of Spermatogenesis." *Proc. R.S.E.*, vol. xiii, 1886, pp. 803–823. By Patrick Geddes and J. Arthur Thomson.

But the strength of Geddes's genius did not lie so much in analysis as in synthesis. He realised that in every generation there will be the need for workers who, after having passed through the severest disciplines of their science, shall give themselves rather to the elucidation of its larger implications, concerning themselves with its impacts and linkages. In this connection the sweep of Geddes's mind can rarely have been equalled, while some of the practical aids in elaboration of his ideas have proved as fertile in development as the ideas themselves. His combination of the teaching of history and geography as exhibited and expounded in the Outlook Tower in Edinburgh, his establishment of residential halls for students, and his partnership in the origination of summer schools are but a few of the wide-ranging issues of his synthetic thinking that will have lasting value. Yet he was no born Schoolman, but, as it happened, the son of an Army officer, with countless other calls to service, and many other visions competing for the support and patronage of his tireless energy.

In the realm of the material and objective, *e.g.* in town-planning, Geddes was conservative, seeking to preserve the old whenever reasonable excuse could be found for doing so. In the realm of the mind he was liberal and revolutionary, ready always to experiment with new ideas. He had a rare gift of imagination, which served him well in every one of his multifarious adventurings. It was an unusual education to sit beside him and watch him devising graphs, formulæ, and diagrams in aid of the thesis he happened to be elaborating.

Those who studied Botany under him during his tenure of that Chair in University College, Dundee, have told how in the course of practical instruction in the garden, they would be as much impressed with his vivid sociological excursuses as with his treatment of the set preoccupation of the hour. His outlook was neo-Lamarckian rather than neo-Darwinian, and there was a temperamental touch of the mystic about him, although he would have been the last to admit the fact. He was very generous in

sympathetic understanding of, and interest in, the many peculiar situations and problems upon which men consulted him, and on occasion at critical points in individual careers gave counsel the wisdom of which became apparent with the years. He enjoyed the company of his friends, and many will treasure his memory as one of their chief possessions.

J. Y. S.

Eugène Gley (1857–1930).

Eugène Gley, who died on 24th October 1930, was born in 1857 at Épinal, where his father, Gérald Gley, was Professor of Latin. Gérald Gley came from Gérardmer in the Vosges: the name Gley is said to indicate a Celtic origin. Eugène Gley began to study medicine at Montpellier but transferred to Nancy, where he obtained the doctorate in 1881. Probably he had already determined to pursue Physiology, for his thesis was on a physiological subject: "Étude expérimentale sur l'état du pouls carotidien pendant le travail intellectuel"; the gist of it was published in the same year in the *Archives de physiologie normale et pathologique.* At the age of 23 he proceeded to Paris. Here he at first worked with Marey, whom he was a few years later to eulogise in an allocution pronounced on behalf of the Société de Biologie on the occasion of the presentation to Marey of a medal commemorative of the completion of his fiftieth year of scientific work. In 1883 Gley was made "préparateur des travaux pratiques de physiologie" in the Faculty of Medicine, and in 1889 became "professeur agrégé." In 1893 he was appointed to the Muséum d'Histoire Naturelle, where he continued actively to pursue physiological researches. His reputation as an experimental physiologist was now established: he had already published more than 130 papers dealing with different aspects of the subject. During the next few years he continued to produce an astonishing amount of work both alone and in collaboration with others. Included amongst these are many distinguished names—Richet, Lapicque, Charrin, Phisalix, Thiroloix, Pachon, Langlois. As there was no vacant professorship in Physiology in Paris, the Collège de France created in 1908 a Chair of General Biology which he was invited to occupy and which he retained until his death.

The most striking feature of Gley's scientific life is its activity. There were few subjects in Physiology which were not included in his investigations, and these continued without intermission during fifty years. A list of his publications from 1881 to 1929 embraces 524 titles!* His work is characterised by accuracy of observation and clarity of exposition; to which must be added a critical restraint in the premature formulation of conclusions which is a somewhat rare feature in scientific literature.

Considering the number of subjects he dealt with and the masterly

* "Liste chronologique des travaux et écrits scientifiques de Eugène Gley"—*Arch. inter. de pharmacodyn. et de thér.*, xxxviii, 1928.

manner in which they are treated, it is a little difficult to select those upon which his reputation is mainly based. But on the whole one would be inclined to pick out his work on the internal secretions as the most important. This began with observations on the results of removal of the thyroid and parathyroid, was continued on the effects of extracts of these and other endocrine glands, and culminated in a series of important experimental observations on the secretion of adrenaline, most of these last having been carried out with the collaboration of A. Quinquaud. A still more recent series of observations on the internal secretion of the sex-glands, undertaken with the co-operation of pupils and colleagues (Pézard, Caridroit, Champy), have served materially to advance our knowledge regarding this important branch of endocrinology.

In an early investigation, carried out in collaboration with L. Camus upon the serum of the eel (which Mosso had found to be highly toxic for other animals), Gley was able to show that the toxicity could be antagonised by the administration of very small doses gradually increased, enabling the cells of the organism to produce an antitoxin which prevents the otherwise toxic effects of the serum. In the course of this research the effect of hæmolysins upon the blood-corpuscles was also studied. It was found that this also could be antagonised by vaccinating an animal with minute doses of hæmolysin, thereby causing the production of antihæmolysins. These experiments helped materially to place upon a physiological basis the doctrine of the production of immunity by antibodies. Other subjects of Gley's researches were the coagulation of blood-plasma, the anticoagulant action of peptones and of the liver, the character of sensations, especially that of taste, the relation of the nervous system to secretion, the action of drugs, especially those acting upon the heart and blood-vessels, the production of anaphylaxis and of tachyphylaxis, the external and internal secretions of the pancreas, the pathogeny of goitre, the presence of a "vagal" substance in blood, and the influence of calcium on nerve excitability.

It is not infrequently found that scientific observers, having once worked at and apparently exhausted a subject, dismiss it from their future programme. But on looking through the chronological list of Gley's works we find him again and again reverting to subjects previously investigated, with the view of throwing further light on points which, in his judgment, he had incompletely elucidated.

Gley's books, as distinguished from his scientific articles and his memorial and other discourses, were not numerous, the principal being the *Traité élémentaire de physiologie*, published in 1906–1909 in three parts,

and in subsequent editions appearing in one volume, the 7th and last in 1928; *Les sécrétions internes*, a small eminently readable work which first appeared in 1914, passed through two or three editions and has been translated into several languages; and another small book, *Les grands problèmes de l'endocrinologie*, which appeared in 1926 and is distinguished by the same breadth of view that characterises his other publications.

A fluent speaker and a notable orator, Gley was frequently selected to deliver allocutions and eulogies on behalf of the Collège de France, or the Académie de Médecine—of which he occupied at one time the Presidential Chair—or the Société de Biologie, of which for some years he was secretaire general. The list of his writings includes, therefore, many discourses pronounced at the obsequies or centenaries of eminent biologists or at the jubilee celebrations of distinguished physiologists. No one will easily forget the brilliancy and fervour of his eloquence on such occasions. To British physiologists he was *persona grata*, and they on the other hand were sure of a warm welcome from him both in his laboratory and in his hospitable home. A prominent figure at all the International Congresses of Physiology, he was long the representative of French physiology on the International Committee. He was a frequent visitor to this country, and made a point of being present in London at the dinner in celebration of the fiftieth anniversary of the Physiological Society. We little thought that his memorable speech on that occasion was to be the last we should have an opportunity of hearing. The sincerity of his character and the charm of his personality endeared him to his friends, who will long retain a pleasant memory both of himself and of his gracious consort, whose death in 1923 he had to deplore. He leaves behind him a son (Pierre) who, after having obtained distinction in the War, is now following in his distinguished father's footsteps, having already been the author of more than one paper of special physiological interest.

Eugène Gley was elected an Honorary Fellow of the Society in 1916.

E. A. S.-S.

William Gordon, B.Sc. (Eng.), Ph.D., M.I.Mech.E.

Dr William Gordon, Lecturer on Strength of Materials in the University of Edinburgh, died unexpectedly on 22nd May 1932, after a short illness.

William Gordon had inherited his engineering tendencies from his father, who was a civil engineer in Messrs Stevenson's office, Edinburgh. His professional education began in Lockie's Academy of Engineering, Leith, before the foundation of Leith Technical College. For a brief period he was a student of engineering in the University of Edinburgh, in Professor Armstrong's time, and served his apprenticeship at Messrs James Milne & Sons (Ltd.), general engineers. He devoted himself to the mechanical side of the profession, and found his chief interest in the strength of materials. He joined the staff of Leith Technical College, and soon became chief lecturer in Mechanical Engineering. He held this appointment for over thirty years, throughout the famous days of Principal Bolam, and produced a high standard of work.

Returning to Edinburgh University fairly late in life, he graduated B.Sc.(Eng.) in 1911, under Professor Sir Thomas Hudson Beare. The years 1911–13 were occupied by research work on the strength of materials, under, and in conjunction with, Dr Gulliver. When Dr Gulliver left Edinburgh University to become head of Kirkaldy's testing laboratories (Southwark, London), Mr Gordon succeeded him as assistant in Engineering, and was appointed Lecturer in Forest Engineering.

Mr Gordon visited Germany in 1914, immediately before the outbreak of war, and on his return, though he was unfit for active service in the field, he applied for and was granted a commission in the Royal Field Artillery (T.F.). During the War, in addition to his University duties, he did valuable work with the University Battery of the Officers' Training Corps.

In 1919, when the Engineering Department was reorganised and extended after the War, he resigned Forest Engineering, and was appointed University Lecturer in Strength of Materials, with charge of the laboratory. This position he held to the end of his life. He had continued his research work, and became Ph.D. in 1925. When the Engineering Department was transferred from High School Yards to King's Buildings in 1931, Dr Gordon was in charge of the arrangements for the new Strength of Materials Laboratory. During his University career he carried out—in addition to

his ordinary duties—a vast amount of testing of the materials of construction for all sorts of purposes.

Dr Gordon published various papers, some of which were in conjunction with Dr Gulliver, and one with Sir Thomas Hudson Beare. These appeared in *Proc. Roy. Soc. Edin.*, the *Brit. Assoc.*, and the *Revue de Métallurgie.* The subjects dealt chiefly with the Strength of Materials. He was specially interested in the behaviour of flat steel bars, and got, with some trouble, an 'ideal' structural steel cast specially for his investigations. Accuracy in measurement appealed to him, and he endeavoured to base his work on mathematical methods.

He was a splendid teacher and full of enthusiasm for his subject. His devotion to the University was whole-hearted, it was the chief interest of his life. Combining as he did the two positions of lecturer in a University and in a Technical College, he was a thorough believer in the importance of each, but held strongly that Universities and Technical Colleges supplied different needs, and that their aims and spheres of action should be kept distinct.

Gordon combined the intense practicality of the engineer with a speculative and philosophic mind. This kept him far from narrow views. He read widely, and in his later years became much interested in architecture and archæology. A versatile talker, his conversation was enhanced by an amazing gift of mimicry. His death is deeply regretted by a wide circle of friends, and by none more than by his colleagues in the Department of Engineering.

He was elected a Fellow of the Society in 1920.

E. M. H.

John Walter Gregory, D.Sc., LL.D., F.R.S.

Dr John Walter Gregory, Professor of Geology in the University of Glasgow, was drowned in the Urubamba River, at the Megantoni Rapids, on 2nd June 1932. About six months previously he had started from Britain with a small expedition which included Miss Mackinnon Wood and Messrs Coverley-Price and Landa, their object being to explore the geology of certain parts of Chili, especially with reference to changes of level, earth movements, and faulting. The other members of the party have returned in safety.

J. W. Gregory was born in London in 1864. He was the son of a wool merchant, and on leaving Stepney Grammar School studied at London University. Thereafter for a time he assisted his father in business, and paid a visit to Russia.

At the age of twenty-three he was appointed Assistant in the Geological Department of the British Museum (Natural History). Much of his time was spent in the study of fossil Bryozoa, and in 1896, 1899, and 1908 three volumes of a Catalogue of Fossil Bryozoa appeared from his pen. In 1900 he published a monograph on the Jurassic Corals of Cutch, and at various periods subsequently he made contributions to our knowledge of these groups and of the fossil Echinodermata.

In 1892 he accompanied an expedition to East Africa as scientific member. Difficulties were experienced and the project was abandoned, but Gregory, with great courage and perseverance, fitted out a small party and completed a journey which yielded notable scientific results. His journey to Mount Kenya involved the passage through the Masai country, then in a very unsettled condition. On his return he published his volume on the Great Rift Valley, which is still the best known and most readable of his travel books. At a much later period he again visited East Africa, and his theories of the structure and history of that province have received much attention from geographers and geologists.

In 1896 he accompanied Sir Martin Conway to Spitzbergen. In 1900 he was appointed Professor of Geology at Melbourne. Two years later he became Director of the Geological Survey of Victoria. During his stay in Australia he led an expedition to Lake Eyre, and devoted much attention to questions of artesian water-supply, which were discussed in a fascinating

manner in his book *The Dead Heart of Australia.* He also visited Tasmania, and published a valuable report on the copper deposits of Mount Lyell.

In 1904 he became Professor of Geology in the University of Glasgow, but he did not cease to travel widely and visited many quarters of the globe to study problems of general and of economic geology. Among these we may mention Cyrenaica, Angola, India, Canada, Rhodesia, the Transvaal, the West Indies, and Chinese Tibet. As he had a fluent pen and a wide knowledge of the varied aspects of his subject his travels furnished him with the materials of a vast number of original papers which are estimated to amount to over two hundred and fifty. Among his most important and famous contributions we may instance his papers on the gold deposits of the Rand, the nickel mines of Canada, and the copper deposits of Germany; but, though a practical geologist of great renown, he was also intensely devoted to the study of the larger questions of geology and geomorphology, and in this he was an apt disciple of Professor Suess, whose theories had much influence on Gregory at an early period in his geological career. His work on the Rift Valleys is an example of this, and we may quote also his support of the "tetrahedral" theory of the earth, his studies of the tectonics of Asia, and his luminous addresses to the Geological Society of London on the history of the Atlantic and Pacific Oceans.

As became a skilled and philosophic geographer he interested himself also in problems of social development, and this aspect of his work is reflected in his books *The Menace of Colour*, *Human Migration and the Future*, and *The Story of the Road*, a study of the development of communications.

Gregory was elected a Fellow of the Royal Society of London in 1901. In 1919 he was awarded the Bigsby Medal of the Geological Society of London and the Victoria Medal of the Royal Geographical Society, in 1922 the Gold Medal of the Scottish Geographical Society and the Gallois Medal of the Société Géographique de Paris, in 1924 the Keith Medal of the Royal Society of Edinburgh. He was Honorary LL.D. of the Universities of Glasgow and Liverpool, President of the Geological Society of London 1928–1930, and presided over Section C (Geology) of the British Association in 1907 and also in the Centenary Meeting in London in 1931. He was also President of Section E (Geography) in 1924.

Few geologists have exhibited a wider range of accomplishments than J. W. Gregory. He had made valuable contributions to every department of his subject. His vast knowledge, acquired not only from very diligent reading but also from extensive travel, made him a very conspicuous figure

among British geologists. Although he did not seem robust he was most energetic, and his courage and perseverance never failed. As a teacher he was both competent and popular, and by his teaching and example he encouraged many young men to follow in his footsteps. His tragic death has removed from the scene one of the most fascinating and inspiring personalities of the British scientific world.

He was elected a Fellow of this Society in 1905.

J. S. F.

Alexander James, M.D., F.R.C.P.Ed.

Dr James died on 7th April 1932, in his eighty-second year, at his home in Haddington, to which he had retired only three years ago. Apart from increasing deafness, he was possessed of a mental ability which rendered him a most interesting companion capable of discussing all sorts of subjects.

After graduating in Medicine at Edinburgh University he studied at Leipzig, and later settled in practice in Edinburgh. He lectured as an extra-mural teacher, first on Physiology and later on Practice of Medicine.

He was associated with the Royal Infirmary, Edinburgh, as Resident Physician, Assistant Physician, Physician in charge of Wards, and finally as Consulting Physician. Dr James was much beloved by his patients, on behalf of whom he spared no pains in ministering to their needs. He was a popular and successful teacher and particularly so when, as Physician, he lectured on Clinical Medicine.

For a number of years he acted as Consulting Physician to the Edinburgh City Hospital, and in this capacity he proved a shrewd and active helper to those on the staff.

Dr James was a remarkable personality in many ways. He held strong views upon many subjects, but, though fond of controversy, he listened sympathetically to the opinions of those with whom he did not agree, and an argument with him was always a real pleasure to both parties. Dr James made several contributions to the literature of phthisis and other subjects, but it was rather as a teacher that he impressed the younger medical men of his day. He wrote many topical songs, and his *Divulgations of a Doctor*, published as recently as 1924, contains a number of articles which give the clearest picture of the Doctor as many of us knew him. A truly lovable man, he will be long remembered for his great social gifts.

Dr James was elected a Fellow of the Society in 1889 and some of his work appears in the Society's *Transactions*.

R. A. F.

Malcolm Laurie, B.A., D.Sc., F.L.S.

My brother Malcolm was born in Edinburgh in 1866, and educated at the Edinburgh Academy. He developed a keen interest in Biology while still a schoolboy, and I remember well a dead snake which he obtained from a travelling menagerie and concealed in a cupboard. The snake gave rise to considerable trouble, until the source of the appalling odour was discovered and the concealed treasure removed.

From the Edinburgh Academy he went to the University of Edinburgh, taking his B.Sc. degree in Biology, and subsequently in 1894 he obtained his D.Sc. degree, the subject of his thesis being "Studies in Arachnid Morphology." He entered as a student at King's College, Cambridge, in 1889, obtaining a second class in the Science Tripos, part 2, in 1892. He also spent some time studying Biology in Tübingen University. He was appointed Professor of Biology in St Mungo's Medical College, Glasgow, in 1894, and resigned on leaving Glasgow for Edinburgh, where he was appointed Lecturer on Biology in the Edinburgh School of Medicine of the Royal College, and held the appointment until he resigned on account of ill-health in 1918. He was also appointed part-time Lecturer in the Church of Scotland Training College and afterwards the Provincial Training College, and lectured in the Heriot-Watt College on Zoology. He held these appointments up to 1918.

He then went to live at Harpenden near Rothamsted, and undertook some research work on Wire-worms at the Experimental Station, but soon had to give it up owing to eye-strain and continued ill-health.

He was a Fellow of the Linnean Society, and of the Royal Society of Edinburgh, and a member of the original committee of the Scottish Marine Biological Station, in which he took the greatest interest.

Like so many Edinburgh students my brother could not resist the lure of Geology, and naturally took up the palæontological side of the subject. Professor Geikie once said to me that if the Almighty had consulted him at the Creation as to the construction of an area for geological research, he could not have suggested any improvements on the district within a twenty-mile radius of Edinburgh.

The rare Eurypterids of the Pentland Hills had for my brother an absorbing fascination, and many laborious days were spent searching for them among the Silurian Shales.

He was, I believe, the first to publish sections of these fossils revealing the internal organs. They were rubbed down on plate glass with fine emery powder, and each layer photographed. The photographs were then thrown on a screen and the details drawn for publication. From these again he built up models, one of which was shown at the British Association Meeting at Glasgow. The exact thickness of each layer removed was controlled by a specially constructed piece of apparatus on the principle of the microtome.

Never very robust in health, he suffered like so many others from the extra strain of war work in addition to carrying on his teaching, and so was compelled to give up his work and live a quiet life at Harpenden. He had the genius for research, and accomplished what he did in spite of poor health and laborious work as a teacher.

He was elected a Fellow of the Society in 1894, and published several papers in its *Transactions*, 1891, 1893, 1899; other papers appearing in *Quart. Journ. Micr. Sci.*, 1889, 1890, 1891; *Zool. Anz.*, 1892; *Journ. Linn. Soc.*, 1894; *Brit. Assoc. Report*, 1893, 1894, 1912; *Ann. Mag. Nat. Hist.*, 1896; *Proc. Roy. Phys. Soc. Edin.*, 1897; and *Nature*, 1911. He was one of the Editors of *Fauna, Flora, and Geology of the Clyde Area*, published by the British Association in 1901, and contributed the article "Actinozoa."

He died on 16th July 1932.

A. P. L.

Graham Lusk, 1866–1932.

Graham Lusk, Professor of Physiology in Cornell University in New York, was born at Bridgeport, Connecticut, in 1866. His father, William Thompson Lusk, played, as a young man, a distinguished part in the American Civil War: his letters from the front, privately printed in 1911, form a volume of singular interest. After the war he took up Medicine, and eventually settled in New York where he became Professor of Obstetrics in Bellevue Hospital Medical College.

Graham was not educated for the medical profession, but devoted his attention mainly to Chemistry, and at the age of twenty-one his father sent him to Munich to work at Physiological Chemistry under Voit, the collaborator and successor of Pettenkofer. Voit inspired in him a devoted attachment and admiration to which he frequently gave expression. His first paper—on diabetes—was founded on work performed in Voit's laboratory; his last paper, published in 1931, was concerned with the same subject. In the meantime he had worked steadily and written much; chiefly on carbohydrate metabolism, but also on general metabolism and nutrition. On these matters he came to be recognised as one of the first authorities, so that in the Great War he was appointed to represent the United States on the Inter-Allied Scientific Food Commission, which was concerned with the distribution of foodstuffs not only to the military forces but also to the civil population of the allied nations. His knowledge of the general subject of nutrition was indeed encyclopædic; his work *Elements of the Science of Nutrition*, which has gone through four editions, is a monument to his grasp of the subject, and is remarkable for the energy with which he defends any position he takes up on controversial subjects, of which the doctrine of "specific dynamic action" of foodstuffs may be cited as a prominent example.

He was an Honorary Member of the Physiological Society, and only this year was elected to the Foreign Membership of the Royal Society. He would doubtless long since have received a similar honour from the Royal Society of Edinburgh had he not been already an Ordinary Fellow for many years (since 1900).

Deafness, from which he suffered during most of his life, was a severe handicap. But he managed largely to overcome the difficulties it

presented, and his audiphone placed on the table before him was a familiar object at the scientific meetings he attended. Gentle and unassuming in character, he had many friends on both sides of the Atlantic, and his memory will be long cherished by them. He was married in 1899 to May Tiffany, who, with two sons and a daughter, survives him. No one could be more happy in his home life, nor will anyone who visited their home on Long Island easily forget the charm of its surroundings and the hospitable welcome of its inhabitants.

E. S.-S.

The Very Rev. Dugald Mackichan, M.A., D.D., LL.D.

WITH great regret the Society records the death on 7th April 1932 of this distinguished missionary, who was elected a Fellow of the Society in 1926.

Born in Glasgow in 1851, Dugald Mackichan, after passing through the High School of that city, proceeded to the University, and there in 1869 graduated M.A. with Honours. As a student he showed remarkable aptitude in Natural Philosophy. In 1873, when holding the position of Thomson (Lord Kelvin) Research Scholar, he communicated to the Royal Society of London a paper which carried out the determination of the ratio of Electrostatic and Electromagnetic units and was published in the *Philosophical Transactions* of that Society, vol. clxiii, pp. 409–427. From various quarters he was encouraged to follow a career in science, and throughout his life he remained an eager student of science in various departments. He had set his heart, however, on being a missionary, and it was in the teaching of Science in the Missionary College in Bombay that his early attainments found an outlet. After studying Theology in the Glasgow Free Church College and also at Leipzig and Berlin, he was ordained as a missionary to India in 1874, and in 1875 was appointed to the staff of the Wilson College, Bombay. There, with remarkable rapidity, he rose to a position of commanding influence. Within three years he was elected a Fellow of Bombay University, and four years later, at the age of thirty-one, he became Principal of the Wilson College. In 1888, at the age of thirty-seven, he was appointed Vice-Chancellor of Bombay University for a period of three years. This office was no sinecure, for it laid upon him the chief responsibility for the shaping of the policy of the University which controlled all the higher education in Western India. To this office, later in his career, he had the unique honour of being thrice recalled for similar periods of service, and to this day the University of Bombay bears the stamp of his statesman-like discernment and rare administrative gifts. In 1902 he was appointed a member of the Indian Universities Commission set up by Lord Curzon; and again and again he was called to serve on similar Commissions. In these and many other ways, for the long period of forty-five years, with untiring devotion, Dr Mackichan rendered invaluable service to the cause of Higher Education in India alike in its scholastic and in its Christian aspects. That cause for the last hundred years has been nobly served by the Missionary Colleges of the Scottish

Church, and the name of Dr Mackichan will take rank alongside the names of Alexander Duff of Calcutta, John Wilson of Bombay, and William Miller of Madras—the great personalities of an earlier generation.

When Dr Mackichan began his work it was still the day of small things. There were but 30 students in the College and about 160 pupils in the School. When he retired in 1920 there were over 800 students in the College and 500 pupils in the High School. Under his able administration handsome new buildings were soon erected on a valuable site, and from time to time additional equipment was provided, all without cost to the Home Church beyond the initial contribution of £6000 collected in Scotland by Dr Mackichan himself during his first furlough.

In the midst of all these varied activities as statesman and educationalist, Dr Mackichan remained, first and last, the Christian Missionary. Possessed of rare linguistic gifts, he made himself familiar both with the ancient Sanscrit and with the vernacular of the people among whom he lived. His knowledge of the Marathi language opened avenues of service to him which he used to splendid purpose in aiding the progress of the Marathi Christian Church. At the request of the British and Foreign Bible Society, he took a leading part as Chairman of Committee in the revision of the earlier Marathi version of the Bible. To the Church at home, also, he gave valuable service as Lecturer on Evangelistic Theology and as Chalmers Lecturer; and in 1917 he was called to the Moderator's Chair of the General Assembly. When he left India in 1920 remarkable public testimony was borne to the high appreciation in which he was held alike by Europeans and by Indians. After his retiral he maintained his keen interest in the missionary enterprise; and to the close of his long life his strong personality, his unflinching courage, his sound judgment, and his deep convictions won for him a high place in the esteem of his brethren and a position of commanding influence in the counsels of the Church.

It is fitting to add here that in 1877 Dr Mackichan married the daughter of the Rev. M. MacRitchie. Mrs Mackichan, who died in 1920, was a true helpmeet to her husband, and in Bombay—the Gateway of India—exercised a gracious hospitality for which her memory is still cherished alike by the missionaries of the Church and by many other visitors to the East. They are survived by two daughters, Mrs Williamson and Mrs Service, with whom, in their bereavement, the members of the Royal Society record their sincere sympathy.

R. F.

Mr James Malloch, M.A.

He was first Executive Officer to the National Committee for the Training of Teachers, and died on 3rd April 1932, about a month before reaching his seventy-second birthday. Mr Malloch was a native of Perth. He graduated Master of Arts at the University of Edinburgh in 1886, distinguishing himself particularly in English Literature and Philosophy. After various teaching appointments in Edinburgh, Arbroath, and Hamilton, he joined the staff of Harris Academy, Dundee. With this city a large part of his life was most closely associated. After a period as Second Master at Harris Academy, Dundee, he was appointed Headmaster of Brown Street School, and later of Blackness School. His remarkable success as a Headmaster led to his appointment as Lecturer on Education to the students in University College, Dundee.

In 1907, when the Provincial Committees for the Training of Teachers were first established, he was clearly marked for higher office, and became Director of Studies to the St Andrews and Dundee Provincial Committee. With the Dundee centre Mr Malloch's name will always be closely associated. He established for it a great reputation, which he was zealous to sustain through many vicissitudes of fortune. The building which stands in Park Place, Dundee, to-day is a permanent memorial to his pertinacity and enterprise. Prior to the erection of this Training College Mr Malloch visited America, and there gained many hints in building construction which were used to great advantage in the Dundee College.

In 1920 the responsibility for the professional training of teachers in Scotland passed into the hands of a National Committee for the Training of Teachers, established by Minute of the Committee of Council on Education in Scotland, and Mr Malloch received the important appointment of Executive Officer to this new Committee, with headquarters in Edinburgh. Here he had a task which demanded all his powers of organisation and required tact and resourcefulness in the handling of a situation that contained vexatious elements. His supreme good sense enabled him, along with the late Professor Darroch as his Chairman, to solve on rational lines many difficult problems of an *imperium in imperio.*

He retired from this position in 1925 and went back to Dundee, where his practical interest in education continued to be manifested by his membership of the Dundee Education Authority.

His death removes one who had the supreme gift of foresight. He could anticipate future developments and was never content merely to mark time. With his students he was at all times popular. He had a rare gift of friendship, and in every educational quarter in Scotland he will long be remembered as a man who brought to his daily task a high sense of purpose, great cheerfulness and force of character, to which were allied conspicuous abilities in administration.

He was elected a Fellow of the Society in 1925.

J. R. P.

Emeritus Professor David Henry Marshall, M.A.

THE passing of this lovable personality awakens a sense of loss among admirers at home and abroad. He had spent a quarter of a century in retirement at his beautiful residence by the shore of Lake Ontario, where he died on Sunday, 13th March 1932.

Son of John Marshall, a Jury Court Macer, David Henry was born at No. 31 Howe Street, Edinburgh, 9th April 1848. Taught for two years by Mr Robert B. Crowe, of the Rev. Dr Andrew Thomson School at No. 10 Queensferry Street, he was nominated for admission to Donaldson's Hospital by a distinguished trio, the Hon. Lord Curriehill, the Rev. Dr Candlish, and Sir William Gibson-Craig, Bart., of Riccarton. A contemporary at Donaldson's, his senior by four years, was Sir Alexander Oliver Riddell, LL.D., proprietor of Craiglockhart. Marshall proceeded later to the University of his own romantic town, where he secured medals in Mathematics and Natural Philosophy, obtaining at twenty-one the degree of M.A. with First Class Honours in Mathematics.

Having gained the Bruce of Grangehill Mathematical Scholarship as well as the Drummond, he became class-assistant to Professor Peter Guthrie Tait, Senior Wrangler, one of the eminent physicists of the nineteenth century. On Tait's recommendation, he was in 1873 appointed Professor of Mathematics in the Imperial College of Engineering, the College having been founded that very year by the Government of Japan. He landed in the Far East on 6th June, accompanied by Dr Henry Dyer, Whitworth Scholar and first Principal of the Institution.

Completing an engagement of five years, Professor Marshall remained in Tokyo, succeeding William E. Ayrton on his retiring from the Chair of Natural Philosophy and Telegraphy in the Imperial College. Ayrton's wife, it may be remembered, was the only woman member of the Institution of Electrical Engineers; she it was who carried out electric-arc experiments and water-motion researches.

On his return to Scotland in 1881, D. H. Marshall assisted his old chief (Tait) for one winter, thereafter obtaining the Professorship of Physics in Queen's University, Canada. He reached Kingston in the autumn of 1882 his companion on the voyage being the Very Rev. George M. Grant, D.D., five years later Principal of Queen's. Marshall resigned the Chair in 1907.

His recreations are described as "travelling and star-gazing." He

enjoyed life to the full; he gloried in his walking powers. He saw the most attractive parts of Japan, speaking the language fluently. Twice he climbed Fujiyama (12,365 feet). Ten days of a Christmas vacation he spent encircling the base of Fuji; the experience has been written in *The Land of the Morning* by his fellow-traveller, Dr William G. Dixon, Professor of English Literature. Twice also he ascended Asamayama, an active volcano of 8100 feet.

Fond of travel, he sailed round the world no fewer than four times, on one occasion acquiring at Torres Strait a specimen of the Great Clam (*Tridacna gigas*), the valves weighing 582 lb. in all.

Visiting Samoa in September 1894, he called at Vailima. In receiving the former tutor, R. L. S. spoke a few words of welcome, drinking from a wooden bowl, thereafter handing it to the guest. The bowl contained *Kava* (or *Kawa*), the beverage which the South Sea Islanders extract from the root of a species of pepper. The sederunt of two hours was interrupted by Mrs Stevenson, who, judging from the hearty laughter, said that her husband had no doubt enjoyed the *tête-à-tête*, although he seemed to neglect his doctor's orders. The professor was saddened to find the author of *Treasure Island* a wreck whose end was approaching. Stevenson, it will be recalled, died suddenly three months later, namely, on 3rd December 1894.

From boyhood D. H. Marshall loved astronomy, living to be one of the very few scientists who had observed two transits of the planet Venus.

His intimates mourn a friend warm-hearted and intensely human.

He was elected a Fellow in 1882.

R. T. S.

Alexander Philip, M.A., LL.B. (Edin.).

ALEXANDER PHILIP was the son of the Reverend Alexander Philip, Minister of Cruden, who went out at the Disruption. He practised as a solicitor at Brechin. He was a man of active mind and many varied interests. Many of his holidays were spent abroad, and where it was practicable he preferred to travel on foot. He was the author of several books, among which are his numerous writings on the Calendar referred to below, and his contributions to epistemology. Of the last the best known is his *Essay towards a Theory of Knowledge.*

Among Philip's many interests the chief was the reform of the Calendar. From the point of view of modern life the calendar is inconvenient in many respects. The months are unequal, no respect has been shown to possible simplifications, "movable feasts" govern a large and important part of the year. Still all of these anomalous features have their defenders, or at least are so embedded in antiquity that they are difficult to remove. A calendar reformer must be proof against wet blankets and active opposition. Another obstacle is the zeal of those who are oblivious to the claims of the past. Between these two dangers Philip steered a course, with insistence and imperturbable good humour, which won respect from those who could not feel enthusiastic. First and last he advocated many schemes. Even in his last pamphlet, *The Essentials of Calendar Reform* (Routledge, 1930), he offers two, one of which he had sponsored in his book *The Calendar: Its History, Structure, and Improvement*, and the other in *The Improvement of the Gregorian Calendar.* But in spite of these manifest difficulties he worked indefatigably at the subject, studying its history, corresponding, interviewing individuals and Chambers of Commerce, and attending international gatherings.

He was born at Portobello in 1858; he was elected a Fellow of the Society in 1913, and died on 21st January 1932.

R. A. S.

Col. Sir Bruce Gordon Seton.

By the death of Sir Bruce Gordon Seton, Ninth Baronet of Abercorn, on 3rd July 1932, the Society loses a Fellow who throughout a life of activities of very diverse nature was impelled by a single motive—to study the individual, and, with the knowledge so gained, to help him. He was born in 1868, and in 1892 passed first into the Indian Medical Service. After active service in Waziristan and in the Tochi Valley he was selected for administrative work, and eventually was appointed Deputy Director-General of the Service. As officer in charge of personnel, he was in a position which gave scope to the qualities which he possessed in so high a degree. On the outbreak of the Great War he was placed in charge of the Kitchener Hospital at Brighton for soldiers of the Indian Army. By the aid of an elaborate system of card indexing, which later was widely adopted, he was enabled to identify the individual and to study his needs. He retired from the Service in 1917 with the distinctions of Honorary Physician to the Viceroy and Companionship of the Bath, and found a new sphere of work in the engineering firm of Stewarts & Lloyds. As assistant secretary he was called upon to deal with a large staff of employees, and shortly afterwards he was elected secretary of the Scottish Tubemakers' Wages Association. The words of the Vice-Chairman of that Society summarise a story fraught with humour and pathos. "He came into close contact with the men's leaders and delegates when delicate and difficult matters relating to rates of wages and conditions of work had to be negotiated. By his tactful and courageous handling of these matters he secured the confidence and respect, not only of the employers, but also of the men. To his skilful handling of these questions we can largely attribute the fact that during the period of his office the many difficult problems which arose were in the end settled to the satisfaction of both sides, and in no case did any strike or stoppage occur."

It was almost natural that the study which attracted him was that of biographical history. For the Scottish History Society he prepared *The Prisoners of the '45*, in three volumes, a work of permanent and very great value. He completed and edited the late General Mahon's *Tragedy of Kirk o' Field*. *The Pipes of War*, a large volume, relates largely to the part played by pipers in the Great War. He edited Cordara's *Commentary on the Expedition of Prince Charles Edward Stuart*, and also contributed

numerous articles to the *Scottish Historical Review*, including one on the orderly book of Lord Ogilvie's Regiment, and another on "Wemyss of Bogie." Shortly before his death he completed a history of the Seton family from the twelfth century onwards, which he whimsically entitled *Lost Causes*. This remains unpublished. He was elected a Fellow of the Society in 1926 on account of his historical researches. The charm of his personality attracted men of all classes, and to each he gave his whole-hearted sympathy. When he gave his friendship, he gave it with both hands.

A. G. M.

Sir William Robert Smith, M.D., D.Sc., F.R.C.S.E.

Sir William Robert Smith was born at Plumstead, Kent, on 28th May 1850. His parents were in very poor circumstances, and though they did their best to give him an education which would serve to improve his lot in life, it was found necessary for him to start work at the early age of thirteen.

He obtained his first post in the Chemistry Department of the Royal Arsenal, Woolwich, and during his spare time continued his studies at the Royal Arsenal Science School. As a result of his earnest application to his work he won, at the age of nineteen, the Gold Medal for Animal Physiology awarded by the Science and Art Department of the Government. By dint of hard work and strict economy, he was able to save enough to take up the study of medicine when he was seventeen years of age. He entered St Bartholomew's Hospital, and two years later passed the Final Examinations.

His first post after obtaining his Medical Qualification was that of House Surgeon at the Brompton Hospital, but he soon realised that, in order to make the progress he desired, he would have to obtain a University degree. As a result of further careful saving, he was ultimately able to take the University course which he had contemplated, and in 1876 he graduated M.B. and C.M. in Aberdeen University.

He then decided to devote himself to the study of Preventive Medicine, and visited the Hygienic Institutes of Paris, Berlin, Vienna, Moscow, and Munich, etc., where he obtained a thorough grasp of the methods in use on the Continent. Returning to this country, he took up special work in Hygiene in University College, London, and later at Edinburgh.

In 1881 he obtained the Diploma in Public Health of the University of Cambridge. Two years later he graduated B.Sc. in the University of Edinburgh, and in 1886 he received the degree of D.Sc. from the same University.

From this date his real active work in the cause of Public Health began, and realising that an intimate knowledge of the relationship between the Law and State Medicine would be of great help in his work, he took the necessary course of legal studies at the Middle Temple, and was called to the Bar in 1888.

In 1889 he was appointed to the Chair of Forensic Medicine and

Toxicology in King's College, University of London, a position which he occupied for some years.

In 1886 he founded the College of State Medicine, in which institution he was appointed Professor of Hygiene and Public Health. After some years, during which important work was carried out, the College became the British Institute of Preventive Medicine, and later the Lister Institute of Preventive Medicine.

In 1889 Sir William was appointed the first Medical Officer of Health to the town of Woolwich, where his genius and untiring energy obtained full scope. Thereafter he received an appointment as Public Analyst to the Boroughs of Woolwich and Windsor, and later he took up the position of Medical Officer of Health to St Albans.

After a hard fight, he was instrumental in establishing, in 1890, the post of Medical Officer of Health to the School Board for London, a post which he himself was the first to hold.

Sir William, in 1874, obtained a Commission in the Army, in which he served till 1913. He returned to the Army on the outbreak of War in 1914, as Specialist Sanitary Officer of the 67th Division of the Southern Army, and on his retiral in December 1917 he was specially thanked by the Secretary of State for War for his valuable services, and had the honour of Knighthood conferred on him in 1919.

Probably Sir William Smith's greatest and happiest achievement was the founding of the Public Health Medical Society, now known as the Royal Institute of Public Health. Commencing in 1886 with a small group of medical men, all possessing qualifications in Public Health, the Society steadily advanced until, in 1897, the Institute came under the patronage of Her Majesty Queen Victoria, and was thereafter known as the Royal Institute of Public Health.

Sir William's work in the cause of Public Health was known all over the world, and his great services to Medicine were recognised by the conferment of honours by almost every country in Europe.

He was elected a Fellow of the Society in 1880, and died on 17th March 1932.

S. S.

Sir William Somerville.

SIR WILLIAM SOMERVILLE, Professor Emeritus of Rural Economy in the University of Oxford, died on 17th February at the age of seventy-one. For the last six or seven years of his life he was in feeble health, and latterly was living in almost complete retirement; yet he will long be remembered as a man of quite exceptional vigour of mind and as one who made outstanding contributions to agricultural progress.

Somerville was a pupil of the Royal High School, Edinburgh, and studied agriculture under both Wilson and Wallace at Edinburgh University. He was Class Medallist in Agriculture in Wallace's first year as Professor (1885). Thereafter he worked at Munich, mainly on Forestry and Forest Botany, and graduated D.Œc. In 1889 he returned to Edinburgh as the first University Lecturer on Forestry, and two years later was appointed first Professor of Agriculture and Forestry at Armstrong College, Newcastle-on-Tyne. In 1899 he moved on to Cambridge as the first Drapers Professor of Agriculture, and two years later he went to London as Assistant Secretary to the Board of Agriculture. Official routine and confinement in a Government office, however, proved not much to his liking, so that when, in 1906, the old Sibthorpian Professorship at Oxford was made into a full-time appointment, he was glad to accept the invitation of the University to fill the Chair, which he held until his retirement in 1926.

Somerville was a man of extraordinary breadth of interest and knowledge. Nobody else in recent times has been an acknowledged authority, as Somerville was, on both agriculture and forestry. It would be difficult to say whether the woodland or the field of grass interested him the more, so completely conversant was he with both subjects. Moreover, he combined, in a very remarkable degree, a real scientific outlook with the most minute knowledge of practice. The result was that he won the complete confidence and trust both of the farmer and of the practical forester. No other scientific man—not even Sir John Lawes—did so much to inspire the farmer's confidence in education and research.

Somerville's best known work was done during his eight years at Newcastle. While there he started the Northumberland County Council's experimental farm at Cockle Park, and initiated a vast amount of field experimental work not only on the farm itself but on others throughout

the north of England. The name of Cockle Park he made famous in a very few years, and the work which he began was so ably carried on by his successors (Sir Thomas Middleton and Professor Gilchrist) that it had far-reaching consequences in farm practice throughout the whole country. The most striking results were obtained in increasing the productivity of pastures, but much other valuable work is recorded in Reports which he wrote at this time and which run to some 800 pages.

Somerville had an immense capacity for work. Apart from his Reports, he wrote a good deal on both his subjects; he translated several German works on Forest Botany; lectured very often to farmers, and was a most popular and successful teacher. He was an unfailing source of encouragement and inspiration not only to his own pupils but to all and sundry young men who were working at either of his subjects.

To know Somerville and to see him at his best one had to spend a day with him in the open air. Whether he were shooting or fishing, botanising or farming, hill-climbing or bird-watching, his interest never flagged and his body never seemed to tire. Most remarkable of all was his never-failing kindness and geniality, which must have cheered on many a companion who was ready to drop with weariness before the end of the day. Not even in his last years (when he was condemned to comparative idleness and was able to see very few friends) did his cheery optimism desert him or interest in life fail.

He was elected a Fellow of the Society in 1889.

J. A. S. W.

James Stuart Thomson, M.Sc. (Manchester), Ph.D. (Berne).

JAMES STUART THOMSON was for many years Lecturer in the Zoology Department, Victoria University, Manchester, under Professor S. J. Hickson, D.Sc., LL.D., F.R.S. He was born at Pilmuir, East Lothian, on 21st July 1868, youngest son of the Rev. Arthur Thomson, Free Church minister there. He got his scientific education in the Universities of Edinburgh, Freiburg, and later Berne, and at various marine biological stations. After holding various teaching posts, *e.g.* at Plymouth and in Edinburgh (as assistant to his brother, J. Arthur Thomson, then lecturing on Zoology in the extramural School of Medicine), he became in 1903 Assistant Government Biologist at the Cape of Good Hope. He had also worked for some time in the *Challenger* Office in Edinburgh and at Granton Marine Station. After some experience as Lecturer in the South African College and in the University of Bristol, he was appointed in 1910 Senior Lecturer on Zoology in the University of Manchester. He held this post till 1929, when he had to retire for health reasons; and after this he lived quietly and happily at Cirencester. He died suddenly on the 28th August 1932. Dr Thomson published numerous zoological papers, most of them on Alcyonarian Corals, to the investigation of which he had been prompted by Professor Studer and encouraged by Professor Hickson. Notable among his researches on other lines was his pioneer work on the markings on the scales of fishes as indication of periodic growth, and thus of age. Latterly he worked out a detailed account of the structure of the tortoise, which is being published by the Royal Dublin Society.

He was elected a Fellow of the Society in 1906.

J. A. T.

Francis John Allan, M.D., C.M. (Edin.), D.P.H. (Camb.), was a Scotsman by birth. Graduating at the University of Edinburgh, he became an authority on questions of Public Health, his first appointment as Medical Officer of Health being at Harrow-on-the-Hill. He served on the Councils of the Society of Medical Officers of Health and the Royal Sanitary Institute, and was President of the former in 1915–16. In 1900 the City of Westminster was formed, and Dr Allan was appointed Medical Officer of Health. Here he did great service in respect to tuberculosis, the provision of sanatorium treatment, and in maternity and child welfare. He took a great interest in Freemasonry, and held at various times important masonic offices.

He was the author of *Aids to Sanitary Science*, 2nd Ed., 1903 (with C. E. Allan, LL.B.); *Housing of Working Classes Acts*, 4th Ed., 1916; and *Practical Guide to Disinfection*, 1903.

Dr Allan was elected a Fellow of the Society in 1894, and died on 28th July 1932.

Walter Leonard Bell, M.D. (Edin.), F.S.A.Scot., late Surgeon, Lowestoft Hospital; Assistant Physician, District Asylum, Larbert; and House Surgeon, Cancer Hospital, Brompton, was elected a Fellow of the Society in 1915, and died on 24th July 1932.

Major-General Sir David Bruce, K.C.B., M.B., C.M. (Edin.), LL.D., F.R.S., Chairman, Governing Body of the Lister Institute, was born in Melbourne on 29th May 1855, and educated at Stirling High School and the University of Edinburgh. He joined the Army Medical Service in 1883. He was a pioneer in the study of Tropical Medicine, acting as Director of the Royal Society Commissions: Uganda, 1903; Malta, 1904–1906; Uganda, 1908–10; and Nyasaland, 1911–14. Bruce was President of the Toronto Meeting of the British Association in 1924.

He was elected an Honorary Fellow of the Society in 1927, and died on 27th November 1931. (For details of his life and work see Notices in *Obituary Notices of Fellows of Roy. Soc. Lond.*, No. I, 1932, pp. 79–85; *Nature*, vol. 129, 1932, pp. 84–88; *Times*, 28th November 1931, etc.)

James Roberton Christie, O.B.E., K.C., M.A. (Glas.), LL.B. (Glas. and Edin.), Advocate at the Scottish Bar, and Clerk of Justiciary in

Scotland from 1918, was born in 1866. He was educated at Albany Academy and Hutchisons' Grammar School, Glasgow, and Glasgow and Edinburgh Universities. He was a member of the University Court, Edinburgh. Mr Christie was one of the Counsel for the Free Church of Scotland in its successful litigation with the United Free Church of Scotland. His publications include *Parochial Ecclesiastical Law of Scotland* (with Wm. George Black, LL.D.), and contributions to the *Encyclopædia of Scots Law.*

He was elected a Fellow of the Society in March 1932, and died on 21st August of that year.

THOMAS WILLIAM DEWAR, M.D. (Edin.), F.R.C.P.E., Member of the Royal Institution of Great Britain, held the following appointments: Physician, Hospital for Consumption, Dunblane; late District Medical Officer, Provident Department, Metropolitan Hospital, London; Physician, Royal Public Dispensary, Edinburgh; and Lecturer in Anatomy, Extra-mural School of Medicine, Edinburgh. He was the author of papers in the *Brit. Med. Journal*, 1905; *Trans. Edin. Med.-Chir. Soc.*, 1905, 1909; *Trans. Int. Cong. Tuberculosis*, Paris, 1905; and *Glas. Med. Journal*, 1911.

He was elected a Fellow of the Society in 1906, and died on 24th November 1931.

ANDREW FREELAND FERGUS, M.D., LL.D. (Glas.), F.R.F.P.S.G., was born in Glasgow in 1858, and educated in his native city. After graduating he continued his studies at Paris and Utrecht, and returned to Glasgow to build up an extensive practice as an ophthalmic surgeon. He held the following appointments: Hon. Surgeon, Glasgow Eye Infirmary; Lecturer in Physics in Anderson's Medical College, Glasgow (in 1909 he returned to the College as Lecturer on Ophthalmic Medicine and Surgery); Ophthalmic Surgeon for the Corporation of Glasgow. In 1915 he was appointed Major in the 4th Scottish General Hospital, and was specially mentioned for his services. From 1918 to 1921 he was President of the Faculty of Physicians and Surgeons of Glasgow, and was also a Past President of the Greenock Faculty of Medicine, the Royal Chirurgical Society, and the Royal Philosophical Society of Glasgow. Among the subjects dealt with in his numerous publications were miners' nystagmus and seamen's eyesight.

Dr Fergus was elected a Fellow of the Society in 1899, and died on 24th October 1932.

KARL RITTER VON GOEBEL (born 1855), Professor of Botany in Munich University, is best known for his researches in Cryptogamic Botany. He was the author of an important work on this subject, which was translated into English and edited by the late Sir Isaac Bayley-Balfour under the title *Organography of Plants, especially of the Archegoniatæ and Spermaphyta.* It was published in two volumes, the first in 1900, the second in 1905.

He was elected a Foreign Honorary Fellow of the Society in 1910, and died on 10th October 1932. (For details of his life and work see *Obituary Notices of Fellows of the Roy. Soc. Lond.*, 1933 (not yet published); *Nature*, 29th October 1932, pp. 653–654.)

WILLIAM CARMICHAEL M'INTOSH, M.D., D.Sc., LL.D., F.R.S., Professor of Natural History in the University of St Andrews from 1882 to 1917, was born in St Andrews on 10th October 1838. He spent his school-days at Madras College, and studied at the University of St Andrews from 1853 to 1857. He then proceeded to Edinburgh, where he received the degree of M.D. in 1860. Professor M'Intosh published a large number of faunistic papers. The groups to which his attention was specially devoted were the nemertine and polychæte worms. A series of papers on these led up to his great *Monograph of the British Marine Annelids*, published in four volumes by the Ray Society, of which the first two parts appeared in 1873 and 1874, and the final part in 1923. Another important work on the Polychæta was the report on the collection obtained by the *Challenger* Expedition, which appeared in two large quarto volumes in 1885. His publications on British marine fishes and on *The Resources of the Sea* resulted from another series of his investigations.

He received many academic and other honours.

Professor M'Intosh was elected a Fellow of this Society in 1869, having been awarded its Neill Medal in 1868. From 1885 to 1888 he served as a Councillor, and from 1927 to 1930 as a Vice-President. He contributed a number of papers to the Society's *Transactions* and *Proceedings*. He died on 1st April 1931 in his ninety-third year. (For details of his life and work, see *Proc. Roy. Soc. Lond.*, vol. 110, B, 1932, pp. xxiv–xxviii; and *Nature*, vol. 127, 1931, pp. 673–674.)

ALEXANDER DISNEY LEITH NAPIER, M.D. (Aberdeen and Adelaide), C.M., M.R.C.P., was educated at the Universities of Aberdeen and London. He was formerly Senior Gynæcologist, General Hospital, Adelaide, and Physician to the Royal Maternity Charity of London, and author of *The Menopause and its Disorders*, 1897; *The Thermometer in Obstetrics and Gynæcology*, 1889.

He was elected a Fellow of the Society in 1888, and died on 13th June 1926.

Sir Ronald Ross, K.C.B., K.C.M.G., M.D., F.R.C.S., Hon. D.Sc., LL.D., F.R.S. (Colonel, I.M.S., retired); Director-in-Chief, Ross Institute and Hospital for Tropical Diseases; Nobel Laureate, Physiology and Medicine, 1902, who discovered the exact relation between human malaria and mosquitoes, was born on 13th May 1857, at Almora, India. After education at a private school he studied medicine at St Bartholomew's Hospital, and joined the Indian Medical Service in 1881.

He was elected an Honorary Fellow of the Society in 1921, and died on 16th September 1932. (For details of his life and work see *Obituary Notices of Fellows of the Roy. Soc. Lond.*, 1933 (not yet published); *Nature*, vol. 130, 1932, pp. 465–467; *Times*, 17th September 1932; and *Science Progress*, No. 107, January 1933.)

John Bolton Thackwell, M.B., C.M. (Edin.), D.P.H. (Camb.), late Surgeon, Cloncurry and Ravenswood District Hospitals, Queensland, received his medical education at the University and School of Medicine, Edinburgh, and at University College, London.

He was elected a Fellow of the Society in 1892, and died on 9th March 1932.

APPENDIX.

CONTENTS.

PROCEEDINGS OF THE STATUTORY GENERAL MEETING

Beginning the 149th Session, 1931-1932.

At the Statutory General Meeting of the Royal Society of Edinburgh, held in the Society's Rooms, 24 George Street, on Monday, October 26, 1931, at 4.30 P.M.,

Sir E. A. Sharpey-Schafer, F.R.S., President, in the Chair,

the Minutes of the Statutory Meeting held on October 27, 1930, were read, approved, and signed.

Dr J. D. SUTHERLAND, Dr ALEX. NELSON, and Mr R. A. ROBB signed the Roll, and were formally admitted Fellows.

The GENERAL SECRETARY submitted the following Report:—

SECRETARY'S REPORT, OCTOBER 26, 1931.

By request of the Council addresses were delivered on "Recent Work in the Bio-chemistry of Muscle," by PHILIP EGGLETON, M.Sc., on November 3, 1930; "Recent Advances in Our Knowledge of the Higher Fungi," by Professor A. H. R. BULLER, F.R.S., on June 15, 1931; and on "James Clerk Maxwell (1831–1879) and Mechanical Descriptions of the Universe," by Professor E. T. WHITTAKER, F.R.S., on July 6, 1931, being the BRUCE-PRELLER lecture. 34 papers were read, as compared with 35 in the previous session. The papers were divided among subjects as follows: Mathematics, 8; physics, 5; geology and mineralogy, 3; palæontology, 5; botany, 3; zoology, 6; animal genetics, 3; and physiology, 1. 8 papers have been published or are being published in the *Transactions*, and 21 in *Proceedings*. 4 papers were read, but have not yet been submitted in final form, and 3 were declined for publication.

The Society has lost by death 22 Ordinary Fellows and 2 Foreign Honorary Fellows, and 3 Ordinary Fellows have resigned. 33 Ordinary Fellows were elected.

Invitations were received, and the Society was represented as follows on the occasions mentioned:—

1. Memorial Service in St Giles, loss of Airship R 101, October 10, 1930. Sir E. A. SHARPEY-SCHAFER, President; Dr A. CRICHTON MITCHELL, and Mr A. H. R. GOLDIE.
2. Collège de France. 4th Centenary. Paris, June 18–20, 1931. Sir E. A. SHARPEY-SCHAFER, President.
3. Royal Dublin Society. Bi-Centenary. June 23–26, 1931. Sir E. A. SHARPEY-SCHAFER, President.
4. British Association. Centenary Meeting, London, September 23–30, 1931. Professor R. A. SAMPSON, General Secretary.
5. Faraday Centenary Celebrations, London, September 21–25, 1931. Professor R. A. SAMPSON, General Secretary.
6. British Museum (Natural History). 50th Anniversary. End of September 1931. Professor J. H. ASHWORTH.
7. Clerk Maxwell Centenary Celebrations, Cambridge, October 1 and 2, 1931. Professor C. G. DARWIN and Professor R. A. SAMPSON, General Secretary.
8. Memorial Service of Dr JAMES CURRIE, November 7, 1930. Professor T. J. JEHU and the GENERAL SECRETARY.
9. Memorial Service of Sir F. G. OGILVIE. Principal J. C. SMAIL.

Dr A. CRICHTON MITCHELL was re-elected to represent the Society on the Governing Body of Heriot-Watt College.

The undernoted Gentlemen were nominated as representatives of the Society on National Committees:—Astronomy, Sir A. S. EDDINGTON; Geography, Professor A. G. OGILVIE; Geodesy and Geophysics, Dr A. CRICHTON MITCHELL; Mathematics, Professor E. T. WHITTAKER; Physics, Professor E. TAYLOR JONES; Radio-telegraphy, Professor F. G. BAILY; Biology, Professor J. GRAHAM KERR.

Professor F. G. BAILY and the GENERAL SECRETARY, representing the Council, attended meetings convened by the Royal Scottish Society of Arts, to discuss a project for a group of buildings dedicated to the furtherance of Edinburgh's intellectual interests. The chief part of the project is the housing, under one roof, of the principal scientific societies of the City.

The Rooms of the Society were opened to the delegates attending the Congress of the

Universities of the Empire, held in July 1931, so that they could read the reviews, consult journals, write letters, etc.

The Feu Charter conveying to the Society the ground on which the Peach and Horne Memorial stands at Inchnadamph has been received from Major-General STEWART of Assynt and has been recorded.

Attention is called to the List of Periodicals, received by exchange and purchase, published in the *Proceedings*, vol. 50, part 4.

The issue of *Proceedings* has been increased from 1460 to 1500 owing to the increase in membership, etc.

The Hume MSS. in the possession of the Society have been collated and calendared by Dr J. Y. T. GREIG, Armstrong College, Newcastle-on-Tyne. It is hoped to print the Calendar in the *Proceedings*. The collection of 13 volumes, which includes—

7 volumes of correspondence,
3 volumes, miscellaneous,
2 volumes, History of England,
1 volume, Dialogues on Natural Religion,

have been bound by Messrs HENDERSON & BISSET, Edinburgh. The thanks of the Society are due to Dr J. Y. T GREIG for his valuable services in rearranging the letters and in preparing the Calendar.

A statuette of DAVID HUME has been presented to the Society by Dr PITTENDRIGH MACGILLIVRAY, R.S.A. It has been cast in bronze, and will be set up in the Rooms. The statuette is the original model of the figure of HUME in stone on the North-west corner of the National Portrait Gallery in Queen Street.

During the session, at the request of the Air Ministry, the Society has continued its collaboration with the Royal Society of London in regard to the arrangements of the Second Polar Year (1932–1933). A proposal was considered to have observations made at Fort Rae in Canada, and to conduct a high-level observing station on Ben Nevis. The Air Ministry intends to allot £10,000 as a Grant in Aid to the British part of the programme. The Ben Nevis station was found to be impracticable at short notice, but it is intended to proceed with the other portions.

The MAKDOUGALL-BRISBANE PRIZE for the period 1928 to 1930 was awarded to Dr NELLIE B. EALES of the University of Reading.

It was agreed to devote the BRUCE-PRELLER LECTURE for 1931 to a commemoration of the Centenary of the birth of JAMES CLERK MAXWELL, and Professor Sir HORACE LAMB, F.R.S., was invited to deliver the lecture, but through illness was unable to do so. Professor E. T. WHITTAKER, F.R.S., at the briefest notice, gave an address on July 6, on "James Clerk Maxwell (1831–1879) and Mechanical Descriptions of the Universe," as recorded above, and was appointed BRUCE-PRELLER lecturer in the place of Sir HORACE LAMB.

The acknowledgment of the Society is due to the Carnegie Trust for the Universities of Scotland for grants to authors towards the cost of illustrations of papers published by the Society, amounting to £126, 12s. 9d., and to the Royal Society of London for a sum of £250 from the Government Publication Grant, in aid of the cost of printing the Society's *Transactions* and *Proceedings* during the session 1930–1931.

A photographic enlargement of a portrait of Lord Kelvin has been secured by the Society. It has been hung in the Back Saloon (Street Floor).

During the Session a supply of hot water has been provided in the Lavatory (Street Floor).

During the year, painting, etc. has been renewed in the Back Saloon, the Council Room, Secretary's Room, and the upper flat.

TREASURER'S REPORT :—

Dr JAMES WATT, Treasurer, in submitting the Annual Financial Statement, entered into a comparison of the detailed figures of the past two years.

The PRESIDENT in the course of his remarks gave particulars of the Fellows, Honorary and Ordinary, who died during the session, and made reference to his visits, as the delegate of the Society, to the Fourth Centenary Celebration of the Collège de France, held in Paris from June 18–20, 1931, and to the Bi-Centenary Celebration of the Royal Dublin Society, held in Dublin from June 23–26, 1931.

The PRESIDENT nominated as Scrutineers of the Ballot, Mr J. A. INGLIS and Principal J. C. SMAIL.

The Ballot for the Election of Council and Office-Bearers was then taken.

Mr J. GALL INGLIS moved the adoption of the Reports and the reappointment of Messrs LINDSAY, JAMIESON, and HALDANE, C.A., as auditors for the ensuing Session. These motions were seconded and approved.

The Scrutineers reported that the Ballot Papers were in order, and the PRESIDENT declared that the following Office-Bearers and Members of Council had been duly elected:—

Professor Sir E. A. SHARPEY-SCHAFER, M.D., D.Sc., LL.D., F.R.S., President.
Professor F. G. BAILY, M.A., M.Inst.E.E.
Professor T. J. JEHU, M.A., M.D., F.G.S.
Professor J. H. ASHWORTH, D.Sc., F.R.S.
ARTHUR LOGAN TURNER, M.D., LL.D., F.R.C.S.E.
J. B. CLARK, M.A., LL.D., J.P.
Professor JAMES RITCHIE, M.A., D.Sc.
} Vice-Presidents.
Professor R. A. SAMPSON, M.A., D.Sc., LL.D., F.R.S., General Secretary.
Professor C. G. DARWIN, M.A., F.R.S.
Professor F. A. E. CREW, M.D., D.Sc., Ph.D.
} Secretaries to Ordinary Meetings.
JAMES WATT, W.S., F.F.A., LL.D., Treasurer.
Professor D'ARCY W. THOMPSON, C.B., D.Sc., D.Litt., F.R.S., Curator of Library and Museum.

ORDINARY MEMBERS OF COUNCIL.

Professor JAMES DREVER, M.A., B.Sc., D.Phil.
A. H. R. GOLDIE, M.A., B.A.
ROBERT ALEX. HOUSTOUN, M.A., Ph.D., D.Sc.
The Hon. LORD SANDS, Kt., K.C., LL.D., D.D.
MURRAY MACGREGOR, M.A., D.Sc.
A. CRICHTON MITCHELL, D.Sc.
Professor P. T. HERRING, M.D., F.R.C.P.E.
Principal Sir THOMAS H. HOLLAND, K.C.S.I., K.C.I.E., Hon. D.Sc., LL.D., F.R.S.
Professor JAMES KENDALL, M.A., D.Sc., F.R.S.
Professor THOMAS M. MACROBERT, M.A., D.Sc.
Professor GODFREY H. THOMSON, D.Sc., Ph.D.
MALCOLM WILSON, D.Sc., A.R.C.Sc., F.L.S.

The PRESIDENT, before closing the Meeting, thanked the Scrutineers for their services.

PROCEEDINGS OF THE ORDINARY MEETINGS, Session 1931–1932.

FIRST ORDINARY MEETING.

Monday, November 2, 1931.

Professor J. H. Ashworth, D.Sc., F.R.S., Vice-President, in the Chair.

The Minutes of the previous Meeting were taken as read.

Dr E. L. Ince signed the Roll of Fellows.

The following Communications were submitted: —

1. The Petrology of Iceland. By G. W. Tyrrell, A.R.C.Sc., D.Sc., and Martin A. Peacock, B.Sc., Ph.D. Part II: The Petrology of the Acidic and Intermediate Intrusives and Extrusives. By Martin A. Peacock. Communicated by G. W. Tyrrell.
2. The Primitive Conducting Mechanisms of the Vertebrate Heart: An Introduction to the Study of their Appearance and Development in *Lepidosiren paradoxa*. By Tudor Jones, M.B., Ch.B. *Trans.*, vol. 57, pp. 225–240.
3. An X-ray Examination of *d*-Mannitol and *d*-Mannose. By George W. McCrea, B.Sc., A.I.C., Ph.D. Communicated by Professor James Kendall, M.A., D.Sc., F.R.S. *Proc.*, vol. 51, pp. 190–197.
4. The Absorption Spectra of Cyanogen and the Cyanogen Halides. By R. B. Mooney, M.A., B.Sc., Ph.D., Carnegie Research Fellow, and H. G. Reid, B.Sc. Communicated by E. B. Ludlam, M.A., D.Sc. *Proc.*, vol. 52, pp. 152–158.
5. An Operational Method for the Solution of Linear Partial Differential Equations. By W. O. Kermack, M.A., D.Sc., and W. H. McCrea, M.A., Ph.D., B.Sc. *Proc.*, vol. 51, pp. 176–189. (Read by title.)
6. On the Definition of Spatial Distance in General Relativity. By H. S. Ruse, B.A. *Proc.*, vol. 52, pp. 183–194. (Read by title.)

SECOND ORDINARY MEETING.

Monday, December 7, 1931.

Professor Sir E. A. Sharpey-Schafer, F.R.S., President, in the Chair.

The Minutes of the previous Meeting were taken as read.

By the request of the Council, Professor R. A. Sampson, LL.D., F.R.S., delivered an address on "David Hume and Modern Science."

Volumes of the Hume MSS. in the possession of the Society, which had lately been collated by Dr J. Y. T. Greig and rebound, were exhibited, as was also a statuette of David Hume, presented to the Society by Dr Pittendrigh Macgillivray, R.S.A.

In the name of the Society the President thanked Professor Sampson for his address.

Professor Kemp Smith spoke of Hume as a philosopher.

THIRD ORDINARY MEETING.

Monday, January 11, 1932.

Professor Sir E. A. Sharpey-Schafer, F.R.S., President, in the Chair.

The Minutes of the previous Meeting were taken as read.

The following Communications were submitted:—

1. The Employment of Intra-cardiac Injections of Adrenaline in Asphyxia. By Sir E. A. Sharpey-Schafer, F.R.S., President, and W. A. Bain, B.Sc. *Proc.*, vol. 52, pp. 139–151.

2. Studies on Maternal Behaviour and its Endocrine Basis. By B. P. WIESNER, Ph.D., and Miss N. M. SHEARD.

3. Graphical Classification of Carbonaceous Minerals: the Place of the Constituents of Common Coal. By Professor HENRY BRIGGS, D.Sc., Ph.D. *Proc.*, vol. 52, pp. 195–199.

The undernoted Papers were read by title:—

4. On the Orthogonal Polynomials in Frequencies of Type B. By A. C. AITKEN, M.A., D.Sc. *Proc.*, vol. 52, pp. 174–182.

5. Isohedral and Isogonal Generalizations of the Regular Polyhedra. By Professor D. M. Y. SOMMERVILLE, M.A., D.Sc. *Proc.*, vol. 52, pp. 251–263.

FOURTH ORDINARY MEETING.

Monday, February 1, 1932.

Professor Sir E. A. Sharpey-Schafer, F.R.S., President, in the Chair.

The Minutes of the previous Meeting were taken as read.

The following Communications were submitted:—

1. Differentiation in the Sills of Northern Trotternish (Skye). By FREDERICK WALKER, M.A., Ph.D., D.Sc. *Trans.*, vol. 57, pp. 241–257.

2. A Study of the San Andreas Rift and Adjacent Features near Redlands, California. By GEORGE A. CUMMING, B.Sc., Ph.D. Communicated by Dr FREDERICK WALKER.

3. Contributions to the Stratigraphy of Eastern Spitzbergen. By G. W. TYRRELL, A.R.C.Sc., D.Sc., F.G.S.

4. Tectonic Relations and Petrology of the Dolerites of Spitzbergen. By G. W. TYRRELL, A.R.C.Sc., D.Sc., F.G.S., and K. S. SANDFORD, M.A., Ph.D.

FIFTH ORDINARY MEETING.

Monday, February 15, 1932.

Professor Sir E. A. Sharpey-Schafer, F.R.S., President, in the Chair.

The Minutes of the previous Meeting were taken as read.

The following Communications were submitted:—

1. On the Structure of Vertebraria. By Professor J. WALTON, M.A., D.Sc., and JESSIE A. R. WILSON, B.Sc. *Proc.*, vol. 52, pp. 200–207.

2. The Effect of Malnutrition on Root Structure. By G. BOND, B.Sc., Ph.D. Communicated by Professor J. WALTON, D.Sc. *Proc.*, vol. 52, pp. 159–173.

3. The Development of the Sorus in some Species of *Nephrolepis*, together with some Observations on Points of Anatomical Interest. By ISOBEL M. CASE, M.A., B.Sc., Ph.D. Communicated by S. WILLIAMS, M.Sc., Ph.D. *Trans.*, vol. 57, pp. 259–276.

4. The Pigmentary System and the Dopa Reaction. By EVLYN BOYD, M.Sc., Ph.D. Communicated by Professor F. A. E. CREW, M.D. *Proc.*, vol. 52, pp. 218–235.

SIXTH ORDINARY MEETING.

Monday, March 7, 1932.

Professor Sir E. A. Sharpey-Schafer, F.R.S., President, in the Chair.

The Minutes of the previous Meeting were taken as read.

The following were elected Ordinary Fellows of the Society (Messrs R. T. SKINNER and ROBERT GRANT acting as Scrutineers of the election):—

EUSTACE CECIL BARTON-WRIGHT, SOHAN LAL BHATIA, JAMES COUPER BRASH, EDWARD PROVAN CATHCART, VERE GORDON CHILDE, JAMES ROBERTON CHRISTIE, Sir THOMAS CLARK,

Bart., David Clouston, Leybourne Stanley Patrick Davidson, Alexander Murray Drennan, Sir John Edmund Ritchie Findlay, Bart., Andrew Hunter, James Balfour Lockhart, Archibald Gordon MacGregor, John James Rickard Macleod, Charles M'Neil, William Maxwell, John Murdoch Murray, Matthew Young Orr, James Nichol Pickard, Gorakh Prasad, Thomas Slater Price, John Pringle, John Michael Robson, William Fleetwood Sheppard, Alexander Rudolph Barbour Simpson, John Baird Simpson, Jean-Baptiste Octave Sneeden, William Elgin Swinton, Harold William Thompson, Henry Wade.

By request of the Council, Dr N. A. Mackintosh, A.R.C.S., D.Sc., of the Royal Research Ship *Discovery II*, delivered an address on "Research on Whales in the Antarctic."

SEVENTH ORDINARY MEETING.

Monday, May 2, 1932.

Professor Sir E. A. Sharpey-Schafer, F.R.S., President, in the Chair.

The Minutes of the previous Meeting were taken as read.

Professor E. P. Cathcart, Professor V. G. Childe, Sir Thomas Clark, Dr David Clouston, Professor A. Murray Drennan, Professor Andrew Hunter, Mr J. B. Lockhart, Mr A. G. MacGregor, Dr J. N. Pickard, Professor T. Slater Price, Dr John Pringle, Dr J. M. Robson, and Mr J. B. Simpson signed the Roll, and were duly admitted by the President.

The President announced that :—

The KEITH PRIZE for the period 1929–1931 has been awarded by the Council to Dr Alan William Greenwood, Lecturer in the Institute of Animal Genetics, University of Edinburgh, for his papers on the Biology of the Fowl, several of which had appeared in the *Proceedings* of the Society during the period under review.

The NEILL PRIZE for the period 1929–1931 has been awarded by the Council to Dr Charles Henry O'Donoghue, Reader in Zoology in the University of Edinburgh, for his papers on the Blood Vascular System, one of which had appeared in the *Transactions* of the Society during the period under review. Attention was paid, also, to the earlier work of Dr O'Donoghue on the Morphology of the *corpus luteum*.

The following Communications were submitted :—

1. The Genetic Basis of Alcaptonuria. By Professor Lancelot Hogben, R. L. Worrall, and I. Zieve. *Proc.*, vol. 52, pp. 264–295.
2. The Significance of Familial Incidence in Relation to Human Disease. By J. A. Fraser Roberts, M.A., B.Sc.
3. The Fæcal Pellets of the Anomura. By Hilary B. Moore, B.Sc. Communicated by Professor J. Graham Kerr, F.R.S. *Proc.*, vol. 52, pp. 296–308.
4. Graphical Analysis of Internal Combustion Indicator Diagrams. By Professor A. R. Horne. *Proc.*, vol. 52, pp. 208–217.
5. Pressure Effects in the Secondary Spectrum of Hydrogen. By W. G. Guthrie, M.A., B.A. Communicated by Professor H. S. Allen, F.R.S. *Proc.*, vol. 52, pp. 315–322.
6. The Adrenal Gland of *Xenophus lævis*. By H. Zwarenstein and I. Schrire, University of Cape Town. Communicated by Sir E. A. Sharpey-Schafer, F.R.S., President R.S.E. *Proc.*, vol. 52, pp. 323–326.

The undernoted Papers were read by title :—

7. The Zeta Function of Jacobi. By Professor L. M. Milne-Thomson. Communicated by Professor B. B. Baker. *Proc.*, vol. 52, pp. 236–250.
8. The Geodesics in Einstein's Unified Field Theory. By A. Blackwell, M.Sc. Communicated by Professor E. T. Whittaker, F.R.S. *Proc.*, vol. 52, pp. 327–330.

EIGHTH ORDINARY MEETING.

Monday, June 6, 1932.

Professor Sir E. A. Sharpey-Schafer, F.R.S., President, in the Chair.

The Minutes of the previous Meeting were taken as read.

The following were formally admitted Members :—Professor J. C. Brash, Mr J. Robertson Christie, Mr M. Y. Orr, and Dr W. F. Sheppard.

The President announced that :—

The Joint Committee of the Royal Society of Edinburgh, the Royal Physical Society, and the Royal Scottish Geographical Society has awarded the BRUCE PRIZE for the period 1930–1932 to Mr HENRY G. WATKINS, for important contributions to the topography of Spitzbergen, Labrador, and East Greenland, and investigation of the ice cap of Greenland.

Mr WATKINS was Leader of the Cambridge Expedition to Edge Island, Spitzbergen, 1927; Leader of the Cambridge Expedition to Labrador, 1928–1929; Leader of the British Air Route Expedition to East Greenland, 1930–1931; and will be Leader of the projected expedition to Coats Land, 1932.

The following Communications were submitted :—

1. The Genera *Dictyoconoides* Nuttall, *Lockhartia* nov., and *Rotalia* Lamarck; their Type Species, Generic Differences, and Fundamental Distinction from the *Dictyoconus* Group of Forms. By Lieut.-Col. L. M. DAVIES, R.A., F.G.S. *Trans.*, vol. 57, pp. 397–428.

2. Submarine Faulting in Kimmeridgian Times: East Sutherland. By Professor E. B. BAILEY, M.C., F.R.S., and J. WEIR, D.Sc., F.G.S. *Trans.*, vol. 57, pp. 429–467.

3. The Tertiary Floras of Ireland and Scotland. A Comparative Account: Part I. By Em. Professor T. JOHNSON, D.Sc.

The following Papers were read by title :—

4. A Study of the Tyrosinase of Potato Tubers. By IAN M. ROBERTSON, B.Sc., Ph.D. Communicated by ALEXANDER LAUDER, D.Sc. *Proc.*, vol. 52, pp. 309–314.

5. Scottish Carboniferous Ostracoda. By MARY H. LATHAM, M.A. Communicated by Professor E. B. BAILEY, M.C., F.R.S. *Trans.*, vol. 57, pp. 351–395.

NINTH ORDINARY MEETING.

Monday, June 20, 1932.

Professor J. H. Ashworth, F.R.S., Vice-President, in the Chair.

The Minutes of the previous Meeting were taken as read.

By request of the Council, Professor PIERRE RIJLANT, University of Brussels, delivered an address on "Conduction and Automatism in the Mammalian Heart."

A short discussion followed, to which Dr BAIN, Professor RITCHIE, Dr WIESNER, and Dr ROBSON contributed. Professor RIJLANT replied to the questions asked.

The Vice-President conveyed the thanks of the Society to Professor RIJLANT in fitting and gracious terms.

The Vice-President announced that the KEITH, NEILL, and BRUCE PRIZES would be presented at the July 4 Meeting.

The following Communications were submitted by title :—

1. Filial and Fraternal Correlations in Sex Linked Inheritance. By Professor LANCELOT HOGBEN. *Proc.*, vol. 52, pp. 331–336.

2. On the Structure and Function of the Alimentary Canal of the Limpet. By ALASTAIR GRAHAM, M.A., B.Sc. Communicated by Professor J. H. ASHWORTH, F.R.S. *Trans.*, vol. 57, pp. 287–308.

3. Notes on Lower Old Red Sandstone Plants from Callander, Perthshire. By S. M. K. HENDERSON, B.Sc. Communicated by Professor E. B. BAILEY, M.C., F.R.S. *Trans.*, vol. 57, pp. 277–285.

TENTH ORDINARY MEETING.

Monday, July 4, 1932.

Professor Sir E. A. Sharpey-Schafer, F.R.S., President, in the Chair.

The Minutes of the previous Meeting were taken as read.

Dr SNEEDEN and Professor CHARLES M'NEIL signed the Roll of Fellows.

The President referred in fitting terms to the loss that Science and the Society had endured through the tragic death of Professor J. W. GREGORY, F.R.S.

The President then presented the following Prizes: the KEITH PRIZE, 1929–1931, to Dr A. W. GREENWOOD:—

Dr A. W. GREENWOOD'S first important contribution to the literature of biology was a paper which appeared in 1923 on marsupial spermatogenesis. In 1925 the first paper of a series dealing with the fowl appeared and furnished an answer to the question as to whether grafts of gonadic tissue on to the chorio-allantoic membrane of the incubating egg influence the sex characters of the developing chick. His next contribution of importance dealt with gonad cross-transplantation in Leghorn and Campine which appeared in the *Proceedings of the Royal Society*, B, 1927. The results of the exchange of gonadic tissue between these two breeds, in one of which - the Campine - the male is henny feathered, upon the plumage were demonstrated. In the same year, in collaboration, Dr GREENWOOD described for the first time the developmental capon and poularde: individuals which exhibited all the signs of the agonadic condition in spite of the fact that gonads were present. Another paper dealt with the quantitative relation of comb size and gonad activity.

In 1929 he dealt with the relation between the gonads and thymus involution, and as to whether the thymus is concerned in the manufacture of the shell, and in the same year, in collaboration, presented a series of papers, two of which have appeared in the *Proceedings* of the Society, dealing with developmental abnormalities associated with embryonic mortality. He further demonstrated that the injection of egg yolk into the ovariectomised hen leads to the assumption of female plumage.

In 1930 a paper by GREENWOOD and BLYTH on the results of testicular implants on brown Leghorn hens appeared in the *Proceedings of the Royal Society*. More recently Dr GREENWOOD has turned to the study of the action of the testicular hormones upon the comb growth in capons, and, in collaboration with Professor DODDS, has published a number of papers dealing with this subject.

Dr GREENWOOD has spent more than eight years on the intensive study of the factors which influence the sex and plumage characters of the fowl, an experimental material which is being largely used in the field of physiological zoology. He is a skilful experimentalist with a profound knowledge of his material. It is intended that this award of the Keith Prize shall furnish an acknowledgment of his work and stimulate him to further efforts.

The NEILL PRIZE, 1929–1931, to Dr C. H. O'DONOGHUE:—

The Neill Prize for the period 1929–1931 is awarded to Dr CHARLES HENRY O'DONOGHUE in recognition of his published work on the comparative anatomy of the vascular system in the lower Vertebrates and on the *corpus luteum* in Marsupials.

Of a dozen papers on the vascular system three may be especially selected. The last, published in the *Transactions* of the Society for 1931, is a review of the recorded instances of vascular abnormalities in the Frog, together with suggestions as to the significance—embryological or evolutionary—of the conditions described. His description—with EILEEN BULMAN—of the vessels of the Dogfish, 1928, provided an opportunity for elucidating the homologies of some of the main vessels, and his memoir in the *Philosophical Transactions*, 1920, on the vascular system of Sphenodon contains a discussion of its relationships with the corresponding system of other reptiles and shows that in a number of important respects Sphenodon exhibits the most primitive vascular system in the Reptilia.

When Dr O'DONOGHUE, more than twenty years ago, commenced the study of the *corpus luteum* in Marsupials, its structure had been described in only one member of the Order. In several papers from 1911 to 1916 he gave an account of its development in six species, and was among the early workers who correlated the histological changes in it with the growth of the mammary glands.

The BRUCE PRIZE, 1930–1932, to Mr HENRY GINO WATKINS (*in absentiâ*):—

The Joint Committee of the Royal Society of Edinburgh, the Royal Physical Society, and the Royal Scottish Geographical Society has awarded the Bruce Prize for the period 1930–1932 to Mr HENRY GINO WATKINS, for important contributions to the topography of Spitsbergen, Labrador, and East Greenland, and investigation of the ice cap of Greenland.

Mr WATKINS was Leader of the Cambridge Expedition to Edge Island, Spitsbergen, 1927; Leader of the Cambridge Expedition to Labrador, 1928–1929; Leader of the British Air Route Expedition to East Greenland, 1930–1931; and will be Leader of the projected expedition to Coats Land.

He is in the front rank of Arctic Explorers, and is the youngest man who has received a Royal Medal from the Royal Geographical Society. The Bruce Prize is presented *in absentiâ*, but it is hoped that on some future date Mr WATKINS may be present and address the Society.

The President requested the Secretary to forward the Prize to Mr WATKINS.

SCOTT CENTENARY ADDRESS.

By the request of the Council, Professor H. J. C. GRIERSON, M.A., LL.D., delivered an address on "Scott and the Historical Novel," in celebration of the CENTENARY OF SIR WALTER SCOTT.

The following Papers were submitted by title:—

1. Studies in the Physiology of the Virus Diseases of the Potato: a Comparison of the Carbohydrate Metabolism of Normal with that of Leaf Roll Potatoes. By EUSTACE BARTON-WRIGHT, M.Sc., and ALAN M'BAIN, B.Sc. *Trans.*, vol. 57, pp. 309–349.

2. Adrenaline and the Œstrous Cycle in the Mouse. By J. M. ROBSON, M.D., B.Sc. *Proc.*, vol. 52, pp. 434–444.

3. A Study of the Foliar Endodermis of the Plantaginaceæ. By GEORGE TRAPP, M.A., B.Sc. Communicated by Professor J. WALTON, M.A., D.Sc. *Trans.*, vol. 57, pp. 523–546.

4. Relative Co-ordinates. By A. G. WALKER, B.A. Communicated by H. S. RUSE, B.A. *Proc.*, vol. 52, pp. 345–353.

5. The Diffusion Coefficients of Bromine-Hydrogen, Bromine-Nitrogen, Bromine-Oxygen, and Bromine-Carbon Dioxide. By JOHN E. MACKENZIE, D.Sc., and HARRY W. MELVILLE. *Proc.*, vol. 52, pp. 337–344.

PROCEEDINGS OF THE STATUTORY GENERAL MEETING
Ending the 149th Session, 1931-1932.

At the Statutory General Meeting of the Royal Society of Edinburgh, held in the Society's Rooms, 24 George Street, on Monday, October 24, 1932, at 4.30 P.M.,

Professor Sir E. A. Sharpey-Schafer, F.R.S., President, in the Chair,

the Minutes of the Statutory Meeting held on October 26, 1931, were read, approved, and signed.

The GENERAL SECRETARY submitted the following Report:—

SECRETARY'S REPORT, OCTOBER 24, 1932.

By request of the Council, addresses were delivered on "David Hume and Modern Science," by Professor R. A. SAMPSON, LL.D., F.R.S., on December 7, 1931; "Research on Whales in the Antarctic," by Dr N. A. MACKINTOSH, A.R.C.S., on March 7, 1932; and on "Conduction and Automatism in the Mammalian Heart," by Professor PIERRE RIJLANT, University of Brussels, on June 20, 1932. On July 4, 1932, in commemoration of the Centenary of the death of Sir WALTER SCOTT, who was President of the Society from 1820 to 1832, Professor H. J. C. GRIERSON, M.A., LL.D., delivered an address on "Scott and the Historical Novel."

40 papers were read, as compared with 34 in the previous session. The papers were divided among subjects as follows: Mathematics, 7; physics, 1; chemistry, 4; engineering, 1; geology and mineralogy, 7; palæontology, 5; botany, 3; plant physiology, 1; zoology, 3; human genetics, 3; animal genetics, 3; physiology, 2. 9 papers have been published or are being published in the *Transactions*, and 23 in the *Proceedings*. 7 papers were read but have not yet been submitted in final form, and 1 was declined for publication. Several papers were received during the session which were found unsuitable for presentation to the Society.

The Society has lost by death 24 Ordinary Fellows and 4 Honorary Fellows, and 6 Ordinary Fellows have resigned. 31 Ordinary Fellows were elected.

Invitations were received, and the Society was represented as follows on the occasions mentioned:—

1. University of Amsterdam, Tercentenary Celebration, June 27, 1932. Professor D'ARCY W. THOMPSON, C.B., F.R.S., presented address.
2. Royal Society of Canada. 50th Anniversary, May 26-28, 1932. Professor J. MEAKINS, M.D., LL.D., presented address.
3. National Academy of Sciences, Washington. 100th Anniversary of the Electrical Discoveries of JOSEPH HENRY. April 25, 1932. Letter sent.
4. Tartu University (Dorpat). Tercentenary of its Foundation, June 30 and July 1, 1932. Professor J. Y. SIMPSON, D.Sc., presented address.
5. Congresses of Entomology, Paris, July 18-23, 1932. Letter sent.
6. International Congress of Mathematicians, Zurich, September 4-12, 1932. Professor E. T. WHITTAKER, F.R.S., Professor H. W. TURNBULL, F.R.S., and Professor J. E. A. STEGGALL, M.A.
7. Spinoza Tercentenary. The Hague, September 1932. Letter sent.
8. National Trust for Scotland. Society's representative, the Hon. LORD SANDS.

A medal to be awarded in 1933, together with a sum of money, as the DAVID ANDERSON-BERRY PRIZE, has been designed by Mr PERCY METCALFE, designer of the coinage of the Irish Free State. The dies have been made at the Royal Mint, and a bronze replica is now in the possession of the Society.

The KEITH PRIZE for the period 1929-1931 was awarded to Dr A. W. GREENWOOD of the University of Edinburgh.

The NEILL PRIZE for the period 1929-1931 was awarded to Dr C. H. O'DONOGHUE of the University of Edinburgh.

The BRUCE PRIZE for the period 1930-1932 was awarded to Mr HENRY GINO WATKINS, who, unfortunately, met his death by drowning in Lake Fjord, Greenland, on August 20, 1932, while leading the British Arctic Air Route Expedition.

After careful consideration by a Committee, a more durable paper has been decided upon for the *Transactions* and *Proceedings*, and a new type has been selected for the *Proceedings*. This will result in a saving of approximately 10 per cent. in the cost of printing per sheet. The *Transactions*, already in monotype, will be charged as formerly.

Reference was made in my last report to the collaboration of the Society with the Royal Society of London, to arrange for the Second Polar Year and to administer the grant of £10,000 for this purpose made by the Air Ministry. The Joint Committee, reinforced by some outside members, has continued its work and has dispatched a party to Fort Rae in Canada and a second one to Tromsö in Norway. The Society, with the approval of the Committee, has itself arranged for Auroral photographs and observations to be made at Urafirth in Shetland, and these are now in progress.

The Council of the Society drew the attention of the Town Council of Edinburgh to the neglected condition of the David Hume monument in the Old Calton Burial Ground. As a consequence steps have been taken to improve its condition.

The Calendar of the Hume MSS. in the possession of the Royal Society of Edinburgh, by Dr J. Y. T. Greig and Harold Benyon, has now been published as Vol. 52, Part I, of the *Proceedings*, and has been circulated.

A number of new exchanges of publications were initiated during the session.

In connection with the Scott Centenary Exhibitions in Edinburgh and Glasgow, it may be mentioned that the Society lent to these exhibitions its portrait of Sir Walter Scott by Graham Gilbert.

The acknowledgment of the Society is due to the Carnegie Trust for the Universities of Scotland for grants to authors towards the costs of the illustrations of papers published by the Society, amounting to £87, 16s. 7d.; to the University of Liverpool Dunning Lawrence Research Fund for a grant of £25 towards the cost of publication of Dr Tudor Jones's recent paper in the *Transactions*; and to the Royal Society of London for a sum of £250 from the Government Publication Grant in aid of the cost of printing the Society's *Transactions* and *Proceedings* during the session 1931–1932.

Treasurer's Report :—

The Treasurer stated that there were no specially outstanding features with regard to the finances of the past year.

There had been an increase in the amount received for contributions, so that while last year there was an adverse balance on the year of over £62, the Accounts of this year show a surplus of over £20.

The President in the course of his remarks drew attention to the fact that the Society had never confined itself to science, but had always taken a prominent part in the encouragement of literature as well. While referring to the value of addresses given from time to time by eminent scientists, instancing especially that of Bohr in the previous session, and in the present session that of Rijlant, who gave the Society a lucid exposition of the researches of Demoor and himself, the President recalled in the literary field the address above referred to on David Hume, and especially that on Walter Scott, with which Professor H. J. C. Grierson had favoured the Society. He also referred to the losses of the Society by the death of Ordinary and Honorary Fellows, details of which may be found in the Obituary Notices already published. To these must be added, though not a Fellow, the name of the Bruce Medallist, Henry Gino Watkins, whose tragic death by drowning in Greenland occurred on August 20.

The President nominated as Scrutineers of the Ballot, Dr Robert Campbell and Mr J. W. Butters.

The Ballot for the Election of Council and Office-Bearers was then taken.

Mr John Bartholomew moved the adoption of the Reports and the reappointment of Messrs Lindsay, Jamieson, and Haldane, C.A., as auditors for the ensuing session. These motions were approved.

The Scrutineers reported that the Ballot Papers were in order, and the President declared that the following Council and Office-Bearers had been duly elected :—

Professor Sir E. A. Sharpey-Schafer, M.D., D.Sc., LL.D., F.R.S., President.

Professor J. H. Ashworth, D.Sc., F.R.S.	Vice-Presidents.
Arthur Logan Turner, M.D., LL.D., F.R.C.S.E.	
J. B. Clark, M.A., LL.D., J.P.	
Professor James Ritchie, M.A., D.Sc.	
Principal Sir Thomas H. Holland, K.C.S.I., K.C.I.E., D.L., D.Sc., LL.D., F.R.S.	
The Hon. Lord Sands, Kt., K.C., LL.D., D.D.	

Professor R. A. Sampson, M.A., D.Sc., LL.D., F.R.S., General Secretary.

Professor C. G. Darwin, M.A., F.R.S.	Secretaries to Ordinary Meetings.
Professor F. A. E. Crew, M.D., D.Sc., Ph.D.	

James Watt, W.S., LL.D., Treasurer.

Professor D'Arcy W. Thompson, C.B., Hon. D.Sc., D.Litt., F.R.S., Curator of Library and Museum.

ORDINARY MEMBERS OF COUNCIL.

Murray Macgregor, M.A., D.Sc.
A. Crichton Mitchell, D.Sc.
Professor P. T. Herring, M.D., F.R.C.P.E.
Professor James P. Kendall, M.A., D.Sc., F.R.S.
Professor Thomas M. MacRobert, M.A., D.Sc.
Professor Godfrey H. Thomson, D.Sc., Ph.D.
Malcolm Wilson, D.Sc., A.R.C.Sc., F.L.S.
Professor E. B. Bailey, M.C., M.A., F.R.S.
Professor J. C. Brash, M.C., M.A., M.D.
Professor A. J. Clark, M.C., B.A., M.D., F.R.S.
Professor A. G. Ogilvie, M.A., B.Sc.
Professor E. M. Wedderburn, M.A., D.Sc., LL.B., W.S.

The President, before closing the meeting, thanked the Scrutineers for their services.

THE KEITH, MAKDOUGALL-BRISBANE, NEILL, GUNNING VICTORIA JUBILEE, JAMES SCOTT, BRUCE, AND DAVID ANDERSON-BERRY PRIZES, AND THE BRUCE-PRELLER LECTURE FUND.

The above Prizes will be awarded by the Council in the following manner:—

I. KEITH PRIZE.

The Keith Prize, consisting of a Gold Medal and from £40 to £50 in Money, will be awarded in the Session 1933–1934 for the "best communication on a scientific subject, communicated,* in the first instance, to the Royal Society of Edinburgh during the Sessions 1931–1932 and 1932–1933." Preference will be given to a paper containing a discovery. (See also Council's resolutions at the end of these regulations.)

II. MAKDOUGALL-BRISBANE PRIZE

(*Amended June* 7, 1926.)

This Prize is to be awarded biennially by the Council of the Royal Society of Edinburgh to such person, for such purposes, for such objects, and in such manner as shall appear to them the most conducive to the promotion of the interests of science; with the *proviso* that the Council shall not be compelled to award the Prize unless there shall be some individual engaged in scientific pursuit, or some paper written on a scientific subject, or some discovery in science made during the biennial period, of sufficient merit or importance in the opinion of the Council to be entitled to the Prize.

1. The Prize, consisting of a Gold Medal and a sum of Money, will be awarded before the close of the Session 1934–1935, for an Essay, Paper, or other work having reference to any branch of scientific inquiry, either material or mental.

2. It is open to all men of science.

3. The specific subjects taken into consideration in the current award are governed by the resolutions of the Council as stated at the end of these regulations.

4. For the current period the Committee is representative of Group B.

5. The Committee will consider papers presented to the Society within the Sessions 1932–1933 and 1933–1934, and will make a recommendation.

It is empowered to recommend either:—

(*a*) An award to the Author of an Essay or Paper considered as above, or

(*b*) That no award be made on the ground that, within its group, no paper of sufficient merit has been presented, or

(*c*) That the Prize be awarded to some distinguished man of learning, who may not have presented a paper to the Society within the period considered, but who is willing to deliver an address.

* For the purposes of this award the word "communicated" shall be understood to mean the date on which the manuscript of a paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

III. NEILL PRIZE.

The Council of the Royal Society of Edinburgh having received the bequest of the late Dr Patrick Neill of the sum of £500, for the purpose of "the interest thereof being applied in furnishing a Medal or other reward every second or third year to any distinguished Scottish Naturalist, according as such Medal or reward shall be voted by the Council of the said Society," hereby intimate:

1. The Neill Prize, consisting of a Gold Medal and a sum of Money, will be awarded during the Session 1933–1934.

2. The Prize will be given for a Paper of distinguished merit, on a subject of Natural History, by a Scottish Naturalist, which shall have been presented * to the Society during the two years preceding the fourth Monday in October 1933,—or failing presentation of a paper sufficiently meritorious, it will be awarded for a work or publication by some distinguished Scottish Naturalist, on some branch of Natural History, bearing date within five years of the time of award. (See also Council's resolutions at the end of these regulations.)

IV. GUNNING VICTORIA JUBILEE PRIZE.

This Prize, founded in the year 1887 by Dr R. H. Gunning, is to be awarded quadrennially by the Council of the Royal Society of Edinburgh, in recognition of original work in Physics, Chemistry, or Pure or Applied Mathematics.

Evidence of such work may be afforded either by a Paper presented * to the Society, or by a Paper on one of the above subjects, or some discovery in them elsewhere communicated or made, which the Council may consider to be deserving of the Prize.

The Prize consists of a sum of money, and is open to men of science resident in or connected with Scotland. The first award was made in the year 1887. The next award will be made in Session 1936–1937.

In accordance with the wish of the Donor, the Council of the Society may on fit occasions award the Prize for work of a definite kind to be undertaken during the three succeeding years by a scientific man of recognised ability.

V. JAMES SCOTT PRIZE.

This Prize, founded in the year 1918 by the Trustees of the James Scott Bequest, is to be awarded triennially, or at such intervals as the Council of the Royal Society of Edinburgh may decide, "for a lecture or essay on the fundamental concepts of Natural Philosophy."

VI. BRUCE PRIZE.

The Royal Society is trustee of a fund, instituted in 1923, to commemorate the work of Dr W. S. Bruce as an explorer and scientific investigator in polar regions.

The Committee of Award is appointed jointly by the Royal Society, the Royal Physical Society, and the Royal Scottish Geographical Society.

* For the purposes of this award the word "presented" shall be understood to mean the date on which the manuscript of a paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

The Prize consists of a Bronze Medal and sum of money. It is open to workers of all nationalities, with a preference, *ceteris paribus*, for those of Scottish birth or origin, and is to be awarded biennially for some notable contribution to Natural Sciences, such as Zoology, Botany, Geology, Meteorology, Oceanography, and Geography; the contribution to be in the nature of new knowledge, the outcome of a personal visit to polar regions on the part of the recipient. The recipient shall preferably be at the outset of his career as an investigator.

The next award will be made in 1934. Papers for the consideration of the Committee should be in the hands of the General Secretary of the Royal Society, 22 George Street, Edinburgh, not later than March 31 of that year.

VII. BRUCE-PRELLER LECTURE FUND.

The Council of the Royal Society of Edinburgh having received in 1929 the bequest of the late Dr Charles Du Riche Preller of the sum of £500, decided that the income thereof be applied by the Council biennially as an honorarium for a special Bruce-Preller Lecture or Address by an outstanding man of science, its subject to be Geology or Electrical or Physical Science, or in the discretion of the Council some other branch of science. The next award will be made in session 1932–1933.

VIII. DAVID ANDERSON-BERRY FUND.

The Council of the Royal Society of Edinburgh having received in the year 1930, free of duty, the capital sum of one thousand pounds (£1000), to be used in terms of the will of the late Dr David Anderson-Berry, dated 23rd April 1926, decided that the income thereof be applied triennially, "in the first place in the presentation of a gold medal, and in the second place in the payment of a sum of money to the winner for the year of such gold medal, the winner being the person who, in the opinion of the Society, shall be the producer for the year of the best essay on the nature of X-rays and their therapeutical effect on human diseases."

The first award will be made in July 1935.

RESOLUTIONS OF COUNCIL IN REGARD TO THE MODE OF AWARDING PRIZES.

(*See Minutes of Meeting of January* 18, 1915.)

I. With regard to the Keith and Makdougall-Brisbane Prizes, which are open to all Sciences, the mode of award will be as follows:—

1. Papers or essays to be considered shall be arranged in two groups, A and B, —Group A to include Astronomy, Chemistry, Mathematics, Metallurgy, Meteorology, and Physics; Group B to include Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, and Zoology.
2. These two Prizes shall be awarded to each group in alternate biennial periods, provided papers worthy of recommendation have been communicated to the Society.

3. Prior to the adjudication the Council shall appoint, in the first instance, a Committee composed of representatives of the group of Sciences which did not receive the award in the immediately preceding period. The Committee shall consider the Papers which come within their group of Sciences, and report in due course to the Council.

4. In the event of the aforesaid Committee reporting that within their group of subjects there is, in their opinion, no paper worthy of being recommended for the award, the Council, on accepting this report, shall appoint a Committee representative of the alternate group to consider papers coming within their group and to report accordingly.

5. Papers to be considered by the Committees shall fall within the period dating from the last award in groups A and B respectively.

II. With regard to the Neill Prize, the term "Naturalist" shall be understood to include any student in the Sciences composing group B, namely, Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, Zoology.

AWARDS OF THE KEITH, MAKDOUGALL-BRISBANE, NEILL, GUNNING, JAMES SCOTT, BRUCE, AND DAVID ANDERSON-BERRY PRIZES, AND THE BRUCE-PRELLER LECTURE FUND.

I. KEITH PRIZE.

1st Biennial Period, 1827–29.—Dr Brewster, for his papers "on his Discovery of Two New Immiscible Fluids in the Cavities of certain Minerals," published in the Transactions of the Society.

2nd Biennial Period, 1829–31.—Dr Brewster, for his paper "on a New Analysis of Solar Light," published in the Transactions of the Society.

3rd Biennial Period, 1831–33.—Thomas Graham, Esq., for his paper "on the Law of the Diffusion of Gases," published in the Transactions of the Society.

4th Biennial Period, 1833–35.—Professor J. D. Forbes, for his paper "on the Refraction and Polarization of Heat," published in the Transactions of the Society.

5th Biennial Period, 1835–37.—John Scott Russell, Esq., for his researches "on Hydrodynamics," published in the Transactions of the Society.

6th Biennial Period, 1837–39.—Mr John Shaw, for his experiments "on the Development and Growth of the Salmon," published in the Transactions of the Society.

7th Biennial Period, 1839–41.—Not awarded.

8th Biennial Period, 1841–43.—Professor James David Forbes, for his papers "on Glaciers," published in the Proceedings of the Society.

9th Biennial Period, 1843–45.—Not awarded.

10th Biennial Period, 1845–47.—General Sir Thomas Brisbane, Bart., for the Makerstoun Observations on Magnetic Phenomena, made at his expense, and published in the Transactions of the Society.

11th Biennial Period, 1847–49.—Not awarded.

12th Biennial Period, 1849–51.—Professor Kelland, for his papers "on General Differentiation, including his more recent Communication on a process of the Differential Calculus, and its application to the solution of certain Differential Equations," published in the Transactions of the Society.

13th Biennial Period, 1851–53.—W. J. Macquorn Rankine, Esq., for his series of papers "on the Mechanical Action of Heat," published in the Transactions of the Society.

14th Biennial Period, 1853–55.—Dr Thomas Anderson, for his papers "on the Crystalline Constituents of Opium, and on the Products of the Destructive Distillation of Animal Substances," published in the Transactions of the Society.

15th Biennial Period, 1855–57.—Professor Boole, for his Memoir "on the Application of the Theory of Probabilities to Questions of the Combination of Testimonies and Judgments," published in the Transactions of the Society.

16th Biennial Period, 1857–59.—Not awarded.

17th Biennial Period, 1859–61.—John Allan Broun, Esq., F.R.S., Director of the Trevandrum Observatory, for his papers "on the Horizontal Force of the Earth's Magnetism, on the Correction of the Bifilar Magnetometer, and on Terrestrial Magnetism generally," published in the Transactions of the Society.

18th Biennial Period, 1861–63.—Professor William Thomson, of the University of Glasgow, for his Communication "on some Kinematical and Dynamical Theorems."

19th Biennial Period, 1863–65.—Principal Forbes, St Andrews, for his "Experimental Inquiry into the Laws of Conduction of Heat in Iron Bars," published in the Transactions of the Society.

20th Biennial Period, 1865–67.—Professor C. Piazzi Smyth, for his paper "on Recent Measures at the Great Pyramid," published in the Transactions of the Society.

21st Biennial Period, 1867–69.—Professor P. G. Tait, for his paper "on the Rotation of a Rigid Body about a Fixed Point," published in the Transactions of the Society.

22ND BIENNIAL PERIOD, 1869–71.—Professor CLERK MAXWELL, for his paper "on Figures, Frames, and Diagrams of Forces," published in the Transactions of the Society.

23RD BIENNIAL PERIOD, 1871–73.—Professor P. G. TAIT, for his paper entitled "First Approximation to a Thermo-electric Diagram," published in the Transactions of the Society.

24TH BIENNIAL PERIOD, 1873–1875.—Professor CRUM BROWN, for his Researches "on the Sense of Rotation, and on the Anatomical Relations of the Semicircular Canals of the Internal Ear."

25TH BIENNIAL PERIOD, 1875–77.—Professor M. FORSTER HEDDLE, for his papers "on the Rhombohedral Carbonates," and "on the Felspars of Scotland," published in the Transactions of the Society.

26TH BIENNIAL PERIOD, 1877–79.—Professor H. C. FLEEMING JENKIN, for his paper "on the Application of Graphic Methods to the Determination of the Efficiency of Machinery," published in the Transactions of the Society; Part II having appeared in the volume for 1877–78.

27TH BIENNIAL PERIOD, 1879–81.—Professor GEORGE CHRYSTAL, for his paper "on the Differential Telephone," published in the Transactions of the Society.

28TH BIENNIAL PERIOD, 1881–83.—THOMAS MUIR, Esq., LL.D., for his "Researches into the Theory of Determinants and Continued Fractions," published in the Proceedings of the Society.

29TH BIENNIAL PERIOD, 1883–85.—JOHN AITKEN, Esq., for his paper "on the Formation of Small Clear Spaces in Dusty Air," and for previous papers on Atmospheric Phenomena, published in the Transactions of the Society.

30TH BIENNIAL PERIOD, 1885–87.—JOHN YOUNG BUCHANAN, Esq., for a series of communications, extending over several years, on subjects connected with Ocean Circulation, Compressibility of Glass, etc.; two of which, viz., "On Ice and Brines," and "On the Distribution of Temperature in the Antarctic Ocean," have been published in the Proceedings of the Society.

31ST BIENNIAL PERIOD, 1887–89.—Professor E. A. LETTS, for his papers on the Organic Compounds of Phosphorus, published in the Transactions of the Society.

32ND BIENNIAL PERIOD, 1889–91.—R. T. OMOND, Esq., for his contributions to Meteorological Science, many of which are contained in vol. xxxiv of the Society's Transactions.

33RD BIENNIAL PERIOD, 1891–93.—Professor THOMAS R. FRASER, F.R.S., for his papers on *Strophanthus hispidus*, Strophanthin, and Strophanthidin, read to the Society in February and June 1889 and in December 1891, and printed in vols. xxxv, xxxvi, and xxxvii of the Society's Transactions.

34TH BIENNIAL PERIOD, 1893–95.—Dr CARGILL G. KNOTT, for his papers on the Strains produced by Magnetism in Iron and in Nickel, which have appeared in the Transactions and Proceedings of the Society.

35TH BIENNIAL PERIOD, 1895–97.—Dr THOMAS MUIR, for his continued communications on Determinants and Allied Questions.

36TH BIENNIAL PERIOD, 1897–99.—Dr JAMES BURGESS, for his paper "on the Definite Integral $\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$, with extended Tables of Values," printed in vol. xxxix of the Transactions of the Society.

37TH BIENNIAL PERIOD, 1899–1901.—Dr HUGH MARSHALL, for his discovery of the Persulphates, and for his Communications on the Properties and Reactions of these Salts, published in the Proceedings of the Society.

38TH BIENNIAL PERIOD, 1901–03.—Sir WILLIAM TURNER, K.C.B., LL.D., F.R.S., etc., for his memoirs entitled "A Contribution to the Craniology of the People of Scotland," published in the Transactions of the Society, and for his "Contributions to the Craniology of the People of the Empire of India," Parts I, II, likewise published in the Transactions of the Society.

39TH BIENNIAL PERIOD, 1903–05.—THOMAS H. BRYCE, M.A., M.D., for his two papers on "The Histology of the Blood of the Larva of *Lepidosiren paradoxa*," published in the Transactions of the Society within the period.

40TH BIENNIAL PERIOD, 1905–07.—ALEXANDER BRUCE, M.A., M.D., F.R.C.P.E., for his paper entitled "Distribution of the Cells in the Intermedio-Lateral Tract of the Spinal Cord," published in the Transactions of the Society within the period.

41ST BIENNIAL PERIOD, 1907–09.—WHEELTON HIND, M.D., B.S., F.R.C.S., F.G.S., for a paper published in the Transactions of the Society, "On the Lamellibranch and Gasteropod Fauna found in the Millstone Grit of Scotland."

42ND BIENNIAL PERIOD, 1909–11.—Professor ALEXANDER SMITH, B.Sc., Ph.D., of New York, for his researches upon "Sulphur" and upon "Vapour Pressure," appearing in the Proceedings of the Society.

43RD BIENNIAL PERIOD, 1911–1913.—JAMES RUSSELL, Esq., for his series of investigations relating to magnetic phenomena in metals and the molecular theory of magnetism, the results of which have been published in the Proceedings and Transactions of the Society, the last paper having been issued within the period.

44TH BIENNIAL PERIOD, 1913–15.—JAMES HARTLEY ASHWORTH, D.Sc., for his papers on "Larvæ of Lingula and Pelagodiscus," and on "Sclerocheilus," published in the Transactions of the Society, and for other papers on the Morphology and Histology of Polychæta.

45TH BIENNIAL PERIOD, 1915–17.—ROBERT C. MOSSMAN, for his work on the Meteorology of the Antarctic Regions, which originated with the important series of observations made by him during the voyage of the "Scotia" (1902–1904), and includes his paper "On a Sea-Saw of Barometric Pressure, Temperature, and Wind Velocity between the Weddell Sea and the Ross Sea," published in the Proceedings of the Society.

46TH BIENNIAL PERIOD, 1917–19.—JOHN STEPHENSON, Lt.-Col. I.M.S., for his series of papers on the Oligochæta and other Annelida, several of which have been published in the Transactions of the Society.

47TH BIENNIAL PERIOD, 1919–21.—RALPH ALLEN SAMPSON, F.R.S., for his Astronomical Researches, including the papers "Studies in Clocks and Time-keeping: No. 1, Theory of the Maintenance of Motion; No. 2, Tables of the Circular Equation," published in the Proceedings of the Society within the period of the award.

48TH BIENNIAL PERIOD, 1921–23.—JOHN WALTER GREGORY, F.R.S., for his papers published in the Transactions of the Society, and in recognition of his numerous contributions to Geology, extending over a period of thirty-six years.

49TH BIENNIAL PERIOD, 1923–25.—HERBERT WESTREN TURNBULL, M.A., for the papers on "Hyper-Algebra," "Invariant Theory," and "Algebraic Geometry," three of which have been published in the Proceedings within the period of award.

50TH BIENNIAL PERIOD, 1925–27.—THOMAS JOHN JEHU, M.A., M.D., F.G.S., and ROBERT MELDRUM CRAIG, M.A., B.Sc., F.G.S., for the joint series of papers which have recently appeared in the Transactions of the Society on the "Geology of the Outer Hebrides."

51ST BIENNIAL PERIOD, 1927–29.—Dr CHRISTINA C. MILLER, B.Sc., for her papers on the "Slow Oxidation of Phosphorus Trioxide," published in the Proceedings within the period of the award, and in consideration of subsequent developments on "Slow Oxidation of Phosphorus," published elsewhere.

52ND BIENNIAL PERIOD, 1929–31.—Dr ALAN WILLIAM GREENWOOD, for his papers on the "Biology of the Fowl," several of which have appeared in the Proceedings within the period of award.

II. MAKDOUGALL-BRISBANE PRIZE.

1ST BIENNIAL PERIOD, 1859.—Sir RODERICK IMPEY MURCHISON, on account of his Contributions to the Geology of Scotland.

2ND BIENNIAL PERIOD, 1860–62.—WILLIAM SELLER, M.D., F.R.C.P.E., for his "Memoir of the Life and Writings of Dr Robert Whytt," published in the Transactions of the Society.

3RD BIENNIAL PERIOD, 1862–64.—JOHN DENIS MACDONALD, Esq., R.N., F.R.S., Surgeon of H.M.S. "Icarus," for his paper "on the Representative Relationships of the Fixed and Free Tunicata, regarded as Two Sub-classes of equivalent value; with some General Remarks on their Morphology," published in the Transactions of the Society.

4TH BIENNIAL PERIOD, 1864–66.—Not awarded.

5TH BIENNIAL PERIOD, 1866–68.—Dr ALEXANDER CRUM BROWN and Dr THOMAS RICHARD FRASER, for their conjoint paper "on the Connection between Chemical Constitution and Physiological Action," published in the Transactions of the Society.

6TH BIENNIAL PERIOD, 1868–70.—Not awarded.

7TH BIENNIAL PERIOD, 1870–72.—GEORGE JAMES ALLMAN, M.D., F.R.S., Emeritus Professor of Natural History, for his paper "on the Homological Relations of the Cœlenterata," published in the Transactions, which forms a leading chapter of his Monograph of Gymnoblastic or Tubularian Hydroids—since published.

8TH BIENNIAL PERIOD, 1872–74.—Professor LISTER, for his paper "on the Germ Theory of Putrefaction and the Fermentive Changes," communicated to the Society, 7th April 1873.

9TH BIENNIAL PERIOD, 1874–76.—ALEXANDER BUCHAN, A.M., for his paper "on the Diurnal Oscillation of the Barometer," published in the Transactions of the Society.

10TH BIENNIAL PERIOD, 1876-78.—Professor ARCHIBALD GEIKIE, for his paper "on the Old Red Sandstone of Western Europe," published in the Transactions of the Society.

11TH BIENNIAL PERIOD, 1878-80.—Professor PIAZZI SMYTH, Astronomer-Royal for Scotland, for his paper "on the Solar Spectrum in 1877-78, with some Practical Idea of its probable Temperature of Origination," published in the Transactions of the Society.

12TH BIENNIAL PERIOD, 1880-82.—Professor JAMES GEIKIE, for his "Contributions to the Geology of the North-West of Europe," including his paper "on the Geology of the Faroes," published in the Transactions of the Society.

13TH BIENNIAL PERIOD, 1882-84.—EDWARD SANG, Esq., LL.D., for his paper "on the Need of Decimal Subdivisions in Astronomy and Navigation, and on Tables requisite therefor," and generally for his Recalculations of Logarithms both of Numbers and Trigonometrical Ratios, —the former communication being published in the Proceedings of the Society.

14TH BIENNIAL PERIOD, 1884-86.—JOHN MURRAY, Esq., LL.D., for his papers "On the Drainage Areas of Continents, and Ocean Deposits," "The Rainfall of the Globe, and Discharge of Rivers," "The Height of the Land and Depth of the Ocean," and "The Distribution of Temperature in the Scottish Lochs as affected by the Wind."

15TH BIENNIAL PERIOD, 1886-88.—ARCHIBALD GEIKIE, Esq., LL.D., for numerous Communications, especially that entitled "History of Volcanic Action during the Tertiary Period in the British Isles," published in the Transactions of the Society.

16TH BIENNIAL PERIOD, 1888-90.—Dr LUDWIG BECKER, for his paper on "The Solar Spectrum at Medium and Low Altitudes," printed in vol. xxxvi, Part 1, of the Society's Transactions.

17TH BIENNIAL PERIOD, 1890-92.—HUGH ROBERT MILL, Esq., D.Sc., for his papers on "The Physical Conditions of the Clyde Sea Area," Part I being already published in vol. xxxvi of the Society's Transactions.

18TH BIENNIAL PERIOD, 1892-94.—Professor JAMES WALKER, D.Sc., Ph.D., for his work on Physical Chemistry, part of which has been published in the Proceedings of the Society, vol. xx, pp. 255-263. In making this award, the Council took into consideration the work done by Professor Walker along with Professor Crum Brown on the Electrolytic Synthesis of Dibasic Acids, published in the Transactions of the Society.

19TH BIENNIAL PERIOD, 1894-96.—Professor JOHN G. M'KENDRICK, for numerous Physiological papers, especially in connection with Sound, many of which have appeared in the Society's publications.

20TH BIENNIAL PERIOD, 1896-98.—Dr WILLIAM PEDDIE, for his papers on the Torsional Rigidity of Wires.

21ST BIENNIAL PERIOD, 1898-1900.—Dr RAMSAY H. TRAQUAIR, for his paper entitled "Report on Fossil Fishes collected by the Geological Survey in the Upper Silurian Rocks of Scotland," printed in vol. xxxix of the Transactions of the Society.

22ND BIENNIAL PERIOD, 1900-02.—Dr ARTHUR T. MASTERMAN, for his paper entitled "The Early Development of *Cribrella oculata* (Forbes), with remarks on Echinoderm Development," printed in vol. xl of the Transactions of the Society.

23RD BIENNIAL PERIOD, 1902-04.—Mr JOHN DOUGALL, M.A., for his paper on "An Analytical Theory of the Equilibrium of an Isotropic Elastic Plate," published in vol. xli of the Transactions of the Society.

24TH BIENNIAL PERIOD, 1904-06.—JACOB E. HALM, Ph.D., for his two papers entitled "Spectroscopic Observations of the Rotation of the Sun," and "Some Further Results obtained with the Spectroheliometer," and for other astronomical and mathematical papers published in the Transactions and Proceedings of the Society within the period.

25TH BIENNIAL PERIOD, 1906-08.—D. T. GWYNNE-VAUGHAN, M.A., F.L.S., for his papers, 1st, "On the Fossil Osmundaceæ," and 2nd, "On the Origin of the Adaxially-curved Leaf-trace in the Filicales," communicated by him conjointly with Dr R. Kidston.

26TH BIENNIAL PERIOD, 1908-10.—ERNEST MACLAGAN WEDDERBURN, M.A., LL.B., for his series of papers bearing upon "The Temperature Distribution in Fresh-water Lochs," and especially upon "The Temperature Seiche."

27TH BIENNIAL PERIOD, 1910-12.—JOHN BROWNLEE, M.A., M.D., D.Sc., for his contributions to the Theory of Mendelian Distributions and cognate subjects, published in the Proceedings of the Society within and prior to the prescribed period.

28TH BIENNIAL PERIOD, 1912-14.—Professor C. R. MARSHALL, M.D., M.A., for his studies "On the Pharmacological Action of Tetra-alkyl-ammonium Compounds."

29TH BIENNIAL PERIOD, 1914-16.—ROBERT ALEXANDER HOUSTOUN, Ph.D., D.Sc., for his series of papers on "The Absorption of Light by Inorganic Salts," published in the Proceedings of the Society.

30TH BIENNIAL PERIOD, 1916-18.—Professor A. ANSTRUTHER LAWSON, for his Memoirs on "The Prothalli of *Tmesipteris Tannensis* and of *Psilotum*," published in the Transactions of the Society, together with previous papers on Cytology and on The Gametophytes of various Gymnosperms.

31ST BIENNIAL PERIOD, 1918-20.—Professor J. H. MACLAGAN WEDDERBURN of Princeton University for his Memoirs in Universal Algebra, etc., published in the Transactions and Proceedings of the Society, and elsewhere.

32ND BIENNIAL PERIOD, 1920-22.—Professor W. T. GORDON, M.A., D.Sc., for his paper on "Cambrian Organic Remains from a Dredging in the Weddell Sea," published in the Transactions of the Society within the period, and for his investigations on the Fossil Flora of the Pettycur Limestone, previously published in the Transactions.

33RD BIENNIAL PERIOD, 1922-24.—Professor H. STANLEY ALLEN, D.Sc., for his papers on the "Quantum and Atomic Theory," published in the Society's Proceedings within the period.

34TH BIENNIAL PERIOD, 1924-26.—Dr CHARLES MORLEY WENYON, C.M G., C.B.E , F.R.S., for his distinguished work in Protozoology extending over a period of twenty-one years.

35TH BIENNIAL PERIOD, 1926-28.—Dr W. O. KERMACK, M.A., for his contributions to Chemistry, published in the Society's Proceedings and elsewhere.

36TH BIENNIAL PERIOD, 1928-30.—Dr NELLIE B. EALES, for her papers in the Society's Transactions on "The Anatomy of a Fœtal African Elephant."

III. THE NEILL PRIZE.

1ST TRIENNIAL PERIOD, 1856-59.—Dr W. LAUDER LINDSAY, for his paper "on the Spermogones and Pycnides of Filamentous, Fruticulose, and Foliaceous Lichens," published in the Transactions of the Society.

2ND TRIENNIAL PERIOD, 1859-62.—ROBERT KAYE GREVILLE, LL.D., for his contributions to Scottish Natural History, more especially in the department of Cryptogamic Botany, including his recent papers on Diatomaceæ.

3RD TRIENNIAL PERIOD, 1862-65.—ANDREW CROMBIE RAMSAY, F.R.S., Professor of Geology in the Government School of Mines, and Local Director of the Geological Survey of Great Britain, for his various works and memoirs published during the last five years, in which he has applied the large experience acquired by him in the Direction of the arduous work of the Geological Survey of Great Britain to the elucidation of important questions bearing on Geological Science.

4TH TRIENNIAL PERIOD, 1865-68.—Dr WILLIAM CARMICHAEL M'INTOSH, for his paper "on the Structure of the British Nemerteans, and on some New British Annelids," published in the Transactions of the Society.

5TH TRIENNIAL PERIOD, 1868-71.—Professor WILLIAM TURNER, for his papers "on the Great Finner Whale; and on the Gravid Uterus, and the Arrangement of the Fœtal Membranes in the Cetacea," published in the Transactions of the Society.

6TH TRIENNIAL PERIOD, 1871-74.—CHARLES WILLIAM PEACH, Esq., for his Contributions to Scottish Zoology and Geology, and for his recent contributions to Fossil Botany.

7TH TRIENNIAL PERIOD, 1874-77.—Dr RAMSAY H. TRAQUAIR, for his paper "on the Structure and Affinities of *Tristichopterus alatus* (Egerton)," published in the Transactions of the Society, and also for his contributions to the Knowledge of the Structure of Recent and Fossil Fishes.

8TH TRIENNIAL PERIOD, 1877-80.—JOHN MURRAY, Esq., for his paper "on the Structure and Origin of Coral Reefs and Islands," published (in abstract) in the Proceedings of the Society.

9TH TRIENNIAL PERIOD, 1880-83.—Professor HERDMAN, for his papers "on the Tunicata," published in the Proceedings and Transactions of the Society.

10TH TRIENNIAL PERIOD, 1883-86.—B. N. PEACH, Esq., for his Contributions to the Geology and Palæontology of Scotland, published in the Transactions of the Society.

11TH TRIENNIAL PERIOD, 1886-89.—ROBERT KIDSTON, Esq., for his Researches in Fossil Botany, published in the Transactions of the Society.

12TH TRIENNIAL PERIOD, 1889-92.—JOHN HORNE, Esq., F.G.S., for his Investigations into the Geological Structure and Petrology of the North-West Highlands.

13TH TRIENNIAL PERIOD, 1892-95.—ROBERT IRVINE, Esq., for his papers on the Action of Organisms in the Secretion of Carbonate of Lime and Silica, and on the solution of these substances in Organic Juices. These are printed in the Society's Transactions and Proceedings.

14TH TRIENNIAL PERIOD, 1895-98.—Professor COSSAR EWART, for his recent Investigations connected with Telegony.

15TH TRIENNIAL PERIOD, 1898-1901.—Dr JOHN S. FLETT, for his papers entitled "The Old Red Sandstone of the Orkneys" and "The Trap Dykes of the Orkneys," printed in vol. xxxix of the Transactions of the Society.

16TH TRIENNIAL PERIOD, 1901-04.—Professor J. GRAHAM KERR, M.A., for his Researches on *Lepidosiren paradoxa*, published in the Philosophical Transactions of the Royal Society, London.

17TH TRIENNIAL PERIOD, 1904-07.—FRANK J. COLE, B.Sc., for his paper entitled "A Monograph on the General Morphology of the Myxinoid Fishes, based on a Study of Myxine," published in the Transactions of the Society, regard being also paid to Mr Cole's other valuable contributions to the Anatomy and Morphology of Fishes.

1ST BIENNIAL PERIOD, 1907-09.—FRANCIS J. LEWIS, M.Sc., F.L.S., for his papers in the Society's Transactions "On the Plant Remains of the Scottish Peat Mosses."

2ND BIENNIAL PERIOD, 1909-11.—JAMES MURRAY, Esq., for his paper on "Scottish Rotifers collected by the Lake Survey (Supplement)," and other papers on the "Rotifera" and "Tardigrada," which appeared in the Transactions of the Society—(this Prize was awarded after consideration of the papers received within the five years prior to the time of award: see Neill Prize Regulations).

3RD BIENNIAL PERIOD, 1911-13.—Dr W. S. BRUCE, in recognition of the scientific results of his Arctic and Antarctic explorations.

4TH BIENNIAL PERIOD, 1913-15.—ROBERT CAMPBELL, D.Sc., for his paper on "The Upper Cambrian Rocks at Craigeven Bay, Stonehaven," and "Downtonian and Old Red Sandstone Rocks of Kincardineshire," published in the Transactions of the Society.

5TH BIENNIAL PERIOD, 1915-17.—W. H. LANG, F.R.S., M.B., D.Sc., for his paper in conjunction with Dr R. KIDSTON, F.R.S., on *Rhynia Gwynne-Vaughani*, Kidston and Lang, published in the Transactions of the Society, and for his previous investigations on Pteridophytes and Cycads.

6TH BIENNIAL PERIOD, 1917-19.—JOHN TAIT, D.Sc., M.D., for his work on Crustacea, published in the Proceedings of the Society, and for his papers on the blood of Crustacea.

7TH BIENNIAL PERIOD, 1919-21.—Sir EDWARD A. SHARPEY-SCHAFER, F.R.S., for his recent contributions to our knowledge of Physiology, and in recognition of his published work extending over a period of fifty years.

8TH BIENNIAL PERIOD, 1921-23.—JOHN M'LEAN THOMPSON, M.A., D.Sc., University of Liverpool, for his series of Memoirs on Staminal Zygomorphy, and on the Anatomy of the Filicales.

9TH BIENNIAL PERIOD, 1923-25.—FREDERICK ORPEN BOWER, F.R.S., for his recent contributions to Botanical knowledge and in recognition of his published work extending over a period of forty-five years.

10TH BIENNIAL PERIOD, 1925-27.—ARTHUR ROBINSON, M.D., M.R.C.S., for his contributions to Comparative Anatomy and Embryology.

11TH BIENNIAL PERIOD, 1927-29.—Professor ED. BATTERSBY BAILEY, M.C., F.R.S., in recognition of his valuable contributions to the Geology of Scotland, two of which have recently appeared in the Transactions of the Society.

12TH BIENNIAL PERIOD, 1929-31.—Dr CHARLES HENRY O'DONOGHUE, for his papers on the "Blood Vascular System," and for his earlier work on the "Morphology of the *corpus luteum*."

IV. GUNNING VICTORIA JUBILEE PRIZE.

1ST TRIENNIAL PERIOD, 1884-87.—Sir WILLIAM THOMSON, Pres. R.S.E., F.R.S., for a remarkable series of papers "on Hydrokinetics," especially on Waves and Vortices which have been communicated to the Society.

2ND TRIENNIAL PERIOD, 1887-90.—Professor P. G. TAIT, Sec. R.S.E., for his work in connection with the "Challenger" Expedition, and his other Researches in Physical Science.

3RD TRIENNIAL PERIOD, 1890-93.—ALEXANDER BUCHAN, Esq., LL.D., for his varied, extensive, and extremely important Contributions to Meteorology, many of which have appeared in the Society's publications.

4TH TRIENNIAL PERIOD, 1893-96.—JOHN AITKEN, Esq., for his brilliant Investigations in Physics, especially in connection with the Formation and Condensation of Aqueous Vapour.

1ST QUADRENNIAL PERIOD, 1896-1900.—Dr T. D. ANDERSON, for his discoveries of New and Variable Stars.

2ND QUADRENNIAL PERIOD, 1900-04.—Sir JAMES DEWAR, LL.D., D.C.L., F.R.S., etc., for his researches on the Liquefaction of Gases, extending over the last quarter of a century, and on the Chemical and Physical Properties of Substances at Low Temperatures: his earliest papers being published in the Transactions and Proceedings of the Society.

3RD QUADRENNIAL PERIOD, 1904-08.—Professor GEORGE CHRYSTAL, M.A., LL.D., for a series of papers on "Seiches," including "The Hydrodynamical Theory and Experimental Investigations of the Seiche Phenomena of Certain Scottish Lakes."

4TH QUADRENNIAL PERIOD, 1908-12.—Professor J. NORMAN COLLIE, Ph.D., F.R.S., for his distinguished contributions to Chemistry, Organic and Inorganic, during twenty-seven years, including his work upon Neon and other rare gases. Professor Collie's early papers were contributed to the Transactions of the Society.

5TH QUADRENNIAL PERIOD, 1912-16.—Sir THOS. MUIR, C.M.G., LL.D., F.R.S., for his series of Memoirs upon "The Theory and History of Determinants and Allied Forms," published in the Transactions and Proceedings of the Society between the years 1872 and 1915.

6TH QUADRENNIAL PERIOD, 1916-20.—C. T. R. WILSON, Esq., F.R.S., in recognition of his important discoveries in relation to Condensation Nuclei, Ionisation of Gases, and Atmospheric Electricity.

7TH QUADRENNIAL PERIOD, 1920-24.—Sir J. J. THOMSON, O.M., F.R.S., in recognition of his great discoveries in Physical Science.

8TH QUADRENNIAL PERIOD, 1924-28.—Professor E. T. WHITTAKER, F.R.S., in recognition of his distinguished contributions to Mathematical Science, and of his promotion of Mathematical Research in Scotland.

V. JAMES SCOTT PRIZE.

1ST AWARD, 1918-22.—Professor A. N. WHITEHEAD, F.R.S., for his lecture delivered on June 5, 1922, "On the Relatedness of Nature."

2ND AWARD, 1922-27.—Sir JOSEPH LARMOR, M.A., D.Sc., LL.D., F.R.S., for his lecture delivered on July 4, 1927, on "The Grasp of Mind on Nature."

3RD AWARD, 1927-30.—Professor NIELS BOHR, for his lecture delivered on May 26, 1930, on "Philosophical Aspects of Atomic Theory."

VI. BRUCE PRIZE.

1ST AWARD 1926.—JAMES MANN WORDIE, M.A., for his Oceanographical and Geological work in both Polar Regions.

2ND AWARD, 1928.—H. U. SVERDRUP, for his contributions to the knowledge of the Meteorology, Magnetism, and Tides of the Arctic, as an outcome of his travels with the Norwegian Expedition in the "Maud" from 1918 to 1925.

3RD AWARD, 1930.—N. A. MACKINTOSH, M.Sc., A.R.C.S., for his researches into the Biology of Whales in the Waters of the Falkland Islands Dependencies

4TH AWARD. 1932.—HENRY GINO WATKINS. for important contributions to the topography of Spitsbergen, Labrador, and East Greenland, and investigation of the ice cap of Greenland.

VII. BRUCE-PRELLER LECTURE FUND.

1ST AWARD, 1931.—Professor E. T. WHITTAKER, F.R.S., for his lecture, "James Clerk Maxwell and Mechanical Descriptions of the Universe."

VIII. DAVID ANDERSON-BERRY FUND.

ABSTRACT

OF

THE ACCOUNTS

OF

THE ROYAL SOCIETY OF EDINBURGH,

SESSION—1st October 1931 to 30th September 1932.

JAMES WATT, LL.D., W.S.,
Treasurer.

I. GENERAL FUND

CHARGE.

1. Arrears of Contributions at 30th September 1931			£100 16 0
2. Contributions for present Session :—			
1. 398 Fellows at £3, 3s. each		£1253 14 0	
2. Fees of Admission and Contributions of thirty new Fellows at £6, 6s. each		189 0 0	
3. Commutations		102 18 0	
			1545 12 0
3. Extra Contributions for 1931-32 under Amended Law, No. 6 :—			
1. Voluntary Contributions		£45 2 0	
2. Commutations		21 0 0	
			66 2 0
4. Royalties on Sang's Logarithms			1 4 0
5. Interest received—			
a. Interest on £445, 10s. 5% War Loan, 1929-47 (R. M. Smith Legacy), Untaxed		£22 5 6	
b. Interest on £751, 16s. 5% War Loan, 1929-47 (Special Subscription Fund), Untaxed		37 11 10	
c. General—			
Interest on £7830 5% War Loan, 1929-47, Untaxed	£391 10 0		
Interest on £2100 2½% Consolidated Stock, Untaxed	52 10 0		
Interest on Deposit Receipts	6 9 4		
		450 9 4	
			510 6 8
6. Transactions and Proceedings sold			162 0 0
7. Grants—Annual Grant from Government		£600 0 0	
Grant from Royal Society, Government Publication Grant		250 0 0	
Gift by Prof. H. W. Thompson towards Library		15 4 2	
			865 4 2
8. "Polar Year" Expenses paid in year 1930-31, now refunded			66 4 6
9. Cash Bonus on Conversion of War Loan :—			
R. M. Smith Legacy		£4 9 1	
Special Subscription Fund		7 10 4	
P. G. Tait Publication Fund		15 10 0	
General Account		78 6 0	
Dr Aitken Fund		4 9 1	
			£110 4 6
Amount of the Charge			**£3427 13 10**

DISCHARGE.

1. **Taxes, Insurance, Coal and Lighting**:—			
Insurance			£23 18 3
Coal, etc., to 28th May 1932			41 19 0
Gas to 9th Sept. 1932			3 7 10
Electric Light to 14th April 1932			13 0 11
Water, 1931-32			4 4 0
Equitable Life Assurance Co., Ltd., for Superannuation Scheme		£50 5 0	
Less—Received from Assistant Secretary		16 15 0	
			38 10 0
			£120 0 0
2. **Salaries**:—			
General Secretary		£100 0 0	
Librarian and Assistant Secretary		335 0 0	
Assistant Librarian		155 0 0	
Office Keeper		151 12 0	
Treasurer's Clerk		35 0 0	
			776 12 0
3. **Expenses of Transactions and Proceedings**:—			
a. **Transactions**:—			
Neill & Co., Ltd., Printers		£259 19 9	
Hislop & Day, Ltd., Engravers		28 8 0	
The Zinco-Collotype Co.		10 0 0	
Gilmour & Dean, Printers		27 12 0	
		£325 19 9	
Less Receipts—			
1. Grants by Carnegie Trustees towards Papers, etc., of Dr Weir, Messrs Geo. Bond and S. Williams	£87 16 7		
2. Grant by Liverpool University towards Paper, etc., of Dr Tudor Jenkyn Jones	25 0 0		
		112 16 7	
		£213 3 2	
b. **Proceedings**:—			
Neill & Co., Ltd., Printers	£919 13 8		
Hislop & Day, Ltd., Engravers	54 7 10		
	£974 1 6		
Less Receipts:—			
Revenue of Publication Fund	104 19 3		
		869 2 3	
			1082 5 5
4. **Books, Periodicals, Newspapers, etc.**:—			
James Thin, Bookseller		£357 3 1	
British Association Reports		0 6 0	
Robertson & Scott, News Agents		7 8 3	
Ray Society, Subscription		1 1 0	
Berwickshire Naturalists' Club, Do.		0 10 0	
Palæontographical Society, Do.		1 1 0	
History of Science Society, Do.		1 8 9	
Edinburgh and Leith Post-Office Directory, Ltd.		0 13 6	
Institution of Civil Engineers, for Abstracts		0 10 0	
Bombay Natural History Society, for *Journal*		2 1 0	
Scientific and Learned Societies' Year Book 1932		0 8 6	
			372 11 1
Carry forward			£2351 8 6

	Brought forward		£2351 8 6
5. OTHER PAYMENTS:—			
Neill & Co., Ltd., Printers	£114 12 10		
A. Cowan & Sons, Ltd.	8 17 9		
S. Duncan & Sons, Tailors (uniform)	9 9 0		
Macvitties, Guest & Co., Ltd.	47 13 3		
Andrew H. Baird	10 13 0		
Lindsay, Jamieson & Haldane, C.A., Auditors	10 10 0		
The Window Cleaning Co., Ltd.	16 9 3		
Orrock & Son, Ltd., Bookbinders	100 10 0		
Federated Superannuation System for Universities	5 0 0		
G. Waterston & Sons, Ltd.	16 7 8		
Telephone Accounts	14 6 2		
Clarendon Press—Hume Calendar	100 0 0		
W. S. Brown & Sons, Upholsterers, etc.	8 0 6		
Cross & Bevan	5 5 0		
Gemmell & Thin	5 5 0		
Miscellaneous Accounts under £5	42 5 1		
Charwoman	64 18 6		
Petty Expenses, Postages, Carriage, etc.	45 19 6½		
Hume Statuette and Pedestal	39 10 0		
"Polar Year" Expenses	77 0 2		
A. Muirhead & Son—Painting	98 15 0		
			841 7 8½
6. ARREARS OF CONTRIBUTIONS outstanding at 30th September 1932:—			
Present Session	£78 15 0		
Previous Sessions	25 4 0		
			103 19 0
7. TRANSFERRED TO WAR LOAN SUSPENSE ACCOUNT			110 4 6
Amount of the Discharge			**£3406 19 8½**

Amount of the Charge	**£3427 13 10**
Amount of the Discharge	**3406 19 8½**
Excess of Charge transferred to Special Subscription Fund	**£20 14 1½**

SPECIAL SUBSCRIPTION FUND

To 30th September 1932.

CHARGE.

Total Subscriptions towards Fund		£1128 17 9
Less—Written off War Loan Investment		7 12 0
		£1121 5 9
Less—Transfers to General Fund to meet Deficits up to 30th September 1931	£394 5 8½	
Less—Surplus for year to 30th September 1932	20 14 1½	
		373 11 7
AMOUNT OF THE CHARGE		£747 14 2

DISCHARGE.

BALANCE OF FUND—	
£751, 16s. 5% War Loan, 1929-47 (assented stock)	£751 16 0
Due by Treasurer	14 4 10
Due by R. Grant & Sons for Sale of *Transactions* and *Proceedings*	162 0 0
	£928 0 10
Less—Due to Union Bank of Scotland, Ltd., on Current Account	180 6 8
AMOUNT OF THE DISCHARGE	£747 14 2

WAR LOAN SUSPENSE ACCOUNT

CHARGE.

Transferred from General Fund Discharge No. 7	£110 4 6

DISCHARGE.

Due by Union Bank of Scotland, Ltd., on Current Account	£110 4 6

II. KEITH FUND

Year to 30th September 1932.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1931		£77 4 0
2. Interest Received :—		
On £650 5% War Loan, 1929-47, Untaxed	£32 10 0	
On Deposit Receipts	1 18 1	
		34 8 1
3. Cash Bonus on Conversion of War Loan		6 10 0
		£118 2 1

DISCHARGE.

1. Alex. Kirkwood & Son, for Gold Medal	£20 0 0
Dr A. W. Greenwood, 1929-31 award	45 0 0
	£65 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1932	53 2 1
	£118 2 1

III. NEILL FUND

Year to 30th September 1932.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1931		£34 12 0
2. Interest Received :—		
On £300 5% War Loan, 1929-47, Untaxed	£15 0 0	
On Deposit Receipts	0 16 5	
		15 16 5
3. Cash Bonus on Conversion of War Loan		3 0 0
		£53 8 5

DISCHARGE.

1. Alex. Kirkwood & Son—Gold Medal	£20 0 0
Dr C. H. O'Donoghue, 1929-31 award	10 0 0
	£30 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1932	23 8 5
	£53 8 5

IV. MAKDOUGALL-BRISBANE FUND

Year to 30th September 1932.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd. on Deposit Receipt at 30th September 1931		£49 9 8
2 Interest Received :—		
On £400 5% War Loan, 1929-47, Untaxed	£20 0 0	
On Deposit Receipts	1 6 1	
		21 6 1
3. Cash Bonus on Conversion of War Loan		4 0 0
		£74 15 9

DISCHARGE.

Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1932	£74 15 9

V. MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND

Year to 30th September 1932.

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1931		£78 4 8
2. INTEREST RECEIVED:—		
On £250 5% War Loan, 1929-47, Untaxed	£12 10 0	
On Deposit Receipts	1 18 1	
		14 8 1
3. CASH BONUS on Conversion of War Loan		2 10 0
		£95 2 9

DISCHARGE.

BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1932	£95 2 9

VI. GUNNING VICTORIA JUBILEE PRIZE FUND

Year to 30th September 1932.

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1931		£132 15 1
2. INTEREST RECEIVED:—		
On £599, 14s. 5% War Loan, 1929-47, Untaxed	£29 19 8	
On Deposit Receipts	3 6 5	
		33 6 1
3. CASH BONUS on Conversion of War Loan		6 0 0
		£172 1 2

DISCHARGE.

BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1932	£172 1 2

VII. JAMES SCOTT PRIZE FUND

Year to 30th September 1932.

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1931		£15 7 8
2. INTEREST RECEIVED:—		
On £247, 10s. 5% War Loan, 1929-47, Untaxed	£12 7 6	
On Deposit Receipts	0 8 8	
		12 16 2
3. CASH BONUS on Conversion of War Loan		2 9 6
		£30 13 4

DISCHARGE.

BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1932	£30 13 4

VIII. PUBLICATION FUND

(Comprising Peter Guthrie Tait Memorial Fund and Dr John Aitken Fund)

Year to 30th September 1932.

CHARGE.

1. Peter Guthrie Tait Memorial Fund:—		
(*a*) Year's Interest on £1550 5% War Loan, 1929-47, Untaxed		£77 10 0
(*b*) Cash Bonus on Conversion of War Loan		15 10 0
		£93 0 0
2 Dr John Aitken Fund:—		
Year's Interest on £445, 10s. 5% War Loan, 1929-47, Untaxed	£22 5 6	
Interest on Deposit Receipts	1 16 8	
Sale of Volumes	3 7 1	
Cash Bonus on Conversion of War Loan	4 9 1	
		31 18 4
		£124 18 4

DISCHARGE.

1. Transferred to General Fund to meet Cost of Publications (see General Fund Discharge, No. 3)	£104 19 3
2. Transferred to War Loan Suspense Account (General Fund Charge, No. 9)	19 19 1
	£124 18 4

IX. DR W. S. BRUCE MEMORIAL FUND

Year to 30th September 1932.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1931		£16 0 9
2. Interest Received:—		
On £233 3½% Conversion Loan, 1961	£8 3 0	
On Deposit Receipts	0 8 4	
		8 11 4
		£24 12 1

DISCHARGE.

1. H. G. Watkins—1932 Award	£10 0 0
A. Kirkwood & Son—Medal	0 15 0
	£10 15 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1932	13 17 1
	£24 12 1

X. BRUCE-PRELLER LECTURE FUND

Year to 30th September 1932.

CHARGE.

1 Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1931		£12 8 7
2. Dividend and Interest Received:—		
On £140, 9s. Royal Bank of Scotland Stock, less Tax £5, 19s. 4d.	£17 18 2	
On Deposit Receipts	0 7 5	
		18 5 7
3. Repayment of Income Tax for two years to December 1931		12 8 10
		£43 3 0

DISCHARGE.

Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1932	£43 3 0

XI. DR DAVID ANDERSON-BERRY FUND

Year to 30th September 1932.

CHARGE.

1. Balance of Revenue due by Union Bank of Scotland, Ltd., on Deposit Receipt, at 30th September 1931		£31 17 9
2. Interest Received :—		
On £1528, 0s. 4d. Local Loans 3% Stock, *less* tax £11, 14s. 9d.	£34 1 11	
On Deposit Receipts	1 0 2	
		35 2 1
3. Income Tax repaid for 2 years to 5th April 1932		16 12 0
		£83 11 10

DISCHARGE.

Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt, at 30th September 1932	£83 11 10

STATE OF THE FUNDS BELONGING TO THE ROYAL SOCIETY OF EDINBURGH

As at 30th *September* 1932.

1 GENERAL FUND—		
1. £7830 5% War Loan, 1929-47 (assented stock)		£7830 0 0
2. £2100 2½% Consolidated Stock at 53%		1113 0 0
3. £445, 10s. 5% War Loan, 1929-47 (assented stock). Mr Robert Mackay Smith, Legacy		445 10 0
4. Arrears of Contributions, as per preceding Abstract of Accounts		103 19 0
5. Balance of Special Subscription Fund—		
£751, 16s. 5% War Loan, 1929-47 (assented stock)	£751 16 0	
Cash due by Treasurer	14 4 10	
Due by Publishers	162 0 0	
	£928 0 10	
Less—Due to Union Bank of Scotland, Ltd., on Current Account	180 6 8	
		747 14 2
6. War Loan Suspense Account—		
Due by Union Bank of Scotland, Ltd., on Current Account		110 4 6
Amount		£10,350 7 8

Exclusive of Library, Museum, Pictures, etc., and Furniture in the Society's Rooms at George Street, Edinburgh.

2. KEITH FUND—	
1. £650 5% War Loan, 1929-47 (assented stock)	£650 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	53 2 1
Amount	£703 2 1

3. NEILL FUND—	
1. £300 5% War Loan, 1929-47 (assented stock)	£300 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	23 8 5
Amount	£323 8 5

4. MAKDOUGALL-BRISBANE FUND—	
1. £400 5% War Loan, 1929-47 (assented stock)	£400 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	74 15 9
Amount	£474 15 9

5. **MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND—**

1. £250 5% War Loan, 1929–47 (assented stock)	£250	0	0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt . .	95	2	9
AMOUNT	£345	2	9

6. **GUNNING VICTORIA JUBILEE PRIZE FUND**—Instituted by Dr Gunning of Edinburgh and Rio de Janeiro—

1. £599, 14s. 5% War Loan, 1929–47 (assented stock)	£599	14	0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt . .	172	1	2
AMOUNT	£771	15	2

7. **JAMES SCOTT PRIZE FUND—**

1. £247, 10s. 5% War Loan, 1929–47 (assented stock)	£247	10	0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt . .	30	13	4
AMOUNT	£278	3	4

8. **PUBLICATION FUND—**

(COMPRISING PETER GUTHRIE TAIT MEMORIAL FUND AND DR JOHN AITKEN FUND)

1. PETER GUTHRIE TAIT MEMORIAL FUND:—						
£1550 5% War Loan, 1929–47 (assented stock)				£1550	0	0
2. DR JOHN AITKEN FUND:—						
£445, 10s. 5% War Loan, 1929–47 (assented stock) . .	£445	10	0			
Deposit Receipt with Union Bank of Scotland, Ltd. . .	71	6	1			
				516	16	1
AMOUNT				£2066	16	1

9. **DR W. S. BRUCE MEMORIAL FUND—**

1. £233 3½% Conversion Loan, at 72⅞% (cost price)	£169	15	11
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt .	13	17	1
AMOUNT	£183	13	0

10. **BRUCE-PRELLER LECTURE FUND—**

1. £140, 9s. Royal Bank of Scotland Stock, taken over at 350%	£491	11	6
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt . .	43	3	0
AMOUNT	£534	14	6

11. **DR DAVID ANDERSON-BERRY FUND—**

1. £1528, 0s. 4d. Local Loans 3% Stock at 65$\frac{7}{16}$% (cost price)	£1000	0	0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt . .	83	11	10
AMOUNT . . .	£1083	11	10

Note 1.—As previously, 5% War Stock, 1929–47, has been uniformly valued at par in the above State of Funds.

2.—Under the Will of the late Prof. Charles Piazzi Smyth and his wife, the Society will, on the expiry of certain liferents, become entitled to payment of the residue to be applied as set out in the will.

EDINBURGH, 17*th October* 1932.—We have examined the preceding Accounts of the Treasurer of the Royal Society of Edinburgh for the Session 1931–1932, and have found them to be correct. The securities for the various Investments, as noted in the foregoing Statement of Funds, have been verified by us as at 30th September 1932.

LINDSAY, JAMIESON & HALDANE, C.A.,
Auditors.

LIST OF VOLUNTARY CONTRIBUTORS who have made a Single Payment under Law VI (end of para. 3), up to 30th September 1932.

Gregory M. Mathews, Esq.,	£10	10	0
Professor E. M. Wedderburn,	10	10	0
Total,	£21	0	0

LIST OF VOLUNTARY CONTRIBUTORS under Law VI (end of para. 3), up to 30th September 1932.

Col. A. F. Appleton,	£1	1	0
Sir James Barr,	1	1	0
Sir T. Hudson Beare,	1	1	0
Frederick A. Black, Esq.,	1	1	0
Professor F. O. Bower, F.R.S.,	1	1	0
His Hon. Judge F. E. Bradley,	1	1	0
Principal O. C. Bradley,	1	1	0
Dr G. S. Brock (1931–32, 1932–33),	2	2	0
J. W. Butters, Esq.,	1	1	0
Professor E. W. W. Carlier,	1	1	0
David Carnegie, Esq.,	1	0	0
Professor E. G. Coker,	1	1	0
Dr J. Haig Ferguson,	1	1	0
Dr R. A. Fleming,	1	1	0
J. S. Ford, Esq.,	1	1	0
Professor J. Strickland Goodall,	1	1	0
Dr E. M. Horsburgh,	1	1	0
Dr W. F. Hume,	1	1	0
Professor T. J. Jehu,	1	1	0
Professor J. Graham Kerr, F.R.S.,	1	1	0
	£22	0	0
Carried forward,	£22	0	0
F. H. Lightbody, Esq.,	1	1	0
Dr A. Veitch Lothian,	1	1	0
Dr P. M'Bride,	1	1	0
James A. Macdonald, Esq.,	1	1	0
Dr George M'Gowan,	1	1	0
Sir W. Leslie MacKenzie,	1	1	0
Dr D. J. Mackintosh,	1	1	0
Principal Devendranath Mallik,	1	1	0
R. Mathieson, Esq.,	2	2	0
Dr Hugh R. Mill,	1	1	0
R. C. Mossman, Esq.,	2	2	0
Sir Thomas Muir, F.R.S.,	1	1	0
Professor Wm. Peddie,	1	1	0
Em. Professor A. G. Perkin, F.R.S.,	1	1	0
A. G. Ramage, Esq.,	1	1	0
Ralph Richardson, Esq.,	1	1	0
Professor R. A. Robertson,	1	1	0
Edward Smart, Esq.,	1	1	0
Dr H. F. Stockdale,	1	1	0
J. C. Wright, Esq.,	1	1	0
Total,	£45	2	0

Single payments,	£21	0	0
Other payments,	45	2	0
Total,	£66	2	0

THE COUNCIL OF THE SOCIETY.

24th October 1932.

PRESIDENT.

PROFESSOR SIR E. A. SHARPEY-SCHAFER, M.D., D.Sc., LL.D., F.R.S.

VICE-PRESIDENTS.

PROFESSOR J. H. ASHWORTH, D.Sc., F.R.S.
ARTHUR LOGAN TURNER, M.D., LL.D., F.R.C.S.E.
J. B. CLARK, M.A., LL.D., J.P.
PROFESSOR JAMES RITCHIE, M.A., D.Sc.
PRINCIPAL SIR THOMAS H. HOLLAND, K.C.S.I., K.C.I.E., D.L., Hon. D.Sc., LL.D., F.R.S.
THE HON. LORD SANDS, Kt., K.C., LL.D., D.D.

GENERAL SECRETARY.

PROFESSOR R. A. SAMPSON, M.A., D.Sc., LL.D., F.R.S.

SECRETARIES TO ORDINARY MEETINGS.

PROFESSOR C. G. DARWIN, M.A., F.R.S.
PROFESSOR F. A. E. CREW, M.D., D.Sc., Ph.D.

TREASURER.

JAMES WATT, W.S., LL.D.

CURATOR OF LIBRARY AND MUSEUM.

PROFESSOR D'ARCY W. THOMPSON, C.B., Hon. D.Sc., D Litt., F.R.S.

COUNCILLORS.

MURRAY MACGREGOR, M.A., D.Sc.
A. CRICHTON MITCHELL, D.Sc.
PROFESSOR P. T. HERRING, M.D., F.R.C.P.Ed.
PROFESSOR JAMES P. KENDALL, M.A., D.Sc., F.R.S.
PROFESSOR THOMAS M. MACROBERT, M.A., D.Sc.
PROFESSOR GODFREY H. THOMSON, D.Sc., Ph.D.
MALCOLM WILSON, D.Sc., A.R.C.Sc., F.L.S.
PROFESSOR E. B. BAILEY, M.C., M.A., F.R.S.
PROFESSOR J. C. BRASH, M.C., M.A., M.D.
PROFESSOR A. J. CLARK, M.C., B.A., M.D., F.R.S.
PROFESSOR A. G. OGILVIE, M.A., B.Sc.
PROFESSOR E. M. WEDDERBURN, M.A., D.Sc., LL.B., W.S.

OFFICE STAFF.

Assistant Secretary and Librarian, G. A. STEWART.
Assistant Librarian, R. J. B. MUNRO.
Housekeeper, SAMUEL HEDDLE.

ALPHABETICAL LIST OF THE ORDINARY FELLOWS OF THE SOCIETY,

Corrected to 24th October 1932.

N. B.—*Those marked* * *are Annual Contributors.*
,, ,, † *have commuted Voluntary Contribution* (*see 3rd Paragraph, Law VI*).

M-B. prefixed to a name indicates that the Fellow has received a Makdougall-Brisbane Medal.
K. ,, ,, ,, ,, Keith Medal.
N. ,, ,, ,, ,, Neill Medal.
V. J. ,, ,, ,, ,, the Gunning Victoria Jubilee Prize.
B. ,, ,, ,, ,, Bruce Medal.
B-P. ,, ,, ,, ,, Bruce-Preller Lectureship.
C. ,, ,, ,, has contributed one or more Communications to the Society's TRANSACTIONS or PROCEEDINGS.

Date of Election.			Service on Council, etc.
1922		* Abernethy, Charles Lawrence, M.A. (Hons.), B.Sc., Research Physicist, "Exnaböe," Craiglockhart Avenue, Slateford, Edinburgh 11	
1925	C.	* Aitken, Alexander Craig, M.A., D.Sc., Lecturer in Mathematics in the University of Edinburgh. Mathematical Institute, 16 Chambers Street, Edinburgh 1	
1889		† Alison, John, M.A., LL.D., formerly Head Master, George Watson's College. 126 Craiglea Drive, Edinburgh 10	
1927	C.	* Allan, Douglas Alexander, Ph.D. (Edin.), D.Sc., Director, City of Liverpool Public Museums, William Brown Street, Liverpool	
1920	C. M-B.	* Allen, Herbert Stanley, M.A. (Cambridge), D.Sc. (London), F.R.S., Professor of Natural Philosophy in the University of St Andrews	1921–24.
1920	C.	* Anderson, Ernest Masson, M.A., B.Sc., F.G.S., 50 Greenbank Crescent, Edinburgh	
1905		Anderson, William, M.A., Head Science Master, George Watson's College, Edinburgh. 6 Lockharton Crescent, Edinburgh 11	
1905		Andrew, George, M.A., B.A., H.M.I.S., Royal Technical College, George Street, Glasgow. Hamewith, Kilmacolm, Renfrewshire	
1881	C.	Anglin, Arthur H., M.A., LL.D., M.R.I.A., formerly Professor of Mathematics, Queen's College, Cork	
1930		* Annan, William, M.A., C.A., F.C.W.A., Professor of Accounting and Business Method in the University of Edinburgh (South Bridge). Tofthill, Ferry Road West, Edinburgh 5	
1915		Anthony, Charles, M.Inst.C.E., M. Am.Soc.C.E., F.R.San.I., F.R.Met.S., F.R.A.S., F.C.S., c/o Royal Society, 22 George Street, Edinburgh 2	
1906		Appleton, Colonel Arthur Frederick, F.R.C.V.S., Nutwell, 34 Shortlands Road, Shortlands, Kent	
1910	C.	Archibald, E. H., B.Sc., Professor of Chemistry, University of British Columbia, Vancouver, Canada	
1921		* Arthur, William, M.A., Lecturer in Mathematics in the University of Glasgow. 148 Carmunnock Road, Cathcart, Glasgow	
1911	C. K.	* Ashworth, James Hartley, D.Sc., F.R.S. (VICE-PRESIDENT), Professor of Natural History, University of Edinburgh (King's Buildings, West Mains Road). "Hillbank," Grange Loan, Edinburgh 10	1912–14, 1915–18, 1927–30. Sec. 1918–23. V-P 1923–26, 1930–
1920		* Bagnall, Richard Siddoway, Hon. D.Sc., Member of the Entomological and other Scientific Societies. c/o Macdonald, Brown & Co., 9 Charles Street, London, S.W. 1	
1920	C. N.	* Bailey, Edward Battersby, M.C., M.A., F.R.S., F.G.S., Professor of Geology in the University of Glasgow	1932–
1896	C.	† Baily, Francis Gibson, M.A., M.Inst.E.E., Professor of Electrical Engineering, Heriot-Watt College, Edinburgh. Newbury, Colinton, Midlothian	1909–12, 1920–23. V-P 1929–32.

Date of Election.			Service on Council, etc.
1931	C.	* Bain, William Alexander, Ph.D., B.Sc., Lecturer in Biophysics, Department of Physiology, University of Edinburgh. 9 Falcon Gardens, Edinburgh 10	
1931		* Baird, William Macdonald, Fellow and Past President of the Faculty of Surveyors of Scotland, F.S.A.Scot., J.P. Dalveen, Barnton Avenue, Davidson's Mains, Edinburgh 4	
1921		* Baker, Bevan Braithwaite, M.A., D.Sc., Professor of Mathematics, Royal Holloway College, Englefield Green, London	
1928	C.	* Baker, Edwin Arthur, D.Sc. (Edin.), Assistant at the Royal Observatory, Edinburgh. 17 Ladysmith Road, Edinburgh 9	
1905	C.	Balfour-Browne, William Alexander Francis, M.A., F.Z.S., F.L.S., F.E.S., Barrister-at-Law, formerly Professor of Entomology at the Imperial College of Science and Technology, South Kensington, London, S.W. 7. Winscombe Court, Winscombe, Somerset	
1923		* Bannerman, David Armitage, M.B.E., M.A. (Cantab.), F.R.G.S., Special Assistant (Ornithology) in the Department of Zoology, British Museum (Natural History), London. 7 Pembroke Gardens, Kensington, London, W. 8	
1928		Barbour, George Brown, M.A. (Edin.), M.A. (Camb.), Ph.D., F.G.S., c/o Dr R. L. Dickinson, 438 West 116th Street, New York City, N.Y., U.S.A.	
1886		Barclay, A. J. Gunion, M.A., 3 Chandos Avenue, Oakleigh Park, London, N.	
1908		Bardswell, Noël Dean, M.V.O., M.D., M.R.C.P. (Ed. and Lond.), New County Hall, Westminster Bridge Road, London, S.E. 1	
1922		* Barger, George, M.A., D.Sc., Dr *h. c.* (Padua), Hon. D.Sc. (Liverp.), Hon. M.D. (Heidelberg), F.R.S., Hon. Mem. Nederl. Chem. Vereen.; Corr. Mem. Bayerische Akad. d. Wissensch. München and Ges. d. Wissensch. Göttingen, Professor of Chemistry (Medical) in the University of Edinburgh (Teviot Place). 48 St Alban's Road, Edinburgh 9	1925-28.
1929		* Barker, Sydney George, Ph.D., D.I.C., F.Inst.P., Director of Research, British Research Association for the Woollen and Worsted Industries. Torridon, Headingley, Leeds	
1914	C.	* Barkla, Charles Glover, M.A., D.Sc., F.R.S., Professor of Natural Philosophy in the University of Edinburgh (Drummond Street), Nobel Laureate, Physics, 1917. The Hermitage of Braid, Edinburgh	1915-18, 1924-27.
1925		* Barlow, Thomas William Naylor, O.B.E., M.R.C.S., D.P.H., Barrister-at-Law, Past President of the Incorporated Society of Medical Officers of Health. 23 North Drive, New Brighton, Cheshire	
1927		* Barnett, John, F.F.A., C.A., Scottish Widows' Fund Life Assurance Society, 9 St Andrew Square, Edinburgh 2	
1904		Barr, Sir James, C.B.E., M.D., LL.D., F.R.C.P., Hindhead Brae, Hindhead, Surrey	
1921		* Bartholomew, John, M.C., M.A., F.R.G.S., Geographical Institute, Duncan Street, Edinburgh. Nairne Lodge, Duddingston	1925-28.
1932	C.	* Barton-Wright, Eustace Cecil, B.Sc., M.Sc. (Lond.), Chief Assist., Scottish Society for Research in Plant Breeding. 30 Hillview Road, Corstorphine, Edinburgh 12	
1927		* Bastow, Stephen Everard, M.Inst.E.E., M.Inst.Min.E., Managing Director, Bruce Peebles&Co., Ltd., Edinburgh. Northwood, Russell Place, Trinity, Edinburgh 5	
1929		* Bath, Frederick, B.Sc., Ph.D., Lecturer in Mathematics, University of St Andrews, Assistant to the Professor of Mathematics, University College, Dundee	
1918		† Beard, Joseph, F.R.C.S. (Edin.), M.R.C.S., L.R.C.P., D.P.H. (Camb.), formerly Medical Officer of Health and School Medical Officer, City of Carlisle. 8 Carlton Gardens, Carlisle	
1888		Beare, Sir Thomas Hudson, Kt., B.A., B.Sc., M.Inst.C.E., J.P., D.L., Professor of Engineering in the University of Edinburgh (Sanderson Engineering Laboratory, Mayfield Road). 10 Regent Terrace, Edinburgh 7	1907-09. V-P 1909-15, 1923-26.
1897	C.	Beattie, Sir John Carruthers, K.B., D.Sc., LL.D., Vice-Chancellor and Principal, The University, Cape Town	
1898	C. M-B.	Becker, Ludwig, Ph.D., Regius Professor of Astronomy in the University of Glasgow. The Observatory, Dowanhill, Glasgow	
1916		* Bell, Robert John Tainsh, M.A., D.Sc., LL.D. (Glas.), Professor of Mathematics in the University of Otago, Dunedin, New Zealand	
1929		* Bennet, George, B.Sc., A.M.I.Mech.E., Lecturer in Mechanical Engineering, 68 Arden Street, Edinburgh 10	
1893	C.	Berry, Sir George A., M.B., C.M., LL.D., F.R.C.S.E., King's Knoll, North Berwick	1916-19. V-P 1919-22.
1897	C.	Berry, Richard J. A., M.D., F.R.C.S.E., Director of Medical Services, Stoke Park Colony, Stapleton, Bristol. Rufford, Canford Lane, Westbury-on-Trym, Bristol	

Date of Election.			Service on Council, etc.
1932		* Bhatia, Sohan Lal, M.C., M.A., M.D. (Camb.), M.R.C.P., Major, Indian Medical Service, Professor of Physiology and Dean, Grant Medical College, Bombay. Two Gables, Mount Pleasant Road, Malabar Hill, Bombay, India	
1880	C.	Birch, de Burgh, C.B., M.D., Em. Professor of Physiology in the University of Leeds	
1907		Black, Frederick Alexander, T.D., Solicitor, 57 Academy Street, Inverness	
1931	C.	* Black, Thomas Purves, M.A. (Edin.), B.Sc. (Lond.), Ph.D. (Edin.), Head of Department of Mathematics, Trinity Academy, Leith. 32 Eastfield, Joppa 6	
1918		* Blight, Francis James, formerly Chairman and Managing Director of Charles Griffin & Co., Ltd., Publishers. Bolstone Tor, 37 Parkside, Mill Hill, London, N.W. 7	
1894		Bolton, Herbert, D.Sc., F.G.S., F.Z.S., formerly Director of the Bristol Museum and Art Gallery, Bristol. 318 Tilehurst Road, Reading, Berks	
1915		* Boon, Alfred Archibald, D.Sc., B.A., F.I.C., Em. Professor of Chemistry, Heriot-Watt College, Edinburgh	
1925		* Borthwick, Albert William, O.B.E., D.Sc., Professor of Forestry in the University of Aberdeen	
1925		* Bose, Sahay Ram, M.A., D.Sc., F.L.S., Professor of Botany in the Carmichael Medical College Belgachia, Calcutta, India	
1886	C. N.	Bower, Frederick Orpen, M.A., D.Sc., LL.D., F.R.S., F.L.S., Em. Regius Professor of Botany in the University of Glasgow. 2 The Crescent, Ripon, Yorks	1887-90, 1893-96, 1907-09, 1917-19. V-P 1910-16. P 1919-24.
1924		* Bowman, Alexander, D.Sc., Scientific Superintendent, Marine Laboratory, Fishery Board for Scotland, Torry, Aberdeen	
1916		Bradley, His Honour Judge (Francis Ernest), M.A., M.Com., LL.D., Member of the Court of Governors of Manchester University. 8 Balmoral Road, St Annes-on-the-Sea	
1903	C.	Bradley, O. Charnock, M.D., D.Sc., Principal, Royal (Dick) Veterinary College, Edinburgh 9	1907-10, 1915-17.
1926		* Braid, Kenneth William, M.A. (Cantab.), B.Sc., Professor of Botany, West of Scotland Agricultural College. 6 Blythswood Square, Glasgow	
1907		Bramwell, Edwin, M.D., F.R.C.P.E., F.R.C.P., Professor of Clinical Medicine in the University of Edinburgh (Royal Infirmary). 23 Drumsheugh Gardens, Edinburgh 3	
1932		* Brash, James Couper, M.C., M.A., M.B., Ch.B. (Edin.), M.D. (Birm.), Professor of Anatomy in the University of Edinburgh (Teviot Place)	1932-
1918		* Bremner, Alexander, M.A., D.Sc., Headmaster, Demonstration School, Training Centre, Aberdeen. 13 Belgrave Terrace, Aberdeen	
1916	C.	* Briggs, Henry, O.B.E., D.Sc., Ph.D., A.R.S.M., James A. Hood Professor of Mining in the University of Edinburgh (79 Grassmarket, Edinburgh 1). 12 Gordon Terrace, Edinburgh 9	1923-26.
1895		Bright, Sir Charles, M.Inst.C.E., M.Inst.E.E., F.R.Aë.S., F.Inst.Radio.E., F.R.A.S., F.R.G.S., Little Brewers', Hatfield Heath, Harlow, Essex, and Athenæum Club, Pall Mall, London, S.W.	
1893		Brock, G. Sandison, M.D., F.R.C.P.E., 53 Cheniston Gardens, Kensington, London, W. 8	
1901	C.	Brodie, W. Brodie, M.D., Camden House, Bletchingley, Surrey	
1907		Brown, Alexander, M.A., B.Sc., Professor of Applied Mathematics, The University, Cape Town	
1928		* Brown, Hugh Wylie, F.I.A., F.F.A., 1 Cobden Crescent, Edinburgh 9	
1885	C.	Brown, J. Macdonald, M.D., F.R.C.S., Oriental Club, Hanover Sq., London, W. 1	
1924		* Brown, Thomas Arnold, M.A., B.Sc., Professor of Mathematics in University College, Exeter	
1923		* Brown, Walter, M.A., B.Sc., Professor of Mathematics, The University, Hong Kong, China	
1921		* Bruce, Alexander, B.Sc. (Edin.), Government Agricultural Chemist and City Analyst, The Laboratory, Turret Road S., Colombo, Ceylon	
1912		* Bruce, Alexander Ninian, D.Sc., M.D., 8 Ainslie Place, Edinburgh 3	
1927		* Bryce, David Lawrence. Vice-President (1918-1925), Quekett Microscopical Club. Ascension House, King's Highway, Plumstead, London, S.E. 18	
1898	C. K.	† Bryce, Thomas Hastie, M.A., M.D. (Edin.), F.R.S., Professor of Anatomy in the University of Glasgow. 2 The College, Glasgow	1911-14, 1922-25. V-P 1925-28.

Date of Election.			Service on Council, etc.
1887		† Burnet, Sir John James, R.A., R.S.A., LL.D., Architect, Killermont, Rowledge, Farnham, Surrey	
1888		Burns, Rev. Thomas, C.B.E., D.D., J.P., F.S.A.Scot., Minister of Lady Glenorchy's Parish Church. Croston Lodge, Chalmers Crescent, Edinburgh 9	
1917		* Burnside, George Barnhill, M.I.Mech.E., Fairhill, Dullatur	
1930	C.	* Burt, David Raitt Robertson, B.Sc. (St Andrews), F.L.S., Lecturer in Zoology, Ceylon University College, Colombo	
1896		Butters, John W., M.A., B.Sc., formerly Rector of Ardrossan Academy. 116 Comiston Drive, Edinburgh 10	
1887	C.	Cadell, Henry Moubray, of Grange, B.Sc., M.Inst.M.E., LL.D. (Edin.), D.L., Linlithgow	1919–22.
1929	C.	* Calder, Alexander, B.Sc., Ph.D., Assistant in the Institute of Animal Genetics, The University (King's Buildings, West Mains Road), Edinburgh. c/o Watson, 4 Salisbury Place, Edinburgh 9	
1910		* Calderwood, Rev. Robert Sibbald, D.D., Minister of Cambuslang, The Old Manse, Cambuslang, Lanarkshire	
1893	C.	Calderwood, W. L., I.S.O., formerly Inspector of Salmon Fisheries of Scotland. New Club, Princes Street, Edinburgh.	1923–26.
1926	C.	* Cameron, Alfred E. Henderson, M.A., D.Sc. (Aberd.), M.Sc. (Vict.), Lecturer in Entomology, Zoology Department, The University (King's Buildings), Edinburgh. 8 West Savile Road, Edinburgh 9	
1905	C.	Cameron, John, M.D., D.Sc., M.R.C.S., formerly Professor of Anatomy, Dalhousie University, Halifax, Nova Scotia. Wingfield, Grosvenor Gardens, Golders Green, London, N.W. 11	
1921		* Campbell, Andrew, Advisory Chemist, Burmah Oil Co., Ltd., and Anglo-Persian Oil Co., Ltd. 15 The Avenue, Beckenham, Kent	
1918		* Campbell, John Menzies, D.D.S. (Toronto), L.D.S. (Glas.), L.D.S. (Ontario), F.I.C.D., 14 Buckingham Terrace, Glasgow, W.	
1915	C. N.	* Campbell, Robert, M.A., D.Sc., F.G.S., Lecturer in Petrology, University of Edinburgh (Grant Institute of Geology, King's Buildings, West Mains Road) Maryton, Colinton	1920–23.
1927	C.	* Cannon, Herbert Graham, M.A., Sc.D. (Cantab.), D.Sc. (Lond.), F.L.S., Beyer Professor of Zoology in the University of Manchester	
1899	C.	Carlier, Edmund W. W., B. ès Sc. (France), M.Sc., M.D., F.E.S., Em. Professor of Physiology, University, Birmingham. Morningside, Dorridge, near Birmingham	
1910		Carnegie, Col. David, C.B.E., M.Inst.C.E., J.P., The Haven, Seasalter, Whitstable	
1931		* Carroll, John Anthony, M.A., Ph.D. (Camb.), Professor of Natural Philosophy, University of Aberdeen. Marischal College, Aberdeen	
1920	C.	* Carruthers, R. G., F.G.S., District Geologist, H.M. Geological Survey, High Barn, Stocksfield-on-Tyne	
1905	C.	Carse, George Alexander, M.A., D.Sc., Reader in Natural Philosophy, University of Edinburgh (Drummond Street). 3 Middleby Street, Edinburgh 9	
1901		Carslaw, Horatio Scott, M.A., Sc.D. (Camb.), D.Sc., LL.D. (Glasg.), Fellow of Emmanuel College, Cambridge, Professor of Mathematics in the University of Sydney, New South Wales	
1925		* Carter, George Stuart, M.A., Ph.D., Corpus Christi College, Cambridge	
1898		Carus-Wilson, Cecil, J.P., F.G.S., F.R.G.S., Altmore, Waldegrave Park, Strawberry Hill. Twickenham, and Sandacres Lodge, Parkstone, Dorset	
1932		* Cathcart, Edward Provan, C.B.E., M.D., D.Sc., LL.D., F.R.S., Professor of Physiology in the University of Glasgow. 28 Hillhead Street, Glasgow	
1899		Chatham, James, Actuary, Ladieside, Melrose.	
1912		Chaudhuri, Banawari Lal, B.A. (Cal.), D.Sc. (Edin.), formerly Superintendent, Natural History Section, Indian Museum. Sherpurtown P.O., India	
1932	C.	* Childe, Vere Gordon, B.A., B.Litt., F.R.A.I., F.S.A., Professor of Prehistoric Archæology, in the University of Edinburgh (14 Chambers Street)	
1932		Christie, James Roberton, K.C., M.A., LL.B. (Glas.), LL.B. (Edin.), Clerk of Justiciary in Scotland. (*Died August* 21, 1932)	
1925	C.	* Chumley, James, M.A., Ph.D., Lecturer on Oceanography, Department of Zoology, University of Glasgow. Thalassa, Thorn Drive, Bearsden, Dumbartonshire	
1928	C.	* Clark, Alfred Joseph, M.C., B.A., M.D., F.R.S., Professor of Materia Medica in the University of Edinburgh (Teviot Place). 67 Braid Avenue, Edinburgh 10	1932–
1891		Clark, John Brown, M.A., LL.D., J.P. (VICE PRESIDENT), formerly Head Master of George Heriot's School. Garleffin, 146 Craiglea Drive, Edinburgh 10	1928–1931. V.P 1931–
1932		* Clark, Sir Thomas, Bart., Publisher, Head of T. & T. Clark, Ltd. 6 Wester Coates Road, Edinburgh 12	

Date of Election.			Service on Council, etc.
1911		* Clark, William Inglis, D.Sc., 1 Belgrave Crescent, Edinburgh 4	
1903		Clarke, William Eagle, I.S.O., LL.D., F.L.S., Honorary Supervisor of the Bird Collection and formerly Keeper of the Natural History Collections in the Royal Scottish Museum, Edinburgh. c/o Royal Scottish Museum, Edinburgh 1	
1909		Clayton, Thomas Morrison, M.D., D.Hy., B.Sc., D.P.H., Medical Officer of Health, Health Department, Greenesfield House, Gateshead-on-Tyne	
1922		* Clerk, Sir Dugald, K.B.E., D.Sc., LL.D., F.R.S., M.Inst.C.E., etc., Lukyns, Ewhurst, Surrey	
1932		* Clouston, David, M.A., B.Sc. (Agric.), D.Sc., C.I.E., formerly Agricultural Adviser to the Government of India, and Director of the Imperial Agricultural Research Institute, Pusa. Forthview, Boswall Road, Edinburgh 5	
1904	C.	Coker, Ernest George, M.A. (Cantab.), D.Sc. (Edin.), Hon. D.Sc. (Sydney and Louvain), M.Sc. (M'Gill), F.R.S., M.Inst.C.E., M.I.Mech.E., Kennedy Professor of Civil and Mechanical Engineering, and Director of the Engineering Laboratories, University of London, University College, Gower Street, London, W.C. 1	
1904		Coles, Alfred Charles, M.D., D.Sc., York House, Poole Road, Bournemouth, W.	
1888	V. J. C.	Collie, John Norman, Ph.D., D.Sc., LL.D., F.R.S., F.C.S., F.I.C., F.R.G.S., Em. Professor of Organic Chemistry in the University College, Gower Street, London. 20 Gower Street, London, W.C. 1	
1909	C.	* Comrie, Peter, M.A., B.Sc., LL.D., Rector, Leith Academy. 19 Craighouse Terrace, Edinburgh 10	
1924	C.	* Copson, Edward Thomas, M.A., D.Sc., Lecturer in Applied Mathematics in the University of St Andrews. Mayfield, St Andrews	
1929		* Coull, George, D.Sc., Pharmaceutical Chemist, Smith's Place House, Leith 6	
1928		* Coutie, Rev. Alexander, B.Sc., Ph.D., South Manse, Fraserburgh, Aberdeenshire	
1914		* Coutts, William Barron, M.A., B.Sc., Senior Lecturer in Range Finding and Optics, Military College of Science, Red Barracks, Woolwich, S.E. 18. 11 Coleraine Road, Blackheath, S.E. 3	
1911		* Cowan, Alexander, Papermaker, Valleyfield, Penicuik, Midlothian	
1931		* Cowan, John Macqueen, M.A., D.Sc. (Edin.), B.A. (Oxon.), F.L.S., Assistant Keeper, Royal Botanic Garden, Edinburgh. 17 Inverleith Place, Edinburgh 4	
1920		Craib, William Grant, M.A. (Aberdeen), Regius Professor of Botany in the University of Aberdeen	
1916	C.	† Craig, E. H. Cunningham, B.A. (Cambridge), Geologist and Mining Engineer, The Dutch House, Beaconsfield	
1903		Craig, James Ireland, M.A., B.A., Woolwich House, The Drive, Sydenham, London, S.E. 26. (At present—Turf Club, Cairo)	
1925	C. K.	* Craig, Robert Meldrum, M.A., D.Sc., F.G.S., Lecturer in Economic Geology in the University of Edinburgh (Grant Institute of Geology, King's Buildings, West Mains Road)	
1903		Crawford, Lawrence, M.A., D.Sc., Professor of Pure Mathematics, The University, Cape Town	
1922	C.	* Crew, Francis Albert Eley, M.D., D.Sc., Ph.D. (SECRETARY TO ORDINARY MEETINGS), Professor of Animal Genetics in the University of Edinburgh, and Director of the Institute of Animal Genetics (King's Buildings, West Mains Road). 10 Salisbury Road, Edinburgh 9	1928-31. Sec. 1931-
1931		* Crichton, John, M.A., B.Sc. (Edin.), Assistant Superintendent, Meteorological Office, Edinburgh. 10 Laverockbank Road, Edinburgh 5	
1870		Crichton-Browne, Sir Jas., Kt., M.D., LL.D., D.Sc., F.R.S., Vice-President and Treasurer of the Royal Institution of Great Britain. 45 Hans Place, London, S.W. 1	
1929		* Cruickshank, Ernest William Henderson, M.D., D.Sc., Ph.D., Professor of Physiology, Dalhousie University, Halifax, Nova Scotia	
1914		* Cumming, Alexander Charles, O.B.E., D.Sc., Roselands, Crescent Road, Blundell Sands, Liverpool	
1928		* Cumming, William Murdoch, D.Sc. (Glasg.), F.I.C., M.Inst.Chem.E., Senior Lecturer on Organic Chemistry, Royal Technical College, Glasgow. "Bonnieblink," 4 Newlands Road, Newlands, Glasgow	
1917		* Cunningham, Brysson, D.Sc., B.E., M.Inst.C.E., Lecturer on Waterways, Harbours, and Docks, University College, London. 141 Copers Cope Road, Beckenham, Kent	
1930		* Cunningham, John, C.I.E., B.A., M.D., Lt.-Colonel, I.M.S. (retired). 2 Murrayfield Avenue, Edinburgh 12	
1904		Cuthbertson, John, Secretary, West of Scotland Agricultural College, 6 Charles Street, Kilmarnock	

Date of Election.			Service on Council, etc.
1885		Daniell, Alfred, M.A., LL.B., D.Sc., Advocate, The Athenæum Club, Pall Mall, London	
1924		* Darwin, Charles Galton, M.A., F.R.S. (SECRETARY TO ORDINARY MEETINGS), Tait Professor of Natural Philosophy in the University of Edinburgh (Drummond Street). 4 Churchhill, Edinburgh 10	1925-28. Sec. 1928-
1932		* Davidson, Leybourne Stanley Patrick, B.A. (Camb.), M.D. (Edin.), F.R.C.P.E., Regius Professor of Medicine in the University of Aberdeen. 55 Queen's Road, Aberdeen	
1930	C.	* Davies, Lewis Merson, F.G.S., F.R.A.I., Lt.-Colonel, Royal Artillery. 8 Garscube Terrace, Murrayfield, Edinburgh 12	
1931		* Dawson, Shepherd, M.A., D.Sc., Principal Lecturer on Psychology, Training College, Glasgow. Hazel Bank, Bearsden, Dumbartonshire	
1928		Dawson, Warren Royal, F.R.S.L., Honorary Librarian of Lloyd's. 28 Grange Road, Barnes, London, S.W. 13	
1917		* Day, T. Cuthbert, F.C.S., Partner of the firm of Hislop & Day. 36 Hillside Crescent, Edinburgh 7	
1923		* Deane, Arthur, M.R.I.A., Curator, Public Art Gallery and Museum, Belfast. Threave, Saintfield Road, Newtownbreda, Belfast	
1894		† Denny, Sir Archibald, Bart., LL.D., 5 St Helen's Place, London, E.C. 4	
1925		Dey, Alexander John, Managing Director of T. & H. Smith, Ltd., Manufacturing Chemists, Blandfield Works, Edinburgh. Rothiemay, Corstorphine, Edinburgh 12	
1924		* Dinham, C. H., B.A., H.M. Geological Survey, 28 Jermyn Street, London, S.W. 1	
1885	C.	Dixon, James Main, M.A., Litt. Hum. Doctor, Professor of English, University of Southern California. University Avenue, Los Angeles, California, U.S.A.	
1923		* Dixon, Ronald Audley Martineau, of Thearne, F.G.S., F.S.A.Scot., F.R.G.S., Thearne Hall, near Beverley	
1897		Dobbie, James Bell, F.Z.S., 12 South Inverleith Avenue, Edinburgh 4	
1881	C.	Dobbin, Leonard, Ph.D., formerly Reader in Chemistry in the University of Edinburgh. Fuladam, Blackshiels, Midlothian	1904-07, 1913-16.
1918		* Dodd, Alexander Scott, B.Sc., Ph.D., F.I.C., F.C.S., City Analyst for Edinburgh. 20 Stafford Street, Edinburgh 3	
1925		* Donald, Alexander Graham, M.A., F.F.A., F.S.A.Scot., Secretary of the Scottish Provident Institution, Edinburgh. 18 Carlton Terrace, Edinburgh 7	
1905		Donaldson, Rev. Wm. Galloway, J.P., F.R.G.S., F.E.I.S., The Manse of Forfar, Forfar	
1882	C.	Dott, David B., F.I.C., Memb. Pharm. Soc., Ravenslea, Musselburgh	
1921	M-B. C.	* Dougall, John, M.A., D.Sc., Publisher's Reader, 47 Airthrey Avenue, Glasgow, W. 4	
1901 & 1918		Douglas, Carstairs Cumming, M.D., D.Sc., Professor of Medical Jurisprudence and Hygiene, Anderson's College, Glasgow. 110 South Brae Drive, Jordanhill, Glasgow	
1910		* Douglas, Loudon MacQueen, Author and Lecturer, Newpark, Mid-Calder, Midlothian	
1932		* Drennan Alexander Murray, M.D. (Edin.), F.R.C.P.E., Professor of Pathology in the University of Edinburgh (Teviot Place)	
1923	C.	* Drever James, M.A., B.Sc., D.Phil., Professor of Psychology, University of Edinburgh (South Bridge). Ivybank, Wardie Road, Edinburgh 5	1929-32.
1901		Drinkwater, Thomas W., L.R.C.P.E., L.R.C.S.E., Chemical Laboratory, Surgeons' Hall, Edinburgh	
1923		* Drummond, J. Montagu F., M.A. (Cantab.), Harrison Professor of Botany in the University of Manchester	1928-31.
1925		* Dryerre, Henry, M.R.C.S., Ph.D., Professor of Physiology, Royal (Dick) Veterinary College; Physiological Biochemist, Animal Diseases Research Association. Kenmore, Lasswade	
1921		* Drysdale, Charles Vickery, C.B., O.B.E., D.Sc. (Lond.), M.I.E.E., F.Inst.P., Director of Scientific Research and Experiment to the Admiralty, S.R.E. Department. Archway Block N., Admiralty, Whitehall, London, S.W. 1	
1904		Dunlop, William Brown, M.A., 4A St Andrew Square, Edinburgh 2	
1892	C.	Dunstan, M. J. R., M.A., F.I.C., F.C.S., formerly Principal, Royal Agricultural College, Cirencester. Windyacre, Wrotham, Kent	
1906	C.	Dyson, Sir Frank Watson, K.B.E., M.A., D.Sc., LL.D., F.R.S., Astronomer Royal, Royal Observatory, Greenwich	1907-10.
1925		* Eastwood, George Samuel, B.Sc., Principal Teacher of Mathematics, Beath Secondary School, Cowdenbeath, Fife. Craigie Lea, Cowdenbeath, Fife	
1904		Edwards, John, LL.D., 4 Great Western Terrace, Kelvinside, Glasgow	

Date of Election.			Service on Council, etc.
1931		* Eggleton, Philip, D.Sc., Lecturer in Biochemistry in the Department of Physiology, University of Edinburgh (Teviot Place). 36 Gillespie Crescent, Edinburgh 10	
1924		* Elliot, Rt. Hon. Walter Elliot, P.C., M.C., M.B., Ch B., D.Sc , LL.D., M.P., Minister of Agriculture, 14 Markham Square, Chelsea, London	
1906	C.	Ellis, David, D.Sc., Ph.D., Professor of Botany and Bacteriology, Royal Technical College, Glasgow	
1924		* Evans, Arthur Humble, M.A., Sc.D., Lecturer in English History (under Special Board), Cheviot House, Crowthorne, Berks	
1924		Evans, William Edgar, B.Sc., Assistant in charge of Herbarium, Royal Botanic Garden, Edinburgh 4	
1879	C. N.	Ewart, James Cossar, M.D., LL.D., F.R.C.S.E., F.R.S., F.Z.S., formerly Regius Professor of Natural History, University of Edinburgh. Craigybield, Penicuik, Midlothian	1882-85, 1904-07. V-P 1907-12.
1902		Ewen, John Taylor, O.B.E., B.Sc., M.I.Mech.E., J.P., H.M. Inspector of Schools (Emeritus), Pitscandly, Forfar	
1878	C.	† Ewing, Sir James Alfred, K.C.B., M.A., D.Sc., LL.D., F.R.S., Hon. Memb. Inst.C.E., J.P., formerly Principal of the University of Edinburgh and Director of Naval Education, Admiralty. 5 Herschel Road, Cambridge	1888-91, 1919-20. V-P 1920-23. P 1924-29.
1900	C.	Eyre, John W. H., M.D., M.S. (Dunelm), D.P.H. (Camb.), Professor of Bacteriology, Guy's Hospital, London, S.E. 1	
1931		* Fairbairn, William Ronald Dodds, M.A., M.D., D.Psych. (Edin.), Consultant Physician and Lecturer in Psychology, University of Edinburgh (South Bridge). 18 Lansdowne Crescent, Edinburgh 12	
1910	C.	* Fairgrieve, Mungo M'Callum, M.A. (Camb. and Glasg.), Master at the Edinburgh Academy. 37 Queen's Crescent, Edinburgh 9	
1907	C.	Falconer, John Downie, M.A., D.Sc., F.G.S., formerly Director of the Geological Survey of Nigeria. The Cedars, Hatton Road, Harlington, Hayes, Middlesex	
1923		* Feldman, William Moses, M.D., B.S., M.R.C.P. (Lond.), M.R.C.S. (Eng.), Physician. St Mary's Hospital for Women and Children, Plaistow. 851 Finchley Road, London, N.W. 11	
1928		* Fenton, Edward Wyllie, M.A., B.Sc. (Aberd.), Head of Botany Department, Edinburgh and East of Scotland College of Agriculture, 13 George Sq., Edinburgh 8	
1907		Fergus, Edward Oswald, c/o Messrs M'Kay & Boyd, Solicitors, 50 Wellington Street, Glasgow	
1904		Ferguson, James Haig, M.D., LL.D., F.R.C.P.E., F.R.C.S.E., 7 Coates Crescent, Edinburgh 3	
1925	C.	* Ferrar, William Leonard, M.A., Fellow and Tutor of Hertford College, Oxford	
1932		* Findlay, Sir John Edmund Ritchie, Bart., B.A. (Oxon.), Proprietor of the *Scotsman*. 18 Lauder Road, Edinburgh 9	
1927	C.	* Finlay, Thomas Matthew, M A., D.Sc. (Edin.), Lecturer in Palæontology in the University of Edinburgh. 11 Dudley Terrace, Leith 6	
1911		† Fleming, John Arnold, F.C.S., etc., Pottery Manufacturer, Locksley, Helensburgh, Dumbartonshire	
1906		Fleming, Robert Alexander, M.A., M.D., LL.D., F.R.C.P.E., Consulting Physician, Royal Infirmary. 10 Chester Street, Edinburgh 3	
1900	C. N.	Flett, Sir John S., K.B.E., M.A., D.Sc., LL.D., F.R.S., Director of the Geological Survey of Great Britain and of the Museum of Practical Geology, 28 Jermyn Street, London, S.W. 1	1916-19.
1872		Forbes, George, M.A., LL.D., F.R.S., M.Inst.C.E., M.Inst.E.E., F.R.A.S., formerly Professor of Natural Philosophy in Anderson's College, Glasgow. 11 Little College Street, Westminster, S.W.	
1892		Ford, John Simpson, F.I.C., 7 Corrennie Drive, Edinburgh 10	
1921		* Forrest, George Topham, F.R.I.B.A., F.G.S , F.S.Arc., Architect to the London County Council, and Superintending Architect of Metropolitan Buildings, The County Hall, Westminster Bridge, London, S.E. 1	
1928	C.	* Forrest, James, M.A., B.Sc. (Glasg.), D.Sc. (St Andrews), Lecturer in Physics, University College, Dundee. "Cumbrae," Oxford Street, Blackness, Dundee	
1920	C.	* Franklin, Thomas Bedford, B.A.(Camb.), Stancliffe Hall, near Matlock, Derbyshire	
1910		* Fraser, Alexander, Actuary, 15 S. Learmonth Gardens, Edinburgh 4	
1929		* Fraser, David Kennedy, M.A., B.Sc., Lecturer in charge of course for training of Teachers of mentally defective children under Scottish National Committee, Psychologist to Glasgow Education Authority. Edge o' the Moor, Milngavie, Dumbartonshire	

Date of Election.			Service on Council, etc.
1928		* Fraser, John, M.C., M.D., Ch.M., F.R.C.S.E., Regius Professor of Clinical Surgery in the University of Edinburgh (Royal Infirmary). 32 Moray Place, Edinburgh 3	
1915		* Fraser, Rev. Joseph Robert, The New Manse, Kinneff, Inverbervie, Montrose	
1928		* Fraser, Kenneth, M.D. (Edin.), D.P.H. (Camb.), D.T.M. (Edin.), County Medical Officer of Health, Cumberland. The Croft, Scotby, near Carlisle	
1914		* Fraser, William, Managing Director, Neill & Co., Ltd., Printers, 212 Causewayside, Edinburgh 9	
1896	C.	Fraser-Harris, David Fraser, B.Sc. (Lond.), D.Sc. (Birm.), M.D., formerly Professor of Physiology in the Dalhousie University, Halifax, Nova Scotia. Grove Park Lodge (3), Chiswick, London, W. 4	
1907		Galbraith, Alexander, "Ravenswood," Dalmuir, Glasgow	
1888	C.	Galt, Alexander, D.Sc., formerly Keeper of the Department of Technology, Royal Scottish Museum, Edinburgh. C/o Clydesdale Bank, 1 Melville Place, Edinburgh 3	
1901		Ganguli, Sanjiban, M.A., Principal, Maharaja's College, and Director of Public Instruction, Jaipur State, Jaipur, India	
1923		* Gardiner, Frederick, M.D., B.Sc., F.R.C.S.E., F.R.S.M., Lecturer in Diseases of the Skin in the University of Edinburgh; Physician to the Royal Infirmary. 35 Manor Place, Edinburgh 3	
1926		* Gardner, John Davidson, B.Sc., Assoc.M.Inst.C.E, Chief Assistant to Messrs D. & C. Stevenson, Civil Engineers, Edinburgh. 23 Ivy Terrace, Edinburgh 11	
1930	C.	* Geddes, Alexander Ebenezer M'Lean, O.B.E., M.A., D.Sc. (Aberd.), Lecturer in Natural Philosophy in the University of Aberdeen. 12 Louisville Avenue, Aberdeen	
1909	C.	* Geddes, Rt. Hon. Sir Auckland C., G.C.M.G., K.C.B., P.C., M.D., LL.D., formerly British Ambassador to the U.S.A. Frensham, The Layne, Rolvenden, Kent	
1909		* Gentle, William, B.Sc., Head Master, George Heriot's School. 10 West Savile Road, Edinburgh 9	
1920	C.	* Ghosh, Sudhamoy, M.Sc. (Cal.), D.Sc. (Edin.), F.C.S., Professor of Chemistry, School of Tropical Medicine and Hygiene, Central Avenue, Calcutta, India	
1914		* Gibb, Sir Alexander, G.B.E., C.B., M.Inst.C.E., Consulting Civil Engineer, Queen Anne's Lodge, Westminster, London, S.W. 1	
1916		* Gibb, Alfred William, M.A., D.Sc., Professor of Geology in the University of Aberdeen. 1 Belvidere Street, Aberdeen	
1910	C.	* Gibb, David, M.A., B.Sc., Lecturer in Mathematics in the University of Edinburgh (16 Chambers Street). 15 South Lauder Road, Edinburgh 9	
1917	C.	* Gibson, Alexander, M.B., Ch.B., F.R.C.S., 620 Medical Arts Buildings, Winnipeg, Canada	
1921		* Gibson, Walcot, D.Sc., F.R.S., F.G.S., formerly Assistant Director, H.M. Geological Survey (Scotland), Pathways, Fairlight Road, Hythe, Kent	
1911		Gidney, Lt.-Col. Sir Henry A. J., Kt., F.R.C.S., D.O., D.P.H., J.P., M.L.A., I.M.S. (retired), Army Specialist Public Health, c/o The Allahabad Bank, Ltd., Calcutta, India	
1925		* Gillies, William King, M.A., B.A., F.E.I.S., LL.D. (Glas.), Rector of the Royal High School, Edinburgh. Davaar, 12 Suffolk Road, Edinburgh 9	
1907		Gilruth, John Anderson, M.R.C.V.S., D.V.Sc. (Melb.), Clowes Street, South Yarra, Melbourne, Australia	
1909		* Gladstone, Hugh Steuart, M.A., M.B.O.U., F.Z.S., Capenoch, Thornhill, Dumfriesshire	
1911		Gladstone, Reginald John, M.D., F.R.C.S., Lecturer and Senior Demonstrator of Anatomy, King's College, University of London. 22 Court Lane Gardens, London, S.E. 21	
1898		Glaister, John, M.D., F.R.F.P.S. (Glas.), D.P.H. (Camb.), LL.D., formerly Regius Professor of Forensic Medicine and Public Health in the University of Glasgow. 3 Newton Place, Glasgow	
1925	C.	* Goldie, Archibald Hayman Robertson, M.A., B.A., Superintendent, Meteorological Office, Air Ministry, Edinburgh, 6 Drumsheugh Gardens, Edinburgh 3	1929-32.
1910		Goodall, Joseph Strickland, M.B. (Lond.), M.R.C.P., F.R.C.S.E., M.S.A. (Eng.), Professor of Physiology and Biology, City of London Hospital. 136 Harley Street, London, W. 1	
1901		Goodwillie, James, M.A., B.Sc., 239 Clifton Road, Aberdeen	
1913	C. M-B.	* Gordon, William Thomas, M.A., D.Sc. (Edin.), M.A. (Cantab.), Professor of Geology, University of London, King's College, Strand, W.C.	
1897		Gordon-Munn, John Gordon, M.D., Croys, Dalbeattie	
1923		* Grabham, George Walter, O.B.E., M.A. (Cantab.), F.G.S., Government Geologist, Anglo-Egyptian Sudan. Box 178, Khartoum	

Date of Election.			Service on Council, etc.
1924		Graham, Robert James Douglas, M.A., D.Sc., Lecturer in Plant Physiology in the University of Edinburgh. Royal Botanic Garden, Edinburgh 4	
1931		* Grant, Robert, Publisher (Oliver and Boyd), Edinburgh; (Gurney and Jackson) London. 6 Kilgraston Road, Edinburgh 10	
1898	C	Gray, Albert A., M.D., 5 Hammersmith Terrace, London, W. 6	
1909	C.	* Gray, James Gordon, D.Sc., Professor of Applied Physics in the University of Glasgow. 11 The University, Glasgow	1913-15.
1918		* Gray, Wm. Forbes, F.S.A.Scot., Editor and Author, 8 Mansionhouse Road, Edinburgh 9	
1927	C. K.	* Greenwood, Alan William, D.Sc. (Melb.), Ph.D. (Edin.), Lecturer in the Institute of Animal Genetics, University of Edinburgh (King's Buildings, West Mains Road)	
1922		* Greenwood, William Osborne, M.D. (Leeds), B.S. (Lond.), L.S.A., Obstetric Surgeon and Physician, Woodroyd, 19 Ripon Road, Harrogate, Yorks	
1925		* Greig, David Middleton, M.B., C.M., F.R.C.S.E., LL.D., Conservator of the Museum of the Royal College of Surgeons of Edinburgh. 12 Abbotsford Crescent, Edinburgh 10	
1906		Greig, Edward David Wilson, C.I.E., M.D., D.Sc., Lt.-Col., H.M. Indian Medical Service, 38 Coates Gardens, Edinburgh 12	
1931		* Greig, John Russell, Ph.D. (Edin.), Director, The Moredun Institute Animal Diseases Research Association. Wedderlie, Kirkbrae, Liberton 9	
1905		† Greig, Sir Robert Blyth, M.C., LL.D., Secretary to the Department of Agriculture for Scotland, York Buildings, Queen Street, Edinburgh 2	1921-24. V-P 1924-27.
1910		* Grimshaw, Percy Hall, F.E.S., Keeper, Natural History Department, The Royal Scottish Museum. 49 Lygon Road, Edinburgh 9	
1899		† Guest, Edward Graham, M.A., B.Sc., J.P., formerly City Treasurer, Edinburgh. 5 Newbattle Terrace, Edinburgh 10	
1927		* Gulland, John Masson, M.A. (Oxon.), D.Sc. (Edin.), Ph.D (St Andrews), Reader in Biochemistry, The Lister Institute of Preventive Medicine, Chelsea Bridge Road, London, S.W. 1. Reader in Biochemistry in the University of London	
1907		Gulliver, Gilbert Henry, D.Sc., A.M.I.Mech.E., 99 Southwark Street, London, S.E.	
1930		* Guthrie, Douglas, M.D., F.R.C.S., Lecturer in Diseases of the Ear, Nose, and Throat, School of Medicine of the Royal Colleges, Edinburgh. 4 Rothesay Place, Edinburgh 3	
1911		* Guy, William, F R.C.S., L.R.C.P., L.D.S.Ed., LL.D., (Penn.), Consulting Dental Surgeon, Edinburgh Royal Infirmary; Dean, Edinburgh Dental Hospital and School; Lecturer on Human and Comparative Dental Anatomy and Physiology. 11 Wemyss Place, Edinburgh 3	
1922		* Hannay, Robert Kerr, M.A., Professor of Ancient History and Palæography in the University of Edinburgh (South Bridge). Historiographer-Royal for Scotland. 5 Royal Terrace, Edinburgh 7	
1923		* Hanneford-Smith, William, Assoc.Inst.C.E., Hon. A.R.I.B.A., Member of the Institute of Metals, 3 The Avenue, Gravesend, Kent	
1918		* Hardie, Patrick Sinclair, M.A., B.Sc., Bruntsfield Private Hotel, Bruntsfield Place, Edinburgh	
1928		* Harding, William Gerald, F.R.Hist.S., F.S.A.Scot, F.E.S., Peckwater House, Charing, Kent	
1923	C.	* Harris, Robert Graham, M.A., D Sc. (Edin.), (Aeronautical) Research Physicist, Lorraine, Manor Road, Farnborough, Hants	
1914		Harrison, Edward Philip, Ph.D., F.Inst.P., Chief Scientist, H.M.S. "Vernon," Portsmouth	
1921		* Harrison, John William Heslop, D.Sc. (Durham), F.R.S., Professor of Botany, Armstrong College, Newcastle. The Avenue, Birtley, Co. Durham	
1926	C.	* Harrower, John Gordon, M.B., Ch.M. (Glas.), F.R.C.S.E., D.Sc. (Edin.), Professor of Anatomy, King Edward VII. Medical College, and Surgeon, General Hospital, Singapore	
1926		* Harvey, William Frederick, C.I.E., M.A., M.B., C.M., D.P.H., Lieut.-Col. I.M.S. (retired), Histologist, Research Laboratory, Royal College of Physicians, Edinburgh. 56 Garscube Terrace, Edinburgh 12	
1893		Hehir, Sir Patrick, K.C.I.E., C.B., C.M.G., F.R.C.P.E., F.R.C.S.E., D.P.H. (Camb.), D.T.M. (Liverpool), Maj.-General I.M.S. (retired). Ennisfarne, Westward Ho, Devon	
1900		Henderson, John, D.Sc., A.Inst.E.E., Kinnoul, Gregory's Road, Beaconsfield, Bucks	
1931		* Henderson, John, F.C.I.I., Manager and Secretary, Edinburgh Assurance Co. Ltd., Member and Past President of the Insurance Society of Edinburgh. Seaforth Cottage, York Road, Trinity, Edinburgh 5	

Date of Election.			Service on Council, etc.
1929		* Henderson, Thomas, J.P., F.S.A.Scot., Actuary of the Savings Bank of Glasgow. 5 Belmont Crescent, Glasgow, W.2	
1908		Henderson, William Dawson, M.A., B.Sc., Ph.D., Lecturer, Zoological Laboratories, University, Bristol	
1925		* Heron, Alexander Macmillan, D.Sc., Officiating Superintendent, Geological Survey of India, Calcutta, India	
1916		* Herring, Percy Theodore, M.D., F.R.C.P.E., Professor of Physiology, University of St Andrews. Linton, St Andrews	1917-20, 1931-
1922		Hindle, Edward, M.A., Sc.D. (Camb.), Ph.D., A.R.C.Sc., London School of Hygiene and Tropical Medicine. 32 Belsize Avenue, Hampstead, London, N.W. 3	
1902		Hinxman, Lionel W., B.A., formerly of the Geological Survey of Scotland. 4 Morant Gardens, Ringwood, Hants	
1904		Hobday, Frederick T. G., C.M.G., F.R.C.V.S., Principal, Royal Veterinary College, Camden Town, London, N.W. 1	
1928	C.	* Hobson, Alfred Dennis, M.A. (Camb.), Professor of Zoology, Armstrong College, Newcastle-upon-Tyne	
1928		* Hodge, William Vallance Douglas, M.A. (Edin.), M.A. (Camb.), Lecturer in Mathematics in the University of Bristol	
1885		Hodgkinson, W. R., C.B.E., M.A., Ph.D., F.I.C., F.C.S., formerly Professor of Chemistry and Physics at the Ordnance College, Woolwich. 89 Shooter's Hill Road, Blackheath, Kent	
1923	C.	* Hogben, Lancelot Thomas, M.A., D.Sc., Professor of Social Biology (London School of Economics), University of London, Houghton St., London, W.C. 2	
1927		Holden, Henry Smith, D.Sc., F.L.S., Professor of Botany, University College, Nottingham	
1930		* Holland, Sir Thomas Henry, K.C.S.I., K.C.I.E., D.L., Hon. D.Sc., LL.D., F.R.S. (Vice-President), Principal of the University of Edinburgh (South Bridge). Blackford Brae, Edinburgh	1931-32. V.P 1932-
1911		Holland, William Jacob, LL.D. (St Andrews), Director-Emeritus, Carnegie Institute, Pittsburg, Pa. 5545 Forbes Street, Pittsburg, Pa., U.S.A.	
1929		* Hora, Sunder Lal, D.Sc. (Punjab et Edin.), F.L.S., F.Z.S., F.A.S.B., Senior Assistant Superintendent, Zoological Survey of India. Indian Museum, Calcutta	
1920	C.	* Horne, Alexander Robert, O.B.E., B.Sc., M.I.Mech.E., A.M.I.C.E., Professor of Mechanical Engineering, Heriot-Watt College, Edinburgh. 31 Queen's Crescent, Edinburgh 9	
1896		Horne, J. Fletcher, M.D., F.R.C.S.E., Shelley Hall, Huddersfield	
1904	C.	Horsburgh, Ellice Martin, M.A., D.Sc., Reader in Technical Mathematics, University of Edinburgh (16 Chambers Street). 11 Granville Terrace, Edinburgh 10	
1897		Houston, Sir Alex. Cruikshanks, K.B.E., C.V.O., M.B., C.M., D.Sc., F.R.S., 20 Nottingham Place, London, W. 1	
1912	C. M-B.	* Houstoun, Robert Alexander, M.A., Ph.D., D.Sc., Lecturer in Physical Optics, University, Glasgow. 45 Kirklee Road, Glasgow	1929-32.
1893		Howden, Robert, M.A., M.B., C.M., D.Sc., LL.D., Em. Professor of Anatomy in the University of Durham. Broomfield, Crieff	
1910		Hume, William Fraser, D.Sc. (Lond.), Director, Geological Survey of Egypt, Helwân, Egypt. The Laurels, Rustington, Sussex	
1927		* Hunt, Owen Duke, B.Sc. (Manch.), "Corrofell," Newton Ferrers, South Devon	
1923		* Hunter, Rev. Adam Mitchell, M.A., D.Litt., Librarian of New College, Edinburgh. 3 Suffolk Road, Edinburgh 9	
1932		* Hunter, Andrew, M.A., B.Sc., M.D., F.R.S.C., Professor of Biochemistry in the University of Glasgow	
1928		* Hunter, Arthur, F.F.A., LL.D. (Edin.), Vice-President and Chief Actuary of the New York Life Insurance Co. 124 Lloyd Road, Montclair, N.J., U.S.A.	
1916		* Hunter, Charles Stewart, L.R.C.P.E., L.R.C.S.E., D.P.H., Cotswold, 36 Streatham Hill, London, S.W. 2	
1911		Hunter, Gilbert Macintyre, M.Inst.C.E., M.Inst.E.S., M.Inst.M.E., formerly Resident Engineer, Nitrate Railways, Iquique, Chile. Auchraig, Cramond Brig, near Edinburgh	
1887	C.	Hunter, William, C.B., M.D., M.R.C.P.L. and E., M.R.C.S., LL.D., 103 Harley Street, London	
1927		* Hyslop, James, M.A. (Glasg.), Ph.D. (Camb.), Lecturer in Mathematics in the University of Glasgow	
1908		Hyslop, Theophilus Bulkeley, M.D., M.R.C.P.E., Stretton House, Church Stretton, Shropshire	
1923	C.	* Ince, Edward Lindsay, M.A. (Cantab.), D.Sc. (Edin.), Lecturer in Mathematics in the Imperial College of Science and Technology, South Kensington. 6 Rutland Gardens, West Ealing, London, W. 13	

Date of Election.			Service on Council, etc.
1920		* Inglis, James Gall, Publisher and Editor of Educational Works, 36 Blacket Place, Edinburgh 9	
1927		* Inglis, John Alexander, of Auchindinny and Redhall, K.C., M.A. (Oxon.), LL.B. (Edin.), King's and Lord Treasurer's Remembrancer. 13 Randolph Crescent, Edinburgh 3	
1912		* Inglis, Robert John Mathieson, M.Inst.C.E., Assistant Chief Engineer, Lond.& N.E. Railway, Liverpool Street Station, London. Dixton, Hadley Common, Barnet	
1904	C.	Innes, Robert T. A., formerly Director, Government Observatory, Johannesburg, Transvaal. 91 Becker Street, Johannesburg, South Africa	
1917		* Irvine, Sir James Colquhoun, Kt., C.B.E., Ph.D. (Leipzig), D.Sc. (St Andrews), Hon. D.Sc. (Liverpool, Princeton), Hon. Sc.D. (Cambridge, Yale, Pennsylvania), Hon. LL.D. (Glasgow, Aberdeen), Hon. D.C.L. (Durham), D.L., F.R.S., Hon. Mem. American Chemical Society, Principal of the University of St Andrews	1920-22. V-P 1922-25.
1930	C.	* Jack, David, M.A., B.Sc. (Edin.), Ph.D. (St Andrews), Lecturer in Natural Philosophy in the United College, University of St Andrews. 22 Grange Road, St Andrews	
1923		* Jack, John Louttit, Solicitor, Assistant Secretary, Department of Health for Scotland, 121A Princes Street, Edinburgh 2	
1901		Jardine, Robert, M.D., M.R.C.S., F.R.F.P.S. (Glas.), Wiston Manse, Lamington	
1912	C.	* Jeffrey, George Rutherford, M.D. (Glasg.), F.R.C.P. (Edin.), Bootham Park Private Mental Hospital, York	
1906	C. K.	Jehu, Thomas John, M.A., M.D., F.G.S., Professor of Geology in the University of Edinburgh (Grant Institute of Geology, King's Buildings, West Mains Road). 35 Great King Street, Edinburgh 3	1917-20, 1923-26. V-P 1929-32.
1900		† Jordan, David Smiles, M.A., D.Sc., Ph.D., 26 Avenue du Château d'Eau, Saventhem, Belgium	
1931		* Johnson, Thomas, D.Sc. (Lond.), Emeritus Professor of Botany, Royal College of Science for Ireland. Tomeg, Hillview Drive, Corstorphine 12	
1925		* Johnston, Christopher Nicholson, The Hon. Lord Sands, Kt., K.C., LL.D., D.D., Senator of the College of Justice (VICE-PRESIDENT). 4 Heriot Row, Edinburgh 3	1929-32. V-P 1932-
1895		Johnston, Col. Henry Halcro, C.B., C.B.E., D.Sc., M.D., F.L.S., late A.M.S., Stromness Hotel, Stromness, Orkney	
1928		* Johnston-Saint, Percy Johnston, M.A. (Camb.), Secretary, Wellcome Historical Medical Museum, 54A Wigmore Street, London, W. 1. 4 Wyndham Place, Bryanston Square, London, W.	
1928		* Johnstone, Robert William, C.B.E., M.A., M.D. (Edin.), F.R.C.S.E., M.R.C.P.E., Professor of Midwifery and Diseases of Women in the University of Edinburgh. 26 Palmerston Place, Edinburgh 12	
1927		* Jones, Edward Taylor, D.Sc. (Lond.), Professor of Natural Philosophy in the University of Glasgow	1927-30.
1888		Jones, John Alfred, M.Inst.C.E., Fellow of the University of Madras, Sanitary Engineer to the Government of Madras. (*Address not known*)	
1930	C.	* Jones, Samuel Griffith, D.Sc. (Univ. Wales), Lecturer in Botany in the University of Glasgow. Broomfield, Kilmacolm, Renfrewshire	
1928	C.	* Jones, Tudor Jenkyn, M.B., Ch.B. (Glasg.), Lecturer in Anatomy in the University of Liverpool	
1922		* Juritz, Charles Frederick, M.A., D.Sc., F.I.C., Chief of the Union Department of Chemistry, Villa Marina, Three Anchor Bay, Cape Town, South Africa	
1925	C.	* Kemp, Charles Norman, B.Sc., Technical Radiologist, Secretary of the Royal Scottish Society of Arts. Ivy Lodge, Laverockbank Road, Edinburgh 5	
1929		* Kendall, James Pickering, M.A., D.Sc., F.R.S., Professor of Chemistry in the University of Edinburgh (King's Buildings, West Mains Road). 14 Mayfield Gardens, Edinburgh 9	1931-
1912		† Kennedy, Robert Foster, M.D. (Queen's Univ., Belfast), M.B., B.Ch. (R.U.I.), Assoc. Professor of Neurology, Cornell University, New York. Ninth Floor, 410 East 57th Street, near First Avenue, New York, U.S.A.	
1927		* Kennedy, Walter Phillips, B.Sc., Ph.D. (Edin.), L.R.C.P. and S.E., A.I.C., Physiology Department, University (Teviot Place), Edinburgh	
1909		Kenwood, Henry Richard, C.M.G., M.B., C.M., Chadwick Em. Professor of Hygiene in the University of London. "Wadhurst," Queen's Road, Finsbury Park, London, N.	
1925	C. M-B.	* Kermack, William Ogilvy, M.A., D.Sc., Chemist to the Research Laboratory of the Royal College of Physicians, 2 Forrest Road, Edinburgh 1	
1908		Kerr, Andrew William, F.S.A.Scot., 81 Great King Street, Edinburgh 3	

Date of Election.			Service on Council, etc.
1903 & 1923	C. N.	Kerr, John Graham, M.A., F.R.S., F.L.S., F.Z.S., Regius Professor of Zoology, University of Glasgow. 9 The University, Glasgow	1904–07, 1913–16, 1924–27. V.-P 1928–31.
1891		Kerr, Joshua Law, M.D., J.P., Beaconsfield, Tasmania	
1913		* Kerr, Walter Hume, M.A., B.Sc., formerly Lecturer on Engineering Drawing and Structural Design in the University of Edinburgh. Glenfriars, Jedburgh	
1926		* Khastgir, Satis Ranjan, M.Sc. (Calcutta), D.Sc. (Edin.), Ph.D. (Edin.), Physics Department, University, Dacca, India	
1907		King, Archibald, M.A., B.Sc., H.M. Inspector of Schools, 10 Leslie Road, Pollokshields, Glasgow	
1925		* King, Leonard Augustus Lucas, M.A., Professor of Zoology in the West of Scotland Agricultural College, Glasgow. 14 Bank Street, Glasgow, W. 2	
1918		* Kingon, Rev. John Robert Lewis, M.A., D.Sc., The Manse, Dundee, Natal Province, South Africa	
1901		Knight, Rev. G. A. Frank, M.A., D.D., F.S.A.Scot., 10 Hillhead Street, Glasgow	
1907		Knight, James, M.A., D.Sc., F.C.S., F.G.S., J.P., Rector, Queen's Park High School, Langside, Glasgow	
1927		* Lambie, Charles George, M.C., M.D., F.R.C.P.E., Bosch Professor of Medicine in the University of Sydney, Australia	
1920	C.	* Lamont, John Charles, Lieut.-Col., I.M.S. (retired), C.I.E., M.B., C.M. (Edin.), M.R.C.S., 7 Merchiston Park, Edinburgh 10	
1925	C. N.	* Lang, William Henry, M.B., C.M., D.Sc., LL.D. (Glas.), F.R.S., Barker Professor of Cryptogamic Botany in the University of Manchester	
1931		* Langrishe, John du Plessis, D.S.O., M.B., B.Ch. (Dub.), D.P.H. (R.Cs.P. and S.), Lt.-Col. R.A.M.C. (retired), Lecturer in Public Health in the University of Edinburgh (Usher Institute of Public Health, Warrender Park Road). 2 South Gillsland Road, Edinburgh 10	
1910	C.	* Lauder, Alexander, D.Sc., Head of Chemistry Department, Edinburgh and East of Scotland College of Agriculture, 13 George Square, Edinburgh 8. Lecturer in Agricultural Chemistry, University of Edinburgh	1917–20. Sec. 1923–28.
1885	C.	Laurie, Arthur Pillans, M.A., D.Sc., LL.D., J.P., formerly Principal of the Heriot-Watt College, Edinburgh. 38 Springfield Road, St John's Wood, London, N.W. 8	1908–11, 1913–16.
1921		* Laurie, The Rev. Albert Ernest, M.C., C.F., D.D., Rector of Old St Paul's, and Canon of St Mary's Cathedral, Edinburgh. Lauder House, Jeffrey Street, Edinburgh 1	
1905		Lawson, David, M.A., M.D., L.R.C.P. and S.E., Druimdarroch, Banchory, Kincardineshire	
1903		Leighton, Gerald Rowley, O.B.E., M.D., D.Sc., Medical Officer, Scottish Board of Health, 121A Princes Street, Edinburgh 2	
1930		* Lelean, Percy Samuel, C.B., C.M.G., F.R.C.S., L.R.C.P., D.P.H., Professor of Public Health in the University of Edinburgh (Usher Institute of Public Health, Warrender Park Road). 2 Barnton Loan, Davidson's Mains, Edinburgh 4	
1910		Lovie, Alexander, F.R.C.V.S., D.V.S.M., Balmae, Manor Road, Littleover, Derby	
1916	C.	* Levy, Hyman, M.A., D.Sc., Professor of Mathematics, Imperial College of Science and Technology, London, S.W. 7. 62 Kenilworth Avenue, Wimbledon Park, London, S.W. 19	
1914	C. N.	Lewis, Francis John, D.Sc., F.L.S., Professor of Biology, University of Alberta, Edmonton South, Alberta, Canada	
1918		* Lidstone, George James, F.F.A., F.I.A., LL.D., formerly Manager and Actuary of the Scottish Widows' Fund Life Assurance Society. Hermiston House, Hermiston, Currie, Midlothian	1919–22.
1905		Lightbody, Forrest Hay, 53 Queen Street, Edinburgh 2	
1931		* Lightfoot, Nicholas Morpeth Hutchinson, M.A. (Camb.), Lecturer in Mathematics, Heriot-Watt College, Edinburgh. 3 Park Gardens, Liberton, Edinburgh 9	
1923		* Lim, Robert Kho Seng, M.B., Ch.B., D.Sc., Ph.D., Peking Union Medical College, Department of Physiology, Peking, China	
1912		* Lindsay, John George, M.A., B.Sc. (Edin.), Rector of Dunfermline High School	
1920	C.	* Lindsay, Thomas A., M.A. (Hons.), B.Sc., Head Master, Higher Grade School, Bucksburn, Aberdeenshire	
1912		* Linlithgow, The Most Honourable the Marquis of, K.T., G.C.I.E., D.L., T.D., Hopetoun House, South Queensferry	
1903		† Liston, William Glen, C.I.E., M.D., Lt.-Col. Indian Medical Service (retired), Milburn Tower, Gogar, Corstorphine, Edinburgh 12	

Date of Election.			Service on Council, etc.
1929		* Little, John Robert, F.C.I.I., F.C.I.S., General Manager and Secretary of the Century Insurance Co., Ltd. 5 Dalrymple Crescent, Edinburgh 9	
1932		* Lockhart, James Balfour, M.A., B.Sc. (Edin.), Mathematical Master at the Edinburgh Academy, Dundas House, Kinnear Road, Edinburgh 4	
1926		* Lorraine, Norman Stanley Rees, M.D., D.P.H. (Edin. and Glasg.), Medical Officer of Health, Shoeburyness Urban District, and Assistant County Medical Officer of Health, County of Essex. 1 Burlescoombe Leas, Burlescoombe Road, Thorpe Bay, Southend-on-Sea	
1898		Lothian, Alexander Veitch, M.A., B.Sc., Glendoune, 996 Great Western Road, Glasgow	
1930		* Low, James Wotherspoon, B.Sc. (Edin.), Ph.D. (Bristol), 5 Palmerston Road, Edinburgh 9	
1923	C.	* Ludlam, Ernest Bowman, M.A., D.Sc., Lecturer in Chemistry in the University of Edinburgh (King's Buildings, West Mains Road)	
1923		* Lyford-Pike, James, M.A., B.Sc., Lecturer in Forestry in the University of Edinburgh. Rosetta, Liberton, Edinburgh 9	
1924		* Lyon, David Murray, M.D., F.R.C.P.E., D.P.H., D.Sc., Professor of Therapeutics and Clinical Medicine in the University of Edinburgh (Royal Infirmary). Druim, Colinton, Edinburgh 11	
1894		Mabbott, Walter John, M.A., formerly Rector of County High School, Duns, Berwickshire. The Hawthorn, Farnham Lane. Haslemere, Surrey	
1917		* Macalister, Sir Donald, Bt., K.C.B., M.D., M.A., B.Sc., Chancellor of the University of Glasgow. Barrmore, Lady Margaret Road, Cambridge	
1929		* M'Arthur, Donald Neil, D.Sc., Ph.D., F.I.C., Professor of Agricultural Chemistry, West of Scotland Agricultural College, Glasgow, C. 2. 35 Kersland Street, Glasgow, W. 2	
1921		* M'Arthur, Neil, M.A., B.Sc., Lecturer in Mathematics, Glasgow University. 1 Holyrood Crescent, Glasgow	
1926		* M'Bride, James Alexander, B.A. (Roy. Univ., Ireland), B.Sc. (Lond.), formerly Rector of Queen's Park Secondary School, Glasgow. Scottish Liberal Club, Princes Street, Edinburgh 2	
1883		M'Bride, Peter, M.D., F.R.C.P.E., 3 St Peter's Grove, York	
1931	C.	* McCallien, William John, D.Sc.(Glas.), Lecturer in Geology, University of Glasgow. Glenorchy, Tarbert, Argyll	
1930		* M'Candlish, Andrew Corrie, B.Sc. (Glasg.), M.S.A., Ph.D. (Iowa), Advisory Officer in Milk Production, West of Scotland Agricultural College. Claunch, Sorbie, Wigtownshire	
1923		* M'Cracken, William, J.P., F.S.I., F.H.A.S., Land Agent and Technical Adviser, Englesea House, Crewe	
1931	C.	* M'Crea, William Hunter, M.A., Ph.D. (Camb.), B.Sc. (Lond.), F.R.A.S., Reader in Mathematics in the University of London. Imperial College of Science and Technology, South Kensington, S.W. 7	
1918		* M'Culloch, Rev. James David, 3 Ardgowan Street, Greenock	
1905		Macdonald, Hector Munro, O.B.E., M.A., F.R.S., Professor of Mathematics, University of Aberdeen. 52 College Bounds, Aberdeen	1908–11.
1897	C.	Macdonald, James A., M.A., B.Sc., formerly H.M. Inspector of Schools. "Rothes," Frankscroft, Peebles	
1920		* M'Donald, Stuart, M.A., M.D., F.R.C.P.E., Professor of Pathology, University of Durham. College of Medicine, Newcastle-on-Tyne	
1928		* MacDonald, Thomas Logie, M.A., B.Sc. (Glasg.), F.R.A.S., 9 Colebrooke Terrace, Glasgow, W. 2	
1904		Macdonald, William, M.S.Agr., Sc.D., Ph.D., D.Sc., Editor, *Agricultural Journal* of South Africa. Rand Club, Johannesburg, Transvaal	
1886		Macdonald, William J., M.A., LL.D., 15 Comiston Drive, Edinburgh 10	
1931		* M'Dougall, John Bowes, M.D. (Glas.), F.R.F.P.S.G., F.R.C.P.E., Medical Director, British Legion Village, Preston Hall, Kent Preston Hall, Aylesford, Kent	
1901	C.	MacDougall, R. Stewart, M.A., D.Sc., LL.D. (Edin.), Em. Professor of Biology, Royal (Dick) Veterinary College, Edinburgh. Ivy Lodge, Gullane, East Lothian	1914–17.
1910		Macewen, Hugh Allen, O.B.E., M.B., Ch.B., D.P.H. (Lond. and Camb.), Local Government Board, Ministry of Health, Whitehall, London, S.W.	
1888	C.	M'Fadyean, Sir John, Kt., M.B., B.Sc., LL.D., formerly Principal and Professor of Comparative Pathology in the Royal Veterinary College, Camden Town, London. Highlands House, Leatherhead	
1885	C.	† Macfarlane, J. M., D.Sc., LL.D., Em. Professor of Botany. 427 West Hansberry Street, Germantown, Pa., U.S.A.	

Date of Election.			Service on Council, etc.
1897		MacGillivray, Angus, M.D., C.M., D.Sc., F.S.A.Scot., 23 South Tay Street, Dundee	
1878		M'Gowan, George, F.I.C., Ph.D., 21 Montpelier Road, Ealing, London, W. 5	
1932		* MacGregor, Archibald Gordon, M.C., B.Sc., Geologist, H.M. Geological Survey (Scotland). 1 Greenbank Terrace, Edinburgh 10	
1922		* Macgregor, Murray, M.A., D.Sc., Assistant Director (Scotland), H.M. Geological Survey, 19 Grange Terrace, Edinburgh 9	1930–
1903		M'Intosh, Donald C., M.A., D.Sc., formerly Education Officer, Elgin. Tomlay, Tomintoul, Banffshire	
1911		M'Intosh, John William, M.R.C.V.S., Dollis Hill Farm, Cricklewood, London, N.W. 2	
1927	C.	* M'Intyre, Donald, M.B.E., M.D. (Glas.), F.R.C.S.E., Assistant Physician, Glasgow Royal Maternity and Women's Hospital, and Glasgow Samaritan Hospital. 9 Park Circus, Glasgow, C. 3	
1912	C.	M'Kendrick, Anderson Gray, M.B., D.Sc., F.R.C.P.E., Lt.-Col., Indian Medical Service (retired), Superintendent, Research Laboratory, Royal College of Physicians, 2 Forrest Road, Edinburgh 1	1924–27.
1914		* M'Kendrick, Archibald, F.R.C.S.E., D.P.H., L.D.S., 12 Rothesay Place, Edinburgh	
1900	C.	M'Kendrick, John Souttar, M.D., F.R.F.P.S. (Glas.), 2 Buckingham Terrace, Hillhead, Glasgow	
1910	C.	* Mackenzie, Alister, M.A., M.D., D.P.H., Principal Medical Officer and Lecturer in Hygiene under the Glasgow Provincial Committee, Training Centre, Jordanhill, Glasgow. 22 Queen's Gate, Dowanhill, Glasgow	
1916	C.	* Mackenzie, John E., D.Sc., Reader in Chemistry in the University of Edinburgh (King's Buildings, West Mains Road) 2 Ramsay Garden, Edinburgh 1	
1905		Mackenzie, Sir Wm. Colin, K.B., M.D., F.R.C.S., Director Australian Institute of Anatomy, Canberra, F.C.T., Australia	
1904	C.	Mackenzie, Sir W. Leslie, M.A., M.D., D.P.H., LL.D., F.R.C.P.E., formerly Medical Member of the Scottish Board of Health. 14 Belgrave Place, Edinburgh 4	
1929	C.	* Mackie, John, M.A., D.Sc., Mathematical Master, Leith Academy. 19 Beresford Avenue, Trinity, Leith 5	
1928		* Mackie, Thomas Jones, M.D., M.R.C.P.E., Professor of Bacteriology in the University of Edinburgh (Teviot Place). 22 Mortonhall Road, Edinburgh 9	
1910		* MacKinnon, James, M.A., Ph.D., LL.D., formerly Professor of Ecclesiastical History, Edinburgh University. 12 Lygon Road, Edinburgh 9	
1904		Mackintosh, Donald James, C.B., M.V.O., D.L., M.B., C.M., LL.D., Supt. Western Infirmary, Glasgow	
1899		Maclean, Sir Ewan John, M.D., D.Sc. (Hon.), F.R.C.P. (Lond.), D.L., J.P., Emer. Professor of Obstetrics and Gynæcology, Welsh National Medical School. 12 Park Place, Cardiff	
1888	C.	Maclean, Magnus, M.A., D.Sc., LL.D., M.Inst.C.E., M.I.E.E., formerly Professor of Electrical Engineering in the Royal Technical College. 108 University Avenue, Glasgow, W. 2	1916–19.
1918		* M'Lellan, Dugald, M.Inst.C.E., Divisional Engineer, L.M. and S. Railway, Buchanan Street Station, Glasgow. 1 Queen's Gate, Dowanhill, Glasgow	
1932		* Macleod, John James Rickard, M.B., Ch.B. (Aberd.), D.Sc. (Toronto), LL.D., F.R.S., Professor of Physiology in the University of Aberdeen	
1916	C.	* M'Lintock, William Francis Porter, D.Sc. (Edin.), Museum of Practical Geology, 28 Jermyn Street, London, S.W. 1	
1923		* Macmillan, Rt. Hon. Lord, LL.D., 44 Millbank, Westminster, S.W. 1	
1932		* M'Neil, Charles, M.A., M.D. (Edin.), F.R.C.P.E., Major, R.A.M.C., Professor of Child Life and Health in the University of Edinburgh (Sick Children's Hospital). 44 Heriot Row, Edinburgh 3	
1917		* Macpherson, Rev. Hector Copland, M.A., Ph.D., F.R.A.S., Guthrie Memorial U.F. Church. 7 Wardie Crescent, Edinburgh 5	
1921		* M'Quistan, Dougald Black, M.A., B.Sc., Associate-Professor of Natural Philosophy, Royal Technical College, Glasgow. 29 Viewpark Drive, Rutherglen	
1921	C.	* MacRobert, Thomas Murray, M.A., D.Sc., Professor of Mathematics in the University of Glasgow. 10 The University, Glasgow	1931–
1921	C.	* M'Whan, John, M.A. (Glasgow), Ph.D. (Gött.), Lecturer in Mathematics in the University of Glasgow. 84 Munro Road, Jordanhill, Glasgow, W. 3	
1927		* Madwar, Mohamed Reda, B.Sc., Ph.D. (Edin.), Assistant, Helwân Observatory, Egypt	
1898	C.	† Mahâlanobis, S. C., B.Sc., (Edin.), Professor of Physiology, University of Calcutta, formerly Professor of Physiology, and sometime Dean, Presidency College, Calcutta. P. 45 New Park Street, Calcutta	

Date of Election.			Service on Council, etc.
1913		Majumdar, Tarak Nath, D.P.H. (Cal.), L.M.S., F.C.S., Health Officer, Calcutta, IV, Calcutta, India. P. 235 Russa Road, P.O. Tollygunge	
1917		* Malcolm, L. W. Gordon, M.Sc. (Cantab.), Conservator, Wellcome Historical Medical Museum, 54 Wigmore Street, London, W. 1	
1908		Mallik, Devendranath, Sc.D., B.A., Principal, Carmichael College, Rungpur, Bengal, India	
1912		Maloney, William Joseph, M.B.E., M.C., M.D. (Edin.), LL.D., formerly Professor of Neurology at Fordham University. 420 Park Avenue, New York City, U.S.A.	
1918		Marchant, Rev. Sir James, K.B.E., LL.D., F.R.A.S., F.L.S., Director, National Council for Promotion of Race-Regeneration. Rhondda House, 60 Gower Street, London, W.C. 1	
1901	C.	Marshall, Francis Hugh Adam, Sc.D., F.R.S., Reader in Agricultural Physiology in the University of Cambridge, Christ's College, Cambridge	
1920	C.	* Marshall, John, M.A., D.Sc. (St Andrews), B.A. (Cantab.), University Reader in Mathematics, Bedford College, Regent's Park, London, N.W. 1. Logan House, 123 Torrington Park, North Finchley, London, N. 12	
1931		* Mason, John Huxley, F.R.C.V.S., Government Veterinary Laboratory, Onderstepoort, Pretoria, South Africa	
1913		Masson, George Henry, M.D., D.Sc., F.R.C.P.E., Port of Spain, Trinidad, British West Indies	
1885	C.	Masson, Sir David Orme, K.B.E., M.A., D.Sc., LL.D., F.R.S., Em. Professor of Chemistry in the University of Melbourne	
1898	C. M-B.	Masterman, Arthur Thomas, M.A., D.Sc., F.R.S., formerly Superintending Inspector, H.M. Board of Agriculture and Fisheries. Royal Auto Club, Pall Mall, London, S.W. 3 Kedale Road, Seaford	1902–04.
1911		† Mathews, Gregory Macalister, H.F.A.O.U., M.B.O.U., Meadway, St Cross, Winchester, Hants	
1921		* Mathieson, John, F.R.S.G.S., late Division Superintendent, Ordnance Survey (retired), 42 East Claremont Street, Edinburgh 7	
1906		Mathieson, Robert, F.C.S., St Serf's, Innerleithen	
1928		* Matthai, George, M.A. (Camb.), F.Z.S., F.L.S., Professor of Zoology, The Government College, Lahore, India	
1924		* Matthews, James Robert, M.A., F.L.S., Professor of Botany in the University of Reading	
1932		* Maxwell, William, Managing Director of R. & R. Clark, Ltd. 14 South Inverleith Avenue, Edinburgh 4	
1917		* Maylard, A. Ernest, M.B., B.Sc. (Lond.), F.R.F.P.S. (Glasgow), Kingsmuir, Peebles	
1922		* Meakins, Jonathan Campbell, M.D., LL.D., F.R.C.P.E., Professor of Medicine and Director of the Department of Medicine, M'Gill University, Montreal, Canada	
1931		* Mears, Frank Charles, F.R.I.B.A., Architect, 3 Forres Street, Edinburgh 3. 14 Ramsay Garden, Edinburgh 1	
1926		* Mekie, David Clark Thomson, M.A., Ph.D., Head Master, Bonnington Road Public School. 11 Minto Street, Edinburgh 9	
1901	C.	Menzies, Alan W. C., M.A., B.Sc., Ph.D., F.C.S., Professor of Chemistry, Princeton University, Princeton, New Jersey, U.S.A.	
1927		* Menzies, Sir Frederick Norton Kay, K.B.E., M.D., F.R.C.P.E., D.P.H. (Lond.), Medical Officer of Health and School Medical Officer, Administrative County of London, County Hall, London, S.E.	
1929		* Mercer, Walter, M.B., Ch.B., F.R.C.S.E., Lecturer in Clinical Surgery in the University of Edinburgh (Royal Infirmary). 12 Rothesay Terrace, Edinburgh 3	
1917		* Merson, George Fowlie, Manufacturing Technical Chemist, St John's Hill Works, Edinburgh 8	
1902	C.	† Metzler, William H., A.B., D.Sc., Ph.D., Corresponding Fellow of the Royal Society of Canada, Dean of the New York State College for Teachers, Albany, N.Y., U.S.A.	
1885	C. M-B.	Mill, Hugh Robert, D.Sc., LL.D., Hill Crest, Dormans Park, E. Grinstead	
1910		* Miller, John, M.A., D.Sc., Professor of Mathematics, Royal Technical College, 2 Northbank Terrace, North Kelvinside, Glasgow	
1930		* Miller, William Christopher, M.R.C.V.S., Lecturer, Institute of Animal Genetics (Sheep Section), King's Buildings, West Mains Road, Edinburgh. Scotiswoode, Alnwickhill Road, Liberton, Edinburgh 9	
1905		Milne, Archibald, M.A., D.Sc., Deputy Director of Studies, Edinburgh Provincial Training College. 38 Morningside Grove, Edinburgh 10	
1905		Milne, C. H., M.A., D.Litt., Head Master, Daniel Stewart's College. 19 Merchiston Gardens, Edinburgh 10	

Date of Election.			Service on Council, etc.
1904	C.	Milne, James Robert, D.Sc., Lecturer in Natural Philosophy in the University of Edinburgh (Drummond Street). 7 Grosvenor Crescent, Edinburgh 12	
1886		Milne, William, M.A., B.Sc., 70 Beechgrove Terrace, Aberdeen	
1899		Milroy, Thomas Hugh, M.D., B.Sc., LL.D., Professor of Physiology in Queen's University, Belfast	
1889	C.	Mitchell, A. Crichton, D.Sc., Hon. Doc. Sc. (Genève), formerly Director of Public Instruction in Travancore, India. 246 Ferry Road, Edinburgh 5. (Society's Representative on Governing Body of Heriot-Watt College)	1915-16, 1930-Cur. 1916-26. V-P 1926-29.
1897		† Mitchell, George Arthur, M.A., 9 Lowther Terrace, Kelvinside, Glasgow	
1900		Mitchell, James, M.A., B.Sc., Islay Lodge, Lochgilphead, Argyll	
1911		Modi, Edalji Manekji, D.Sc., LL.D., Litt.D., F.C.S., etc., Proprietor and Director of Arthur Road Chemical Works, Meher Buildings, Tardeo, Bombay, India	
1906		† Moffat, Rev. Alexander, M.A., LL.D., formerly Professor of Physics, Christian College, Madras.	
1929		Moir, Henry, F.F.A., F.I.A., President, United States Life Insurance Co., in the City of New York, 156 Fifth Avenue, New York City. Upper Montclair, New Jersey	
1890	C.	† Mond, Sir Robert Ludwig, Kt., M.A. Cantab., LL.D., F.C.S., 9 Cavendish Sq., London, W. 1	
1887	C.	Moos, N. A. F., D.Sc., L.C.E., J.P., Director of Bombay and Alibag Observatories (retired). Red Leaf, Pedder Road, Bombay, India	
1896		Morgan, Alexander, O.B.E., M.A., D.Sc., formerly Principal, Edinburgh Provincial Training College. 1 Midmar Gardens, Edinburgh 10	
1930		* Morison, John Miller Woodburn, M.D., F.R.C.P.E., D.M.R. and E., Professor of Radiology in the University of London, and Director of the Radiological Department of the Cancer Hospital, Fulham Road, London, S.W. 3	
1926		* Morris, James Archibald, R.S.A., F.S.A. Scot., Savoy Croft, Ayr	
1919		* Morris, Robert Owen, M.A., M.D., C.M. (Edin.), D.P.H. (Liverpool). King Edward VII Welsh National Memorial Association (Tuberculosis). Hafod-ar-For, Aberdovey, N. Wales	
1892	C.	Morrison, J. T., M.A., B.Sc., Professor of Mathematical Physics, University, Stellenbosch, Cape Colony	
1914		Mort, Spencer, M.S., M.D. (Glas.), F.R.C.S.E., F.C.S., Surgical Director, North Middlesex County Hospital, Edmonton, London, N. 18. Surgical Director, Radium and Röntgen Deep Therapy Centre for the County of Middlesex	
1930		* Morton, James, LL.D., Chairman and Governing Director of the Scottish Dyes, Ltd. Craigiehall, Cramond Bridge, West Lothian	
1901		Moses, O. St John, M.D, D.Sc., F.R.C.S., Lt.-Col. I.M.S. (retired), formerly Professor of Medical Jurisprudence, Medical College, Calcutta. 18 Manstone Road, Cricklewood, London, N.W. 2	
1892	C. K.	Mossman, Robert Cockburn, Lacar 4332, Villa Devoto F.C.P., Buenos Aires, Argentina	
1916		* Muir, Robert, M.A., M.D., Sc.D., LL.D., F.R.S., F.R.C.P.E., Professor of Pathology, University of Glasgow. 30 Victoria Crescent Road, Glasgow, W. 2	
1874	C. K. V. J.	Muir, Sir Thomas, C.M.G., M.A., D.Sc., LL.D., F.R.S., formerly Superintendent-General of Education for Cape Colony, Elmcote, Sandown Road, Rondebosch, South Africa	1885-88. V-P 1888-91.
1907		Muirhead, James M. P., J.P., F.R.S.L., F.S.S., "Carmel," Rouwkoop Road, Rondebosch, South Africa	
1932		* Murray, John Murdoch, B.Sc. (Edin.), Divisional Officer, Forestry Commission. 76 Hillview Terrace, Corstorphine, Edinburgh 12	
1907		† Musgrove, James, M.D., F.R.C.S., F.R.C.S.E., LL.D., Em. Professor of Anatomy, University of St Andrews. The Swallowgate, St Andrews	
1931		* Nelson, Alexander, B.Sc. (Glas.), Ph.D. (Edin.), N.D.A., Lecturer in Plant Physiology and Agricultural Botany, University of Edinburgh. 11 Wardie Avenue, Edinburgh 5	
1924		* Nelson, Philip, M.A., M.D., Ph.D., F.S.A., Beechwood, Calderstones, Liverpool	
1898		Newman, Sir George, K.C.B., M.D., D.C.L., LL.D., F.R.C.P., Chief Medical Officer of the Ministry of Health and the Board of Education, Whitehall, London, S.W. 1	
1928		* Nichols, James Edward, M.Sc. (Dunelm), Ph.D. (Edin.), British Research Association for the Woollen and Worsted Industries, Torridon, Headingley, Leeds. 5 Huby Park, Huby, Leeds	

Date of Election.			Service on Council, etc.
1927		* Noble, Thomas Paterson, M.D. (Edin.), F.R.C.S., Phya Thai Palace, Bangkok, Siam	
1928	C. N.	* O'Donoghue, Charles Henry, D.Sc. (Lond.), Reader in Zoology in the University of Edinburgh (King's Buildings, West Mains Road). 24 Marchhall Crescent, Edinburgh 9	
1925		* Ogg, William Gammie, M.A., B.Sc., Ph.D., Director, Macaulay Institute for Soil Research, Craigiebuckler, Aberdeen	
1923	C.	* Ogilvie, Alan G., M.A., B.Sc. (Oxon.), Professor of Geography in the University of Edinburgh (High School Yards). 40 Fountainhall Road, Edinburgh 9	1932–
1929		* Ogilvie, Frederick Wolff, M.A., Professor of Political Economy, in the University of Edinburgh (South Bridge). 20 Murrayfield Gardens, Edinburgh 12	
1886		Oliver, James, M.D., F.L.S., Physician to the London Hospital for Women, 123 Harley Street, London, W.	
1895	C.	† Oliver, Sir Thomas, Kt., D.L., M.D., LL.D., F.R.C.P., Em. Professor of Medicine in the University of Durham, 7 Ellison Place, Newcastle-upon-Tyne	
1930		* Oliver, William, B.Sc., A.M.I.C.E., Professor of Organisation of Industry and Commerce in the University of Edinburgh (South Bridge), and Director of Mitchell Graham & Sons, Ltd., Electrical and Mechanical Engineers. 70 Netherby Road, Trinity, Edinburgh 5	
1930		* O'Riordan, George Francis, B.Sc.(Eng.), M.Inst.Mech.E., Principal of Battersea Polytechnic, London, S.W. 7. Hesslewood, Cambalt Road, Putney Hill, London, S.W. 15	
1924		* Orr, John Boyd, D.S.O., M.C., M.A., D.Sc., M.D., F.R.S., Director of Rowett Research Institute for Research in Animal Nutrition. Research Lecturer in Physiology of Nutrition in the University of Aberdeen	
1915		* Orr, Lewis P., F.F.A., General Manager of the Scottish Life Assurance Co., 19 St Andrew Square, Edinburgh. 3 Belgrave Place, Edinburgh 4	
1932		* Orr, Matthew Young, Botanist in Government Service. 3 Royston Terrace, Edinburgh 4	
1927		* Owen, William John, Memb. Roy. Soc. Victoria. Director, Department of Histology, Australian Institute of Anatomy, Canberra, Australia	
1908		Page, William Davidge. (Address not known)	
1905		Pallin, Lt.-Col. William Alfred, C.B.E., D.S.O., F.R.C.V.S., (retired), 5 Tower Gardens, Hythe, Kent	
1924		* Parker, Joseph, D.Sc., Principal, Fife Mining School, Cowdenbeath. 128 Stenhouse Street, Cowdenbeath	
1901		Paterson, David, F.C.S., Leewood, Rosslyn Castle, Midlothian	
1918		* Paterson, Rev. William Paterson, D.D., LL.D., Professor of Divinity, University, Edinburgh (South Bridge). 39 George Square, Edinburgh 8	
1927		* Patterson, Charles, M.Inst. Marine Engineers; Lecturer in Mechanical Engineering Design and Theory of Machines in the University of Edinburgh (Sanderson Engineering Laboratories, Mayfield Road). 22 Dudley Terrace, Trinity, Edinburgh 6	
1926		* Patton, Donald, M.A., B.Sc., Ph.D., Lecturer in Botany, Glasgow Provincial College for the Training of Teachers. 15 Jordanhill Drive, Glasgow, W. 3	
1923	C.	* Peacock, Alexander David, D.Sc., Professor of Zoology, University College, Dundee	
1907		Pearce, John Thomson, B.A., B.Sc., Bolton Manse, Haddington, East Lothian	
1914		Pearson, Joseph, D.Sc., F.L.S., Director of the Colombo Museum, and Marine Biologist to the Ceylon Government, Colombo Museum, Ceylon	
1904		Peck, James Wallace, C.B., M.A., Second Secretary, Scottish Education Department, Dover House, Whitehall, London, S.W. 1	1926-28.
1887	C. M-B.	Peddie, Wm., D.Sc., Professor of Natural Philosophy in University College, Dundee. The Weisha, Ninewells, Dundee	1904-07, 1908-11. V-P 1919-22.
1925		* Penman, David, D.Sc., M.Inst.M.E., Principal Dhanbad School of Mines, India. Mines Department, Dhanbad, East Indian Railway, India	
1928		* Percival, George Hector, M.D., M.R.C.P.E., Ph.D., Assistant Physician, Skin Department, Royal Infirmary, Edinburgh. 38 Palmerston Place, Edinburgh 12	
1893		Perkin, Em. Professor Arthur George, D.Sc., F.R.S., F.I.C., Grosvenor Lodge, Hyde Park, Leeds	
1931	C.	* Phemister, James, M.A., D.Sc. (Glas.), Senior Geologist, H.M. Geological Survey. Everland, 2 Denham Green Terrace, Edinburgh 5	
1889		† Philip, Sir Robert William, Kt., M.A., M.D., LL.D., F.R.C.P.E., Professor of Tuberculosis, University of Edinburgh. 45 Charlotte Square, Edinburgh 2	V-P 1927-30.

Date of Election.			Service on Council, etc.
1907	C.	† Phillips, Major Charles E. S., O.B.E., Castle House, Shooters Hill, Woolwich, S.E. 18	
1929	C.	* Phillips, John Frederick Vicars, D.Sc., F.L.S., Professor of Botany, The University of the Witwatersrand, Johannesburg, Union of South Africa	
1932		* Pickard, James Nichol, B.A. (Camb.), Ph.D. (Edin.). The Cottage, Carlops, by Penicuik, Midlothian	
1928		* Pilcher, Robert Stuart, General Manager and Engineer, Manchester Corporation Tramways and Motors. 55 Piccadilly, Manchester	
1914		* Pilkington, Basil Alexander, "Kambla," Davidson's Mains, Edinburgh 4	
1908	C.	Pirie, James Hunter Harvey, B.Sc., M.D., F.R.C.P.E., Research Pathologist and Bacteriologist, The South African Institute for Medical Research. P.O. Box 1038, Johannesburg, South Africa	
1911		* Pirie, James Simpson, M.Inst.C.E., 25 Grange Road, Edinburgh 9	
1906		Pitchford, Herbert Watkins, C.M.G., F.R.C.V.S., Lt.-Col., Victoria Club, Pietermaritzburg, South Africa	
1924		* Ponder, Eric, M.D., D.Sc., Professor of General Physiology, Washington Square College, New York University, New York, U.S.A.	
1919		* Porritt, B. D., M.Sc. (Lond.), F.I.C., Director of Research, Research Association of British Rubber Manufacturers, 105-7 Lansdowne Road, Croydon, Surrey	
1888		† Prain, Sir David, Kt., Lt.-Col., Indian Medical Service (retired), C.M.G., C.I.E., M.A., M.B., LL.D., F.R.S., F.L.S., For. Memb. K. Svensk. Vetensk. Akad.; Hon. Memb. Soc. Lett. ed Arti d. Zelanti, Acireale; Pharm. Soc. Gt. Britain; Corr. Memb. Bayer. Akad. Wiss., etc.; formerly Director, Royal Botanic Gardens, Kew, Surrey. The Well Farm, Warlingham, Surrey	
1932		* Prasad, Gorakh, D.Sc. (Edin.), Reader in Mathematics, University of Allahabad. Beli Road, Allahabad, India	
1926	C.	* Prashad, Baini, D.Sc., Superintendent, Zoological Survey of India, Indian Museum, Calcutta	
1892	C.	Pressland, Arthur J., M.A. (Camb.), 28 Carlyle Road, Cambridge	
1928		Price, Charles Edward, J.P., formerly M.P. for Central Edinburgh, Hon. Freeman of the City of Edinburgh. Westlands, Weston Rhyn, Shropshire	
1915		† Price, Frederick William, M.D., M.R.C.P.E., Physician to the Royal Northern Hospital, London. 133 Harley Street, London, W.	
1932		* Price, Thomas Slater, O.B.E., D.Sc., F.R.S., Professor of Chemistry in the Heriot-Watt College, Edinburgh. 2 Cluny Drive, Edinburgh 10	
1932		* Pringle, John, Hon. D.Sc., F.G.S., Senior Geologist, acting as Palæontologist on Geological Survey (Scotland), Southpark, 19 Grange Terrace, Edinburgh 9	
1911		Purdy, John Smith, D.S.O., M.D., C.M. (Aberd.), D.P.H. (Camb.), F.R.G.S., Town Hall, Sydney, N.S.W., Australia	
1920	C.	* Purser, George Leslie, M.A. (Cantab.), F.Z.S., Lecturer in Embryology, University of Aberdeen	
1898		Purves, John Archibald, D.Sc., Chilliswood, Trull, Taunton	
1899	C.	Ramage, Alexander G., Lochcote, Linlithgowshire	
1904		Ratcliffe, Joseph Riley, M.B., C.M., c/o The Librarian, The University, Birmingham	
1900		Raw, Nathan, C.M.G., M.D., 30 Clarendon Court, Maida Vale, London, W.9	
1927	C.	* Read, Herbert Harold, D.Sc. (Lond.), A.R.C.Sc., F.G.S., George Herdman Professor of Geology, The University of Liverpool	
1929		* Read, Selwyn, B.A., Schoolmaster, Edinburgh Academy. 2 Oxford Terrace, Edinburgh 4	
1902		Rees-Roberts, John Vernon, M.D., D.Sc., D.P.H., 90 Fitzjohns Avenue, Hampstead, London, N.W. 3	
1913		Reid, Harry Avery, O.B.E., F.R.C.V.S., D.V.H., Bacteriologist and Pathologist, Department of Agriculture, Wellington, New Zealand. c/o Bank of New Zealand, 1 Queen Victoria Street, London, E.C.	
1924		* Reid, William Carstairs, Civil Engineer, 23 Saxe-Coburg Place, Edinburgh 3	
1914		Renshaw, Graham, M.D., M.R.C.S., L.R.C.P., L.S.A., Editor of the *Avicultural Magazine*, Sale Bridge House, Sale, Manchester	
1913		* Richardson, Major Harry, M.Inst.E.E., M.Inst.M.E., 16 Stratford Place, London, W. 1	
1908		Richardson, Linsdall, F.G.S., 104 Greenfield Road, Harbourne, Birmingham	
1875		Richardson, Ralph, W.S., 29 Eglinton Crescent, Edinburgh 12	
1927		* Richey, James Ernest, B.A., B.A.I., Trinity College, Dublin, F.G.S., District Geologist, H.M. Geological Survey (Scotland), 19 Grange Terrace, Edinburgh 9	
1930		* Ritchie, Allan Watt, Chief Sanitary Inspector of the City of Edinburgh. 2 Queensferry Terrace, Edinburgh 4	

Date of Election.			Service on Council, etc.
1916	C.	* Ritchie, James, M.A., D.Sc. (VICE-PRESIDENT), Regius Professor of Natural History in the University of Aberdeen	1921-24, 1926-28. Sec. 1928-31. V-P 1931-
1914	C.	* Ritchie, James Bonnyman, D.Sc., Rector, the Academy, Ayr. 28 Carrick Road, Ayr	
1906	C.	Ritchie, William Thomas, M.D., F.R.C.P.E., Professor of Medicine in the University of Edinburgh (Teviot Place). 10 Douglas Crescent, Edinburgh 12	
1929		* Robb, Richard Alexander, M.A., B.Sc., M.Sc., Lecturer in Mathematics, University of Glasgow. 19 Seyton Avenue, Langside, Glasgow, S.1	
1931		* Robb, William, N.D.A., Director of Research, Scottish Society for Research in Plant Breeding. Craigs House, Corstorphine, Midlothian	
1898	C.	Roberts, Hon. Alexander William, D.Sc., F.R.A.S., Lovedale, South Africa	
1919		* Roberts, Alfred Henry, O.B.E., M.Inst.C.E., Superintendent and Engineer, Leith Docks. 40 Buckingham Terrace, Edinburgh 4	
1926		* Roberts, John Alexander Fraser, M.A. (Cantab.), B.Sc., Institute of Animal Genetics, University of Edinburgh (King's Buildings, West Mains Road)	
1928	C.	* Roberts, Owen Fiennes Temple, M.C., M.A. (Camb.), Lecturer in Astronomy and Meteorology in the University of Aberdeen. 20 Belgrave Terrace, Aberdeen	
1902	C.	Robertson, Robert A., M.A., B.Sc., Professor of Botany in the University of St Andrews	
1919		* Robertson, William Alexander, F.F.A., Century Insurance Co., Ltd., 18 Charlotte Square. Kilwarlin, Barnshot Road, Colinton	
1896	C.	Robertson, W. G. Aitchison, D.Sc., M.D., F.R.C.P.E., Barrister-at-law, Lincoln's Inn. St Margarets, St Valerie Road, Bournemouth	
1932	C.	* Robson, John Michael, M.D., B.Sc., engaged in Research in the Institute of Animal Genetics, University of Edinburgh (King's Buildings, West Mains Road). Tayinloan, Loanhead	
1926		* Romanis, William Hugh Cowie, M.A., M.B., M.C. (Cantab.), F.R.C.S., Surgeon to St Thomas's Hospital, London, etc. 120 Harley Street, London, W.1	
1916		* Ronald, David, M.Inst.C.E., Chief Engineer, Scottish Board of Health, 125 George Street, Edinburgh 2	
1909	C.	* Ross, Alex. David, M.A., D.Sc., F.Inst.P., F.R.A.S., Professor of Physics, University of Western Australia, Perth, Western Australia	
1921		* Ross, Edward Burns, M.A. (Edin. and Camb.), Professor of Mathematics in the Madras Christian College, Madras. 41 Liberton Brae, Edinburgh 9	
1931	C.	* Ruse, Harold Stanley, M.A. (Oxon.), D.Sc., Lecturer in Mathematics, University of Edinburgh. The Mathematical Institute, 16 Chambers Street, Edinburgh 1	
1906		Russell, Alexander Durie, B.Sc., Mathematical Master, Falkirk High School. 14 Heugh Street, Falkirk	
1930		Russell, David, LL.D., Paper Manufacturer. Silverburn, Leven, Fife	
1902	C. K.	Russell, James, 22 Glenorchy Terrace, Edinburgh 9	
1925	C.	* Saddler, William, M.A., B.A., Professor of Mathematics, Canterbury College, Christchurch, N.Z.	
1906		Saleeby, Caleb Williams, M.D., 13 Greville Place, Hampstead, London, N.W. 6	
1916	C.	* Salvesen, The Rt. Hon. Lord, P.C., K.C., LL.D., Judge of the Court of Session (retired), Dean Park House, Edinburgh 4	1920-22. V-P 1922-25.
1914		* Salvesen, Theodore Emile, F.R.S.A., F.S.A. Scot., Chevalier de la Légion d'Honneur. 37 Inverleith Place, Edinburgh 4	
1912	C. K.	* Sampson, Ralph Allen, M.A., D.Sc., LL.D. F.R.S. (GENERAL SECRETARY), Astronomer Royal for Scotland, Professor of Astronomy in the University of Edinburgh (South Bridge). Royal Observatory, Edinburgh	1912-15, 1919-21. V-P 1915-18. Sec. 1922-23. Gen. Sec. 1923-
1908		Samuel, Sir John S., K.B.E., D.L., J.P., F.S.A.Scot., 13 Park Circus, Glasgow, W.	
1927	C.	* Sandeman, Ian, M.A., B.Sc., Ph.D. (St Andrews), Education Office, Kandy, Ceylon	
1930		* Sansome, Frederick Whalley, B.Sc.(Agr.), Ph.D., F.L.S., Assistant, John Innes Horticultural Institution, Merton, London. Old Garden, Church Lane, Merton Park	

Date of Election.			Service on Council, etc.
1922		* Sarker, Bijali Behari, M.Sc., D.Sc. (Edin.), Post Graduate Lecturer in Physiology, University, Calcutta. 33/3 Lansdowne Road, Calcutta	
1903		Sarolea, Charles, Ph.D., D.Litt., formerly Professor of French, University of Edinburgh. 21 Royal Terrace, Edinburgh 7	
1927		* Schlapp, Robert, M.A. (Edin.), Ph.D. (Camb.), Lecturer in Applied Mathematics in the University of Edinburgh. 1 Peel Terrace, Edinburgh 9	
1885	C.	† Scott, Alexander, M.A., D.Sc., F.R.S., 34 Upper Hamilton Terrace, London, N.W. 8	
1919		* Scott, Alexander, M.A., D.Sc., 3 Winton Terrace, Stoke-on-Trent	
1919		* Scott, Alexander Ritchie, B.Sc. (Edin.), D.Sc. (Lond.), Principal London County Council, Beaufoy Institute, Prince's Road, Vauxhall Street, London, S.E. 11	
1917		* Scott, Henry Harold, M.D., F.R.C.P., M.R.C.S., D.P.H., Medical Secretary, Colonial Medical Research Committee, Colonial Office, Downing Street, London, S.W. 1. "Bailleul," Albemarle Road, Beckenham, Kent	
1928		* Senior-White, Ronald, F.E.S., Malariologist, Bengal-Nagpur Railway, Kidderpore, P.O., Calcutta, India	
1930		* Shankland, Ernest Claud, F.R.Met.S., River Superintendent to the Port of London Authority. Mariners, Balfour Gardens, Folkestone	
1900	C. N.	Sharpey-Schafer, Sir Edward Albert, Kt., M.D., D.Sc., LL.D., F.R.S. (PRESIDENT), Corresponding Member of the French Academy of Medicine, Professor of Physiology in the University of Edinburgh. Park End, North Berwick	1900–03, 1906–09, 1918–19. V.P 1913–17. P 1929–
1927		* Sharpley, Forbes Wilmot, B.Sc. (Eng.) (Lond.), M.I.E.E., Professor of Electrical and Mechanical Engineering, Indian School of Mines, Dhanbad, Bihar and Orissa, India	
1931		* Shaw, John James M'Intosh, M.A., M.D., F.R.C.S., Lecturer in Surgery and Clinical Surgery, University of Edinburgh. Greenaway, Kinnear Road, Edinburgh 4	
1927		* Shearer, Ernest, M.A., B.Sc. (Edin.), Professor of Agriculture and Rural Economy, Edinburgh University, and Principal of the Edinburgh and East of Scotland College of Agriculture, 13 George Square, Edinburgh 8	
1931		* Shearer, James Fleming, M.A. (Glas.), Lecturer in Natural Philosophy, University of Glasgow. Balmanno, Flloyd Street, Coatbridge	
1932		* Sheppard, William Fleetwood, Sc.D., LL.M., formerly a Senior Examiner to the Board of Education (retired). Cardrona, Berkhamsted, Herts	
1932		* Simpson, Alexander Rudolf Barbour, B.Sc. (Edin.), M.A. (Camb.), F.R.G.S., House Master and Senior Geography Master, Canford School. Captain T.A. Beaufort House, Canford School, Wimborne, Dorset	
1908		Simpson, George Freeland Barbour, M.D., F.R.C.P.E., F.R.C.S.E., J.P., 43 Manor Place, Edinburgh 3	
1900	C.	† Simpson, James Young, M.A., D.Sc., D.Jur., Professor of Natural Science in the New College, Edinburgh. 25 Chester Street, Edinburgh 3	1922–25.
1932		* Simpson, John Baird, B.Sc. (Aberd.), Senior Geologist, H.M. Geological Survey (Scotland), Southpark, 19 Grange Terrace, Edinburgh 9	
1900		Sinhjee, Sir Bhagvat, G.C.I.E., M.D., LL.D. (Edin.), H.H. the Thakur Sahib of Gondal, Gondal, Kathiawar, Bombay, India	
1903		† Skinner, Robert Taylor, M.A., J.P., Head Master, Donaldson's Hospital, Edinburgh 12	
1930	C.	* Slater, Robert Henry, D.Sc., Ph.D. (Edin.), Department of Chemical Pathology, St Mary's Hospital, London, W. 2	
1929		* Smail, James Cameron, Assoc.Inst.E.E., Principal, Heriot-Watt College, Edinburgh. 1 Grange Terrace, Edinburgh 9	
1926		* Small, James, D.Sc., Ph.C., Professor of Botany, Queen's University, Belfast. Ardcolm, Knock, Belfast	
1901		Smart, Edward, B.A., B.Sc., Tillyloss, Tullylumb Terrace, Perth	
1920		* Smellie, William Robert, M.A., D.Sc., Geologist on the Staff of the Anglo-Persian Oil Company. Ardene, Mossend, Lanarkshire	
1928		Smith, Alick Drummond Buchanan, M.A., B.Sc. (Agric.) (Aberd.), M.S.A. (Iowa), Lecturer, Institute of Animal Genetics, University of Edinburgh (King's Buildings, West Mains Road)	
1921		* Smith, Norman Kemp, M.A., D.Phil., D.Litt., LL.D., Professor of Logic and Metaphysics in the University of Edinburgh (South Bridge). Ellerton, Grange Loan, Edinburgh 9	

Date of Election.			Service on Council, etc.
1928		* Smith, Percy James Lancelot, M.A. (Oxon.), F.I.C., F.C.S., Science Master, Loretto School. 47 Dalrymple Loan, Musselburgh	
1911		* Smith, Stephen, B.Sc., Engineer, 31 Grange Loan, Edinburgh 9	
1929		* Smith, Sydney, M.D., M.R.C.P., D.P.H., Professor of Forensic Medicine in the University of Edinburgh (Teviot Place). 10 Oswald Road, Edinburgh 9	
1907	C.	† Smith, William Ramsay, D.Sc., M.D., C.M., Permanent Head of the Health Department, South Australia, Belair, South Australia	
1919		* Smith, Sir William Wright, Kt., M.A., D. ès Sc., Regius Professor of Botany, University of Edinburgh, Regius Keeper of the Royal Botanic Garden, and King's Botanist in Scotland. Inverleith House, Edinburgh 4	Sec. 1923-28. V-P 1928-31.
1932		* Sneeden, Jean-Baptiste Octave, B.Sc., Ph.D. (Glas.), Lecturer on Heat Engines, Royal Technical College, Glasgow. 39 Kingshouse Avenue, Cathcart, Glasgow	
1899		Snell, Ernest Hugh, M.D., B.Sc., D.P.H. Camb., Barrister-at-law, Middle Temple, late Medical Officer of Health, Coventry. 3 Eaton Road, Coventry	
1880		Sollas, William Johnson, M.A., D.Sc., LL.D., F.R.S., Fellow of University College, Oxford, and Professor of Geology and Palæontology in the University of Oxford	
1910		* Somerville, Robert, B.Sc., Lindrum, Queensferry Road, Dunfermline	
1911	C.	* Sommerville, Duncan M'Laren Young, M.A., D.Sc., Professor of Pure and Applied Mathematics, Victoria College, Wellington, New Zealand	
1929		* Southwell, Thomas, D.Sc., A.R.C.S., Lecturer in Helminthology, School of Tropical Medicine, Liverpool. 8 Waverley Road, Sefton Park, Liverpool	
1925		* Staig, Robert Arnot, M.A., Lecturer in Zoology in the University of Glasgow. Glenlea, Lasswade, Midlothian	
1891		Stanfield, Richard, A.R.S.M., M.Inst.C.E., formerly Professor of Mechanics and Engineering in the Heriot-Watt College, Edinburgh. 24 Mayfield Gardens, Edinburgh 9	1926-29.
1923		* Stebbing, Edward Percy, M.A., Professor of Forestry in the University of Edinburgh (George Square)	
1885 & 1915		* Steggall, John Edward Aloysius, M.A., Hon. A.R.I.B.A., Professor of Mathematics at University College, Dundee (St Andrews University). Woodend, Perth Road, Dundee	
1923		* Stenhouse, Andrew G., F.G.S., 191 Newhaven Road, Edinburgh 6	
1929	C.	* Stephen, Alexander Charles, B.Sc., Assistant, Natural History Department, Royal Scottish Museum, Edinburgh. "Eastcroft," Cramond Bridge, Edinburgh 4	
1912	C. K.	† Stephenson, John, C.I.E., M.B., D.Sc. (Lond.), F.R.S., Lt.-Col. I.M.S. (retired), British Museum (Natural History), Cromwell Road, London, S.W. 7. 42 Orsett Terrace, London, W. 2	
1910		* Stephenson, Thomas, D.Sc., F.C.S., Editor of the *Prescriber*, 13 Glencairn Crescent, Edinburgh 12	
1931		* Steven, George Alexander, B.Sc. (Edin.), Assistant Naturalist at the Plymouth Laboratory, Marine Biological Association of the United Kingdom. 28 Lincoln Avenue, Plymouth, Devon	
1925		* Stevens, Alexander, M.A., B.Sc., Lecturer in Geography in the University of Glasgow	
1886	C.	Stevenson, Charles A., B.Sc., M.Inst.C.E., Radella, North Berwick	
1884		† Stevenson, David Alan, B.Sc., M.Inst.C.E., Troqueer, Kingsknowe, Colinton, Midlothian	1928-31.
1919		* Stevenson, David Alan, B.Sc., M.Inst.C.E., 22 Glencairn Crescent, Edinburgh 12	
1931		* Stewart, Corbet Page, Ph.D. (Edin.), Lecturer in General Biochemistry, University of Edinburgh. 150 Craigleith Road, Edinburgh 4	
1925		* Stewart, David Smith, B.Sc., Ph.D., Assoc.M.Inst.C.E., Lecturer on Structural Engineering Drawing in the University of Edinburgh (Sanderson Engineering Laboratories, Mayfield Road). 12 Lasswade Road, Liberton, Edinburgh 9	
1924		* Stiles, Sir Harold Jalland, K.B.E., M.B., F.R.C.S.E., LL.D., formerly Professor of Clinical Surgery in the University of Edinburgh. Whitton Lodge, Gullane, E. Lothian	
1902		Stockdale, Herbert Fitton, LL.D., Director of the Royal Technical College, Glasgow. Clairinch, Upper Helensburgh, Dumbartonshire	
1889	C.	Stockman, Ralph, M.D., LL.D., F.R.C.P.E., F.F.P.S.G., Professor of Materia Medica and Therapeutics in the University of Glasgow	1903-05.
1926		* Stokoe William Norman, B.Sc., Ph.D. (Lond.), Chief Chemist and Works Manager, Craigmillar Creamery Co., Ltd. 8 Cobden Road, Edinburgh 9	
1906		Story, Fraser, O.B.E., formerly Professor of Forestry, University College, Bangor, North Wales. 4K Artillery Mansions, Victoria Street, London, S.W. 1	

Date of Election.			Service on Council, etc.
1907		Strong, John, C.B.E., M.A., LL.D., Professor of Education in the University of Leeds. Devonshire Hall, Headingley, Leeds	
1930	C.	* Stump, Claude Witherington, M.D., D.Sc., Professor of Embryology and Histology in the University of Sydney	
1903		Sutherland, David W., C.I.E., M.D., M.R.C.P., Lt.-Col. I.M.S. (retired), Braeside, Belhaven, Dunbar	
1930		* Sutherland, John Donald, C.B.E., LL.D., F.S.I., Chevalier of the Legion of Honour (France), Assistant Commissioner, Forestry, Scotland. 11 Inverleith Row, Edinburgh 4	
1925		Sutton, Richard L., M.D., D.Sc., LL.D., Professor of Diseases of the Skin in the University of Kansas School of Medicine, U.S.A.	
1932		* Swinton, William Elgin, B.Sc., Ph.D. (Glas.), F.L.S., F.Z.S., F.G.S., Curator of Fossil Reptiles and Amphibia, British Museum of Natural History, South Kensington, London, S.W. 7	
1917	C. N.	* Tait, John, D.Sc., M.D., Professor of Physiology, M'Gill University, Montreal, Canada	
1904		Tait, John W., B.Sc., formerly Rector of Leith Academy, Netherby, Pitkeathly, Bridge of Earn	
1895		† Talmage, James Edward, D.Sc., LL.D., F.R.M.S., F.G.S., F.G.S.A., formerly President and Professor of Geology, University of Utah. 47 East S. Temple Street, Salt Lake City, Utah, U.S.A.	
1890	C.	† Tanakadate, Aikitu, Hon. Professor of Natural Philosophy in the Imperial University of Japan. Koisikawa, Zôsigayamati, 144, Tokyo, Japan	
1870		Tatlock, Robert R., F.C.S., City Analyst's Office, 156 Bath Street, Glasgow	
1899		Taylor, James, M.A., formerly Mathematical Master in the Edinburgh Academy. 18 Hillview, Blackhall, Edinburgh 4	
1917	C.	* Taylor, William White, M.A., D.Sc., formerly Lecturer on Chemical Physiology, University, Edinburgh.	
1885	C.	Thompson, D'Arcy Wentworth, C.B., M.A., Hon. D.Sc. (Witwatersrand), D.Litt., F.R.S., Foreign Hon. Member Amer. Acad. Arts and Sciences (CURATOR OF LIBRARY AND MUSEUM), Professor of Natural History, University, St Andrews. 44 South Street, St Andrews	1892-95, 1896-99, 1907-10, 1912-15, 1922-25. V-P 1916-19. Curator 1926-
1932		Thompson, Harold William, D.Litt. (Edin.), A.M., Ph.D. (Harvard), F.S.A.Scot., Professor of English, N.Y. State College for Teachers. 544 Providence Street, Albany, New York, U.S.A.	
1917	C. N.	* Thompson, John M'Lean, M.A., D.Sc., F.L.S., Professor of Botany, University of Liverpool	
1931		* Thomson, David Cleghorn, M.A. (Edin.), B.A. (Oxon.), Scottish Regional Director, British Broadcasting Corporation. 11 York Place, Edinburgh 1	
1896		Thomson, George Ritchie, C.M.G., M.B., C.M., formerly Professor of Surgery, University of the Witwatersrand, Johannesburg, Transvaal. Hordle Grange, Hordle, Hants	
1903		Thomson, George S., 31 Tooley Street, London, S.E. 1	
1906		Thomson, Gilbert, M.A., M.Inst.C.E., 164 Bath Street, Glasgow, C. 2	
1926		* Thomson, Godfrey Hilton, D.Sc., Ph.D., Professor of the Theory, History, and Practice of Education in the University of Edinburgh (Moray House)	1931-
1887	C.	Thomson, Sir J. Arthur, M.A., LL.D., Em. Regius Professor of Natural History in the University of Aberdeen. St Mary's Lodge, Limpsfield, Surrey	1906-08, 1920-23.
1926	C.	* Thomson, John, M.A., B.Sc., Ph.D. (Glasg.), Lecturer in Plant Physiology in the University of Glasgow. 2 Chartwell Terrace, Bearsden, Glasgow	
1880		Thomson, John Millar, LL.D., F.R.S., Hon. Fellow King's College and Queen's College, London. 6 Douro Place, Kensington, London, W. 8	
1899		Thomson, R. Tatlock, F.C.S., 156 Bath Street, Glasgow	
1912	C.	Thomson, Robert Black, M.B. (Edin.), Aliwal North, Cape Province, S.A.	
1882		Thomson, Sir William, Kt., M.A., B.Sc., LL.D., formerly Principal, University of the Witwatersrand. Dunedin, Glencairn, Simonstown, South Africa	
1917		* Thorneycroft, Wallace, J.P., Strete Ralegh, Whimple, Exeter, Devon	
1920		* Todd, John Barber, B.Sc., Ph.D., M.I.Mech.E., Lecturer in Engineering in the University of Edinburgh. 98 Findhorn Place, Edinburgh 9	
1917		* Tovey, Donald Francis, B.A. (Oxon.), M.Mus. (Hon.), Birmingham, Professor of Music in the University of Edinburgh (Reid School of Music). 18 Buccleuch Place, Edinburgh 8	

Date of Election.			Service on Council, etc.
1914		† Tredgold, Alfred Frank, M.D. (Durham), F.R.C.P. (Lond.), Lecturer on Mental Deficiency at London University, and Bethlem Royal Hospital, "St Martins," Guildford	
1915		* Trotter, George Clark, M.D. (Edin.), D.P.H. (Aberdeen), Medical Officer of Health, Metropolitan Borough, Islington. Braemar, 17 Haslemere Road, Crouch End, London, N. 8	
1922	C. K.	* Turnbull, Herbert Westren, M.A., F.R.S., Professor of Mathematics in the University of St Andrews. 2 Queens Terrace, St Andrews	1928-31.
1905		Turner, Arthur Logan, M.D., LL.D., F.R.C.S.E., (Vice-President), 27 Walker Street, Edinburgh 3	1926-29. V.P 1930-
1925		* Turner, Harry Moreton Stanley, M.B.E., M.D., M.R.C.S., L.R.C.P., D.T.M. and H., Chevalier de l'Ordre Royale du Sauveur de Grèce. Wing-Commander, R.A.F. (retired). 65 Stert Street, Abingdon-on-Thames, near Oxford	
1924		* Turner, Richard, O.B.E., M.B., C.M., Hotel Hydropathic, Peebles	
1895		Turton, Albert H., M.I.M.M., 233 George Road, Erdington, Birmingham	
1918	C.	* Tyrrell, G. W., A.R.C.Sc., D.Sc., F.G.S., Chief Assistant and Lecturer in Petrology, Geological Department, University, Glasgow	1926-29.
1910		Vincent, Swale, M.D. (Lond.), D.Sc. (Edin.), Professor of Physiology in the University of London. 15 Fishpool Street, St Albans, Herts	
1930	C.	* Voge, Cecil Innes Bothwell, B.Sc., Ph.D. (Edin.), Research Chemist. 25A Upper Park Road, Hampstead, N.W. 3	
1932		* Wade, Henry, C.M.G., D.S.O., M.D., Senior Lecturer in Clinical Surgery in the University of Edinburgh (Royal Infirmary). 6 Manor Place, Edinburgh 3	
1926		* Wakeley, Cecil Pembrey Grey, F.R.C.S., Surgeon to King's College Hospital, London, Lecturer in Anatomy, King's College, London. 24 Queen Anne Street, Cavendish Square, London, W. 1	
1925	C.	* Walker, Frederick, M.A., Ph.D., D.Sc., Lecturer in Geology, University, St Andrews	
1891	C. M-B.	Walker, Sir James, Kt., D.Sc., Ph.D. LL.D., F.R.S., formerly Professor of Chemistry in the University of Edinburgh. 5 Wester Coates Road, Edinburgh 12	1903-05, 1910-13, 1922-25. 1928-31. V.P 1916-19.
1931		* Walker, William James, Ph.D. (Edin.), Research Chemist, H.M. Fuel Research Station, East Greenwich, London, S.E. 10. C/o Harrison, 64 Sandtoft Road, Charlton, London, S.E. 7	
1902		Wallace, Alexander G., M.A., 56 Fonthill Road, Aberdeen	
1886	C.	Wallace, Robert, M.A., LL.D., F.L.S., Em. Professor of Agriculture and Rural Economy in the University of Edinburgh. C/o Mrs M'Call, 11 Bruntsfield Crescent, Edinburgh 10	
1898		Wallace, Wm., M.A., Campsie, Alta, Canada	
1920		* Walmsley, Thomas, M.D. (Glasg.), Professor of Anatomy, Queen's University, Belfast	
1931	C.	* Walton, John, M.A.(Camb.), D.Sc.(Manchester), Regius Professor of Botany, University of Glasgow. 4 Doune Gardens, Glasgow, N.W.	
1927	C.	* Wardlaw, Claude Wilson, Ph.D., D.Sc. (Glasg.), Imperial College of Tropical Agriculture, Trinidad, B.W.I.	
1923		* Warren, John Alexander, M.Inst.C.E., M.Cons.E. (Westminster). 74 Balshagray Avenue, Partick	
1901	C.	Waterston, David, M.A., M.D., F.R.C.S.E., Professor of Anatomy, University, St Andrews	1916-19. 1925-28.
1927		* Watson, Charles Brodie Boog, F.S.A.Scot., 24 Garscube Terrace, Edinburgh 12	
1928		* Watson, H. Ferguson, M.D., F.R.F.P.S., Ph.D., D.P.H. (Glasg.), H.M. Senior Deputy Commissioner, General Board of Control for Scotland. 25 Palmerston Place, Edinburgh 12	
1923	C.	* Watson, William, M.A. (Edin.), B.Sc. (Edin.), Lecturer in Physics, Heriot-Watt College, Edinburgh. 17 Braidburn Crescent, Edinburgh 10	
1911		† Watt, James, W.S., F.F.A., LL.D. (Treasurer), Craiglockhart House, Craiglockhart Avenue, Edinburgh 11	1924-26. Treasurer 1926-
1911		* Watt, Rev. Lauchlan MacLean, D.D., The Cathedral, Glasgow	
1928		* Watters, Alexander Marshall, M.A., B.Sc. (Glasg.), Rector of Hawick High School. High School House, Hawick	
1896		† Webster, John Clarence, B.A., M.D., F.R.C.P.E., Professor of Obstetrics and Gynæcology, Rush Medical College, Shediac, N.B., Canada	

Date of Election.			Service on Council, etc.
1907	M-B. C.	† Wedderburn, Ernest Maclagan, M.A., D.Sc., LL.B., W.S., Professor of Conveyancing in the University of Edinburgh (South Bridge). 6 Succoth Gardens, Edinburgh 12	1913-16, 1921-24, 1932-
1903	M-B. C.	† Wedderburn, J. H. Maclagan, M.A., D.Sc., P.O. Box 53, Princeton, N.J., U.S.A.	
1904		Wedderspoon, William Gibson, M.A., LL.D., Indian Educational Service, Senior Inspector of Schools, Burma. The Education Office, Rangoon, Burma	
1930		* White, Adam Cairns, M.B., Ch.B., Ph.D., Assistant Pharmacologist, Wellcome Physiological Research Laboratory, Beckenham, Kent	
1931		* Whitson, Rt. Hon. Sir Thomas Barnby, C.A., LL.D., Lord Provost of the City of Edinburgh. 27 Eglinton Crescent, Edinburgh 12	
1911		* Whittaker, Charles Richard, F.R.C.S.E., F.S.A.Scot., Lynwood, Hatton Place, Edinburgh 9	
1912	C. V. J. B-P.	* Whittaker, Edmund Taylor, M.A., Hon. Sc.D. (Dubl.), LL.D., F.R.S., Foreign Member of the R. Accademia dei Lincei, Rome, Professor of Mathematics in the University of Edinburgh (16 Chambers Street). 48 George Square, Edinburgh 8	1912-15, 1922-25. Sec. 1916-22. V-P 1925-28.
1928	C.	* Whittaker, John Macnaghten, M.A. (Edin.), M.A. (Camb.), D.Sc., University Lecturer in Mathematics, Pembroke College, Cambridge	
1916		* Whyte, Rev. Charles, M.A., LL.D., F.R.A.S., U.F. Church Manse, Kingswells, Aberdeen	
1929	C.	* Wiesner, Bertold Paul, Ph.D., Lecturer in Sex Physiology, Institute of Animal Genetics, University of Edinburgh (King's Buildings, West Mains Road)	
1918		* Wight, John Thomas, M.I.Mech.E., M.I.Mar.E., Joint Managing Director and Vice-Chairman, Messrs MacTaggart, Scott & Co., Ltd., Station Iron Works, Loanhead. Calderwood Villa, Lasswade	
1925		* Wilkie, David Percival Dalbreck, O.B.E., M.D., Ch.M., F.R.C.S., Professor of Surgery in the University of Edinburgh (Royal Infirmary). 9 Ainslie Place, Edinburgh 3	
1926	C.	* Williams, Samuel, M.Sc., Ph.D., Lecturer in Plant Morphology in the University of Glasgow. 27 Lindsay Place, Kelvindale, Glasgow	
1924		* Williams, William Arthur, F.I.C., 1 Lennox Street, Edinburgh 4	
1908		Williamson, Henry Charles, M.A., D.Sc., formerly Naturalist to the Fishery Board for Scotland, Marine Laboratory, Aberdeen. 11 St Mary's Road, Downfield, Dundee	
1928	C.	* Williamson, John, M.A. (Edin.), Ph.D. (Chicago), Associate Professor of Mathematics in Johns Hopkins University, Baltimore, U.S.A.	
1910	C.	* Williamson, William, F.L.S., 7 Ventnor Terrace, Edinburgh 9	
1927	C.	* Williamson, William Turner Horace, B.Sc. (Aberd.), Ph.D. (Edin.), Chief Chemist, Egyptian Ministry of Agriculture, Cotton Research Board, Giza (Branch), Egypt	
1900		Wilson, Alfred C., Bloomfield House, Sadberge, near Darlington	
1911		* Wilson, Andrew, O.B.E., D.L., M.Inst.C.E., 66 Netherby Road, Trinity, Edinburgh 5	
1902	V. J.	† Wilson, Charles T. R., M.A., LL.D., F.R.S., Nobel Prize, Physics, 1927, Jacksonian Professor of Natural Philosophy in the University of Cambridge. Glencorse, Storey's Way, Cambridge	
1922		* Wilson, John, F.R.I.B.A., Fellow of the Inst. of Scottish Architects, Chief Architect, Scottish Department of Health. 20 Lomond Road, Edinburgh 5	
1920	C.	* Wilson, Malcolm, D.Sc. (London), A.R.C.Sc., F.L.S., Reader in Mycology and Bacteriology in the University of Edinburgh (Royal Botanic Garden). Brent Knoll, Kinnear Road, Edinburgh 4	1931-
1924		* Wilson, William, M.A., LL.B., Advocate, Regius Professor of Public Law in the University of Edinburgh (South Bridge). 38 Moray Place, Edinburgh 3	
1895		Wilson-Barker, Sir David, Kt., R.D., R.N.R., F.R.G.S., formerly Captain-Superintendent Thames Nautical Training College, H.M.S. "Worcester." 12 Bolan Street, London, S.W. 11	
1931	C.	* Wishart, John, M.A., B.Sc. (Edin.), M.A. (Camb.), D.Sc. (Lond.), Reader in Statistics in the University of Cambridge. Astræa, 18 Storey's Way, Cambridge	
1922	C. B.	* Wordie, James Mann, M.A. (Camb.), B.Sc. (Glasg.), 52 Montgomery Drive, Glasgow, and St John's College, Cambridge	
1890		Wright, Johnstone Christie, Conservative Club, Edinburgh 2	

Date of Election.			Service on Council, etc.
1896		† Wright, Sir Robert Patrick, LL.D., formerly Chairman of the Board of Agriculture for Scotland. The Heugh, North Berwick, East Lothian	
1911	C.	* Wrigley, Ruric Whitehead, M.A. (Cantab.), Assistant Astronomer, Royal Observatory, Edinburgh	
1882		Young, Frank W., C.B.E., F.C.S., H.M. Inspector of Schools (Emeritus). 35 Pentland Terrace, Edinburgh 10	
1904		Young, R. B., M.A., D.Sc., F.G.S., Professor of Geology and Mineralogy in the South African School of Mines and Technology, Johannesburg, Transvaal	

Number of Ordinary Fellows, 713.

LIST OF HONORARY FELLOWS OF THE SOCIETY.

(*At 24th October* 1932.)

HIS MOST EXCELLENT MAJESTY THE KING.

HIS ROYAL HIGHNESS THE PRINCE OF WALES.

FOREIGNERS (LIMITED TO THIRTY-SIX BY LAW I).

Elected

1916 Charles Eugène Barrois, formerly Professor of Geology and Mineralogy, Université, Lille, France: 37, rue Pascal, Lille.

1923 Henri Bergson, Honorary Professor, College of France, Paris.

1930 Vilhelm Frimann Koren Bjerknes, Professor of Physics, Geophysical Institute, Bergen.

1927 Niels Bohr, Nobel Laureate, Physics, 1922, Professor of Physics, University of Copenhagen.

1927 Jules Bordet, Nobel Laureate, Medicine, 1919, Professor of Bacteriology, University of Brussels.

1923 Marcellin Boule, Professor at the National Museum of Natural History, Laboratory of Palæontology, 3 Place Valhubert, Paris 5^{e}.

1905 Waldemar Christofer Brögger, Professor of Mineralogy and Geology, K. Frederiks Universitet, Oslo, Norway.

1916 Douglas Houghton Campbell, Em. Professor of Botany, Leland Stanford Junior University, California, U.S.A.

1920 William Wallace Campbell, President-Emeritus of the University of California, Berkeley, Director-Emeritus of the Lick Observatory, Mt. Hamilton, California, and President of the National Academy of Sciences, U.S.A.

1930 Walter Bradford Cannon, Professor of Physiology, Harvard University, Cambridge, U.S.A.

1930 Maurice Caullery, Professor of Zoology in the University of Paris. Evolution des Etres Organises Laboratoire, 105 Bould. Raspail, Paris, VI^{e}.

1921 Reginald Aldworth Daly, Professor of Geology, Harvard University, Cambridge, Mass.

1910 Hugo de Vries, Professor of Plant Anatomy and Physiology, Lunteren, Holland.

1927 Albert Einstein, Nobel Laureate, Physics, 1921, Professor of Mathematical Physics, University of Berlin.

1913 George Ellery Hale, Honorary Director of Mount Wilson Observatory (Carnegie Institution of Washington), Pasadena, California, U.S.A.

1921 Johan Hjort, Professor of Marine Biology, University, Oslo.

1923 Arnold Frederik Holleman, Professor of Organic Chemistry, University, Amsterdam. Boekensteijn Parkweg 7, Bloemendaal, Holland.

1923 Tullio Levi-Civita, Professor of Mathematics (Higher Analysis), University, Rome.

1927 Hans Horst Meyer, Emeritus Professor of Pharmacology, University of Vienna.

1923 Arthur Amos Noyes, Institute of California, Pasadena, U.S.A.

1908 Henry Fairfield Osborn, Research Professor of Zoology, Columbia University, and President, American Museum of Natural History, New York, U.S.A., Senior Geologist, U.S.A. Geological Survey.

1908 Ivan Petrovitch Pavlov, Em. Professor of Physiology, Inst. Exper. Med., Leningrad, Nobel Laureate, Physiology and Medicine, 1904: 7, Linia, No. 2, Vassilievsky, Ostrov, Leningrad, Russia.

1920 Ch. Emile Picard, Perpetual Secretary, Academy of Sciences, Paris.

1921 Salvatore Pincherle, Professor of Mathematics in the University of Bologna.

1913 Santiago Ramón y Cajal, Nobel Laureate, Medicine, 1906, formerly Professor of Histology and Pathological Anatomy, University, Madrid, Spain.

1920 Charles Richet, Professor of Physiology, Faculty of Medicine, Paris, Nobel Laureate, Medicine, 1913.

1927 Johannes Schmidt, A Director of the Carlsberg Laboratorium, Copenhagen.

1930 Erik Helge Oswald Stensiö, Professor, Royal Natural History Museum, Stockholm.

1913 Vito Volterra, Professor of Mathematical Physics, Regia Università, Rome, Italy.

1927 Richard Willstätter, Professor of Chemistry, University of Munich, Nobel Laureate, Chemistry, 1915. Munich 27, Moehlstrasse 29.

1923 Edmund Beecher Wilson, Professor of Zoology, Columbia University, New York, U.S.A.

Total, 31.

BRITISH SUBJECTS (LIMITED TO TWENTY BY LAW I).

Elected

1927 Sir William Henry Bragg, O.M., K.B.E., M.A., D.Sc., LL.D., F.R.S., Nobel Laureate, Physics, 1915, Fullerian Professor of Chemistry, Royal Institution, London.

1930 Sir Arthur Stanley Eddington, Kt., M.A., Hon. D.Sc., F.R.S., Plumian Professor of Astronomy and Experimental Philosophy in the University of Cambridge.

1927 Sir John Bretland Farmer, Kt., M.A., D.Sc., LL.D., F.R.S., formerly Professor of Botany, Imperial College of Science and Technology, London.

1900 Andrew Russell Forsyth, M.A., Sc.D., LL.D., Hon. Math.D., F.R.S., Em. Professor of Mathematics in the Imperial College of Science and Technology, London; formerly Sadleirian Professor of Pure Mathematics in the University of Cambridge. Imperial College of Science and Technology, London, S. W. 7.

1910 Sir James George Frazer, O.M., Kt., D.C.L., LL.D., Litt.D., F.R.S., Commandeur de la Légion d'Honneur, Fellow of Trinity College, Cambridge.

1930 Sir William Bate Hardy, Kt., M.A., F.R.S., Director of Food Investigation, Department of Scientific and Industrial Research, 5 Grange Road, Cambridge.

1927 Sir Frederick Gowland Hopkins, Kt., M.A., M.B., D.Sc., LL.D., F.R S., Pres. R.S., Joint Nobel Laureate, Medicine, 1929, Professor of Bio-Chemistry, University of Cambridge.

1930 Sir Arthur Keith, Kt., M.D., LL.D., F.R.S., Hunterian Professor and Conservator of the Museum of the Royal College of Surgeons, London.

1913 Sir Horace Lamb, M.A., Sc.D., D.Sc., LL.D., F.R.S., formerly Professor of Mathematics in the University of Manchester. 6 Selwyn Gardens, Cambridge.

1910 Sir Joseph Larmor, Kt., M.A., D.Sc., LL.D., D.C.L., F.R.S., formerly Lucasian Professor of Mathematics in the University of Cambridge. St John's College, Cambridge.

1930 John Edward Marr, Sc.D., F.R.S., Fellow of St John's College, and Em. Professor of Geology, Cambridge. 126 Huntingdon Road, Cambridge.

1930 Robert Robinson, D.Sc., F.R.S., Waynflete Professor of Chemistry in the University of Oxford. The Dyson Perrins Laboratory, South Parks Road, Oxford.

1921 Rt. Hon. Lord Rutherford of Nelson, O.M., M.A., D.Sc., B.A., LL.D., Past Pres. R.S., Nobel Laureate, Chemistry, 1908, Cavendish Professor of Experimental Physics in the University of Cambridge.

1916 Sir Arthur Schuster, Kt., Ph.D., D.Sc., LL.D., Dr ès Sc. Geneva, Honorary Professor of Physics in the University of Manchester. Yeldall, Twyford, Berks.

1930 Dukinfield Henry Scott, M.A., D.Sc., LL.D., Ph.D., F.R.S., formerly Honorary Keeper of the Jodrell Laboratory, Royal Botanic Gardens, Kew. East Oakley House, Basingstoke, Hants.

1908 Sir Charles Scott Sherrington, O.M., G.B.E., M.A., D.Sc., M.D., LL.D., Past Pres. R.S., Joint Laureate, Nobel Prize, Medicine, 1932, Waynflete Professor of Physiology in the University of Oxford. Physiological Laboratory, Oxford.

1905 Sir Joseph John Thomson, O.M., Kt., D.Sc., LL.D., Past Pres. R.S., Nobel Laureate, Physics, 1906, formerly Cavendish Professor of Experimental Physics, University of Cambridge, Master of Trinity College, Cambridge.

Total, 17.

CHANGES IN FELLOWSHIP DURING SESSION 1931–1932.

ORDINARY FELLOWS OF THE SOCIETY ELECTED.

EUSTACE CECIL BARTON-WRIGHT.
SOHAN LAL BHATIA.
JAMES COUPER BRASH.
EDWARD PROVAN CATHCART.
VERE GORDON CHILDE.
JAMES ROBERTON CHRISTIE.
SIR THOMAS CLARK, BART.
DAVID CLOUSTON.
LEYBOURNE STANLEY PATRICK DAVIDSON.
ALEXANDER MURRAY DRENNAN.
SIR JOHN EDMUND RITCHIE FINDLAY, BART.
ANDREW HUNTER.
JAMES BALFOUR LOCKHART.
ARCHIBALD GORDON MACGREGOR.
JOHN JAMES RICKARD MACLEOD.
CHARLES M'NEIL.
WILLIAM MAXWELL.
JOHN MURDOCH MURRAY.
MATTHEW YOUNG ORR.
JAMES NICHOL PICKARD.
GORAKH PRASAD.
THOMAS SLATER PRICE.
JOHN PRINGLE.
JOHN MICHAEL ROBSON.
WILLIAM FLEETWOOD SHEPPARD.
ALEXANDER RUDOLPH BARBOUR SIMPSON.
JOHN BAIRD SIMPSON.
JEAN-BAPTISTE OCTAVE SNEEDEN.
WILLIAM ELGIN SWINTON.
HAROLD WILLIAM THOMPSON.
HENRY WADE.

ORDINARY FELLOWS DECEASED.

FRANCIS JOHN ALLAN.
WALTER LEONARD BELL.
ALEXANDER G. BURGESS.
JAMES ROBERTON CHRISTIE.
JAMES EDWARD CROMBIE.
THOMAS WILLIAM DEWAR.
ROBERT DRON.
ANDREW FREELAND FERGUS.
SIR PATRICK GEDDES.
WILLIAM GORDON.
JOHN WALTER GREGORY.
ALEXANDER JAMES.
MALCOLM LAURIE.
GRAHAM LUSK.
VERY REV. DUGALD MACKICHAN.
JAMES MALLOCH.
DAVID HENRY MARSHALL.
A. D. LEITH NAPIER.
ALEXANDER PHILIP.
SIR BRUCE GORDON SETON.
SIR WILLIAM SOMMERVILLE.
SIR WILLIAM (ROBERT) SMITH.
JOHN BOLTON THACKWELL.
JAMES STUART THOMSON.

FOREIGN HONORARY FELLOWS DECEASED.

GIULIO FANO.
KARL RITTER VON GOEBEL.

BRITISH HONORARY FELLOWS DECEASED.

SIR DAVID BRUCE.
SIR RONALD ROSS.

ORDINARY FELLOWS RESIGNED.

RASIK LAL DATTA.
JOHN MARTIN MUNRO KERR.
WILLIAM MACKIE.
RT. HON. VISCOUNT NOVAR.
THOMAS STEWART PATTERSON.
JOHN WILLIAM STRUTHERS.

LAWS OF THE SOCIETY.

Adopted July 3, 1916; amended December 18, 1916.

(Laws VIII, IX, and XIII amended May 3, 1920. Law VI amended February 7, 1921. Law XIX amended June 16, 1924. Law VI amended July 2, 1928.)

I.

THE ROYAL SOCIETY OF EDINBURGH, which was instituted by Royal Charter in 1783 for the promotion of Science and Literature, shall consist of Ordinary Fellows (hereinafter to be termed Fellows) and Honorary Fellows. The number of Honorary Fellows shall not exceed fifty-six, of whom not more than twenty may be British subjects, and not more than thirty-six subjects of Foreign States.

Fellows only shall be eligible to hold office or to vote at any Meeting of the Society.

ELECTION OF FELLOWS.

II.

Each Candidate for admission as a Fellow shall be proposed by at least four Fellows, two of whom must certify from personal knowledge. The Official Certificate shall specify the name, rank, profession, place of residence, and the qualifications of the Candidate. The Certificate shall be delivered to the General Secretary before the 30th of November, and, subject to the approval of the Council, shall be exhibited in the Society's House during the month of January following. All Certificates so exhibited shall be considered by the Council at its first meeting in February, and a list of the Candidates approved by the Council for election shall be issued to the Fellows not later than the 21st of February.

III.

The election of Fellows shall be by Ballot, and shall take place at the first Ordinary Meeting in March. Only Candidates approved by the Council shall be eligible for election. A Candidate shall be held not elected, unless he is supported by a majority of two-thirds of the Fellows present and voting.

IV.

On the day of election of Fellows two scrutineers, nominated by the President, shall examine the votes and hand their report to the President, who shall declare the result.

V.

Each Fellow, after his election, is expected to attend an Ordinary Meeting, and sign the Roll of Fellows, he having first made the payments required by Law VI. He shall be introduced to the President, who shall address him in these words:

> *In the name and by the authority of THE ROYAL SOCIETY OF EDINBURGH, I admit you a Fellow thereof.*

PAYMENTS BY FELLOWS.

VI.

Each Fellow shall, before he is admitted to the privileges of Fellowship, pay an admission fee of three guineas, and a subscription of three guineas for the year of election. He shall continue to pay a subscription of three guineas at the beginning of each session so long as he remains a Fellow.

Each Fellow who was elected subsequent to December 1916 and previous to December 1920 shall also pay a subscription of three guineas at the beginning of each session so long as he remains a Fellow.

Each Fellow who was elected previous to December 1916, and who has not completed his twenty-five annual payments, shall, at the beginning of each session, pay three guineas until his twenty-five annual payments are made. Each Fellow who has completed or shall complete his payments shall be invited to contribute one guinea per annum or to pay a single sum of ten guineas.

A Fellow may compound for the annual subscriptions by a single payment of fifty guineas, or on such other terms as the Council may from time to time fix.

VII.

A Fellow who, after application made by the Treasurer, fails to pay any contribution due by him, shall be reported to the Council, and, if the Council see fit, shall be declared no longer a Fellow. Notwithstanding such declaration all arrears of contributions shall remain exigible.

ELECTION OF HONORARY FELLOWS.

VIII.

Honorary Fellows shall be persons eminently distinguished in Science or Literature. They shall not be liable to contribute to the Society's Funds. Personages of the Blood Royal may be elected Honorary Fellows at any time on the nomination of the Council, and without regard to the limitation of numbers specified in Law I.

IX.

Honorary Fellows shall be proposed by the Council. The nominations shall be announced from the Chair at the First Ordinary Meeting after their selection. The names shall be printed in the circular for the last Ordinary Meeting of the Session, when the election shall be by Ballot, after the manner prescribed in Laws III and IV for the Election of Fellows.

EXPULSION OF FELLOWS.

X.

If, in the opinion of the Council, the conduct of any Fellow is injurious to the character or interests of the Society, the Council may, by registered letter, request him to resign. If he fail to do so within one month of such request, the Council shall call a Special Meeting of the Society to consider the matter. If a majority consisting of not less than two-thirds of the Fellows present and voting decide for expulsion, he shall be expelled by declaration from the Chair, his name shall be erased from the Roll, and he shall forfeit all right or claim in or to the property of the Society.

XI.

It shall be competent for the Council to remove any person from the Roll of Honorary Fellows if, in their opinion, his remaining on the Roll would be injurious to the character or interests of the Society. Reasonable notice of such proposal shall be given to each member of the Council, and, if possible, to the Honorary Fellow himself. Thereafter the decision on the question shall not be taken until the matter has been discussed at two Meetings of Council, separated by an interval of not less than fourteen days. A majority of two-thirds of the members present and voting shall be required for such removal.

MEETINGS OF THE SOCIETY.

XII.

A Statutory Meeting for the election of Council and Office-Bearers, for the presentation of the Annual Reports, and for such other business as may be arranged by the Council, shall be held on the fourth Monday of October. Each Session of the Society shall begin at the date of the Statutory Meeting.

XIII.

Meetings for reading and discussing communications and for general business, herein termed Ordinary Meetings, shall be held, when convenient, on the first and third Mondays of each month from November to July inclusive, with the exception that in January the meetings shall be held on the second and fourth Mondays.

XIV.

A Special Meeting of the Society may be called at any time by direction of the Council, or on a requisition to the Council signed by not fewer than six Fellows. The date and hour of such Meeting shall be determined by the Council, who shall give not less than seven days' notice of such Meeting. The notice shall state the purpose for which the Special Meeting is summoned ; no other business shall be transacted.

PUBLICATION OF PAPERS.

XV.

The Society shall publish Transactions and Proceedings. The consideration of the acceptance, reading, and publication of papers is vested in the Council, whose decision shall be final. Acceptance for reading shall not necessarily imply acceptance for publication.

DISTRIBUTION OF PUBLICATIONS.

XVI.

Fellows who are not in arrear with their Annual Subscriptions and all Honorary Fellows shall be entitled gratis to copies of the Parts of the Transactions and the Proceedings published subsequently to their admission.

Copies of the Parts of the Proceedings shall be distributed by post or otherwise, as soon as may be convenient after publication ; copies of the Transactions or Parts thereof shall be obtainable upon application, either personally or

by an authorised agent, to the Librarian, provided the application is made within five years after the date of publication.

CONSTITUTION OF COUNCIL.

XVII.

The Council shall consist of a President, six Vice-Presidents, a Treasurer, a General Secretary, two Secretaries to the Ordinary Meetings (the one representing the Biological group and the other the Physical group of Sciences),* a Curator of the Library and Museum, and twelve ordinary members of Council.

ELECTION OF COUNCIL.

XVIII.

The election of the Council and Office-Bearers for the ensuing Session shall be held at the Statutory Meeting on the fourth Monday of October. The list of the names recommended by the Council shall be issued to the Fellows not less than one week before the Meeting. The election shall be by Ballot, and shall be determined by a majority of the Fellows present and voting. Scrutineers shall be nominated as in Law IV.

XIX.

The President may hold office for a period not exceeding five consecutive years; the Vice-Presidents, not exceeding three; the Secretaries to the Ordinary Meetings, not exceeding five; the General Secretary, the Treasurer, and the Curator of the Library and Museum, not exceeding ten; and ordinary members of Council, not exceeding three consecutive years; provided that the Treasurer may be re-elected for more than ten successive years in cases where the Council declares to the Society that an emergency exists.

XX.

In the event of a vacancy arising in the Council or in any of the offices enumerated in Law XVII, the Council shall proceed, as soon as convenient, to elect a Fellow to fill such vacancy for the period up to the next Statutory Meeting.

* The Biological group includes Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, Zoology; the Physical group includes Astronomy, Chemistry, Mathematics, Metallurgy, Meteorology, Physics.

POWERS OF THE COUNCIL.

XXI.

The Council shall have the following powers:—(1) To manage all business concerning the affairs of the Society. (2) To decide what papers shall be accepted for communication to the Society, and what papers shall be printed in whole or in part in the Transactions and Proceedings. (3) To appoint Committees. (4) To appoint employees and determine their remuneration. (5) To award the various prizes vested in the Society, in accordance with the terms of the respective deeds of gift, provided that no member of the existing Council shall be eligible for any such award. (6) To make from time to time Standing Orders for the regulation of the affairs of the Society. (7) To control the investment or expenditure of the Funds of the Society.

At Meetings of the Council the President or Chairman shall have a casting as well as a deliberative vote.

DUTIES OF PRESIDENT AND VICE-PRESIDENTS.

XXII.

The President shall take the Chair at Meetings of Council and of the Fellows. It shall be his duty to see that the business is conducted in accordance with the Charter and Laws of the Society. When unable to be present at any Meetings or attend to current business, he shall give notice to the General Secretary, in order that his place may be supplied. In the absence of the President his duties shall be discharged by one of the Vice-Presidents.

DUTIES OF THE TREASURER.

XXIII.

The Treasurer shall receive the monies due to the Society and shall make payments authorised by the Council. He shall lay before the Council a list of arrears in accordance with Rule VII. He shall keep accounts of all receipts and payments, and at the Statutory Meeting shall present the accounts for the preceding Session, balanced to the 30th of September, and audited by a professional accountant appointed annually by the Society.

DUTIES OF THE GENERAL SECRETARY.

XXIV.

The General Secretary shall be responsible to the Council for the conduct of the Society's correspondence, publications, and all other business except that which relates to finance. He shall keep Minutes of the Statutory and Special

Meetings of the Society and Minutes of the Meetings of Council. He shall superintend, with the aid of the Assistant Secretary, the publication of the Transactions and Proceedings. He shall supervise the employees in the discharge of their duties.

DUTIES OF SECRETARIES TO ORDINARY MEETINGS.

XXV.

The Secretaries to Ordinary Meetings shall keep Minutes of the Ordinary Meetings. They shall assist the General Secretary, when necessary, in superintending the publication of the Transactions and Proceedings. In his absence, one of them shall perform his duties.

DUTIES OF CURATOR OF LIBRARY AND MUSEUM.

XXVI.

The Curator of the Library and Museum shall have charge of the Books, Manuscripts, Maps, and other articles belonging to the Society. He shall keep the Card Catalogue up to date. He shall purchase Books sanctioned by the Council.

ASSISTANT-SECRETARY AND LIBRARIAN.

XXVII.

The Council shall appoint an Assistant-Secretary and Librarian, who shall hold office during the pleasure of the Council. He shall give all his time, during prescribed hours, to the work of the Society, and shall be paid according to the determination of the Council. When necessary he shall act under the Treasurer in receiving subscriptions, giving out receipts, and paying employees.

ALTERATION OF LAWS.

XXVIII.

Any proposed alteration in the Laws shall be considered by the Council, due notice having been given to each member of Council. Such alteration, if approved by the Council, shall be proposed from the Chair at the next Ordinary Meeting of the Society, and, in accordance with the Charter, shall be considered and voted upon at a Meeting held at least one month after that at which the motion for alteration shall have been proposed.

Additions to the Library—Presentations, etc.—1931-1932.

Aagaard, Bjarne. Fangst og Forskning i Sydishavet. 2 vols. 8vo. Oslo, 1930. (*Presented by Mr W. G. Burn-Murdoch.*)

Acta Physica Polonica. Vol. I, No. 1– . 8vo. Warsaw, 1932. (*Exchange.*)

Annales Françaises de Chronométrie. No. 1– . 8vo. Besançon, 1931– . (*Presented by l'Observatoire National de Besançon.*)

Annales Musei Zoologici Polonici. Tome 1– . 8vo. Warsaw, 1922. (*Exchange.*)

Bailey, I. W., and H. A. Spoehr. The Rôle of Research in the Development of Forestry in North America. 8vo. New York, 1929. (*Exchange.*)

Berichte des Ōhara Instituts für landwirtschaftliche Forschungen in Kuraschiki. Bd. V, Heft 1– . La. 8vo. Kuraschiki, 1931. (*Exchange.*)

Bigelow, Henry B. Oceanography: Its Scope, Problems, and Economic Importance. 8vo. Boston, 1931.

Bollettino della Reale Istituto Superiore Agrario. Series II. Vol. I– . La. 8vo. Milan, 1932. (*Exchange.*)

Bowman, T. Sutton. Report on Boring for Oil in Egypt: Section III, Eastern Desert and adjoining Islands. 4to. Cairo, 1931. (*Presented by Egyptian Government.*)

Brasch, F. E. The Royal Society of London and its Influence upon Scientific Thought in the American Colonies. La. 8vo. New York, 1931.

——— Commemoration of the Bi-centenary of the Death of Newton. La. 8vo. New York, 1928.

British Museum (Natural History):—

- Catalogue of the Rock Collections in the Mineral Department. Part II: America. By W. Campbell Smith. 8vo. London, 1932.
- Diptera of Patagonia and South Chile. Part II, fascs. 4 and 5. Part V, fasc. 3. Part VI, fasc. 2. 8vo. London. 1931–32.
- General Guide to the Exhibition Galleries (Illustrated). 8vo. London, 1931.
- Great Barrier Reef Expedition, 1928–29. Scientific Reports. Vol. I, – . Vol. II, – . Vol. III, – . Vol. IV, – .
- Index Animalium. Parts 25–30. La. 8vo. London, 1931–32.
- Newton, Lily. Handbook of the British Seaweeds. 8vo. London, 1931.
- Robson, G. C. Monograph of the Recent Cephalopoda. Part II: The Octopoda. La. 8vo. London, 1932.

Special Guide No. 8. Guide to an Exhibition Illustrating the Early History of Palæontology. 8vo. London, 1931.

——— No. 9. A Short History of the Collections. 8vo. London, 1931.

Summary Guide to the Exhibition Galleries of the British Museum (Natural History). 4th edition, 1931. 8vo. London, 1931.

(*Exchange and Presentation.*)

Bulletin of the Academy of Sciences. Vol. I- . 8vo. Allahabad, 1932. (*Exchange.*)

Buyssens, T. Langage et pensée, vie et matière : nouvelles hypothèses. 8vo. Anvers, 1928. (*Presented by the Author.*)

Cameron, John. Researches in Craniometry. Vol. II. 8vo. Halifax, Nova Scotia, 1915–1930. (*Presented by the Author.*)

Cameron, Malcolm. Fauna of British India : Coleoptera, Staphylinidæ. Vol. III. 8vo. London, 1932. (*Presented by the Secretary of State for India.*)

Campbell, Charles A. Scepticism and Construction. (Glasgow University Publications, No. XXIII.) 8vo. London, 1931. (*Exchange.*)

Carnegie Institution of Washington :—

(*Exchange.*)

No. 376. Sarton, George. Introduction to the History of Science. Vol. II, Parts I and II.

Part I : From Rabbi Ben Ezra to Ibn Rushd.

Part II : From Robert Grosseteste to Roger Bacon. 8vo. Washington, 1931.

No. 392. Guide to Materials for American History in the Libraries and Archives of Paris. Vol. I : Libraries. By Waldo G. Leland. 8vo. 1932.

No. 403. Contributions to American Archæology. Vol. I. 4to. Washington, 1931.

No. 409. Documents Illustrative of the History of the Slave Trade to America. By Elizabeth Donnan. Vol. II. 8vo. Washington, 1931.

No. 410. Studies on the Variation, Distribution, and Evolution of the Genus *Partula*—the Species inhabiting Moorea. By Henry E. Crampton. 4to. Washington, 1932.

No. 417. New General Catalogue of Double Stars within 120° of the North Pole. 2 vols. By R. G. Aitken. 4to. Washington, 1932.

No. 419. Albany Catalogue of 20,811 Stars for Epoch 1910. Under the direction of Benjamin Boss, Arthur J. Roy, and William B. Varnum. 4to. Washington, 1931.

No. 420. Anthokinetics : The Physiology and Ecology of Floral Movements. By G. W. Goldsmith and A. L. Hafenrichter. 8vo. Washington, 1932.

No. 421. Contributions to the Genetics of Certain Chromosome Anomalies in *Drosophila melanogaster*. By A. H. Sturtevant and T. Dobzhansky. La. 8vo. Washington, 1931.

No. 424. A Preliminary Study of the Ruins of Cobá Quintana Roo, Mexico. By J. Eric Thompson, Harry E. D. Pollock, and Jean Charlot. 4to. Washington, 1932.

No. 425. The Physiology of Large Reptiles with Special Reference to the Heat Production of Snakes, Tortoises, Lizards, and Alligators. By Francis G. Benedict. 8vo. Washington, 1932.

The Development of the Egg—A Symposium. By George L. Streeter, Carl G. Hartman, and C. W. Metz. (Supplementary Publications No. 3.) La. 8vo. Washington, 1931.

Catalogue of Reproductions of Celestial Photographs. Published by the Royal Astronomical Society. 8vo. London, 1925. (*Exchange.*)

Colyer, Sir Frank. Four Lectures on "Abnormal Conditions of the Teeth of Animals in their Relationship to Similar Conditions in Man." 8vo. London, 1931. (*Presented by the Dental Board of the United Kingdom.*)

Consitt, Frances. The Value of Films in History Teaching. 4to. London, 1931. (*Presented by the Carnegie United Kingdom Trustees.*)

Dawson, Warren R. The Nelson Collection at Lloyd's. 4to. London, 1932. (*Presented by the Corporation of Lloyd's.*)

Destruction of Timber by Marine Organisms in the Port of Sydney. By Tom Iredale, R. A. Johnson, and F. A. McNeill. La. 8vo. Sydney, 1932. (*Presented by Sydney Harbour Trust, Sydney.*)

The Fauna of British India :—

Smith, Malcolm A. Reptilia and Amphibia. Vol. I. Loricata, Testudines. 8vo. London, 1931.

Arrow, G. J. Coleoptera, Lamellicornia. Part III (Coprinæ). 8vo. London, 1931.

(*Presented by the Secretary of State for India.*)

Fifty Years' Retrospect, Anniversary Volume, 1882–1932, Royal Society of Canada. La. 8vo. Ottawa, 1932. (*Presented.*)

The Film in National Life, A Report. 4to. London, 1932. (*Presented by the Commission on Educational and Cultural Films.*)

Fox, Cyril. A Survey of McGill University Museums. 8vo. Montreal, 1932. (*Exchange.*)

Fragmenta Faunistica Musei Zoologici Polonici. Tome 1– . 8vo. Warsaw, 1930. (*Exchange.*)

Gedenkboek van het Athenæum en de Universiteit van Amsterdam, 1632–1932. 4to. Amsterdam, 1932. (*Presented by the University.*)

Greig, J. Y. T. The Letters of David Hume. 2 vols. 8vo. Oxford, 1932. (*Presented by the Clarendon Press.*)

Gurney, Robert. British Fresh-Water Copepoda. Vol. I. (Ray Society Publication.) 8vo. London, 1931. (*Purchased.*)

Hadfield, Sir Robert A., Bart. Faraday and his Metallurgical Researches. La. 8vo. London, 1931. (*Presented by the Author.*)

Houstoun, R. A. Vision and Colour Vision. 8vo. London, 1932. (*Presented by the Author.*)

James Clerk Maxwell: a commemoration volume, 1831–1931. 8vo. Cambridge, 1931. (*Purchased.*)

Jenness, Diamond. The Indians of Canada. (Bulletin 65, National Museum of Canada.) La. 8vo. Ottawa, 1932. (*Exchange.*)

Journal of the Faculty of Science, Hokkaido Imperial University. Series 1–4. Vol. I– . La. 8vo. Sapporo, Japan, 1930. (*Exchange.*)

Laird, John. Hume's Philosophy of Human Nature. 8vo. London, 1932. (*Presented by the Author.*)

The Lord Provosts of Edinburgh, 1296–1932. Printed for Sir Thomas B. Whitson. 4to. Edinburgh, 1932. (*Presented by Sir Thomas B. Whitson.*)

Magyar Chemiai Folyóirat. Január 1931– . 8vo. Budapest, 1931. (*Exchange: Royal Society of Natural Sciences, Budapest.*)

Minto, Walter. *See* Stewart, David.

Osborn, Henry Fairfield. Fifty-two Years of Research, Observation, and Publication, 1877–1929. 8vo. New York, 1930. (*Presented by the Author.*)

Ōtani, Ryōkichi. Tadataka Inō: the Japanese Land-Surveyor. Translated by Kazue Sugimura. La. 8vo. Tokyo, 1932. (*Presented by the Institute of Physical and Chemical Research, Tokyo.*)

Periodici Italiani Scientifici Tecnici e di Cultura Generale. III. Edizione Interamente Rifatta, 1931. (Consiglio Nazionale delle Ricerche.) 8vo. Rome, 1931. (*Exchange.*)

Physikalische Zeitschrift der Sowjetunion. Bd. I– . 8vo. Charkow, 1932. (*Exchange.*)

Plymouth Marine Fauna. Compiled from the Records of the Laboratory of the Marine Biological Association. 2nd edition. 8vo. Plymouth, 1931. (*Exchange.*)

Publications of the British School of Archæology in Egypt and the Egyptian Research Account. Vols. XII, XIII, XV, XVII, XVIII, XX, XXI, XXIII–XXXV, XXXVII, XXXVIII, XLI, and XLIV. In all 24 volumes. 4to. London, 1906–1927. (*Presented by Mr A. R. Barbour Simpson.*)

Publications of the Lick Observatory. Vol. XVII, 1931:—

1. A Study of the Solar Chromosphere. By D. H. Menzel.
2. Halley's Comet in its Apparition of 1909–1911. By N. T. Bobrovnikoff. 4to. Berkeley, California, 1932. (*Exchange.*)

Ray Society. *See* Gurney, Robert.

Report : National Research Council, Dominion of Canada. No. 25- . La. 8vo. Ottawa, 1932. (*Exchange.*)

Report on the Scientific Results of the " Michael Sars " North Atlantic Deep-Sea Expedition, 1910. Vol. III, Part 2, 1913–1921. 4to. Bergen, 1932. (*Purchased.*)

Reviews of Modern Physics. Vol. I- . 1929. (Published by the American Physical Society.) La. 8vo. Minneapolis. (*Purchase.*)

Reynolds, F. Esmond. *See* Turner, A. Logan.

Sharpey-Schafer, Sir E. A. Endocrine Physiology, being the John Mallet Purser Lecture, 1931. 8vo. Dublin, 1931. (*Presented by Sir E. A. Sharpey-Schafer.*)

Smith, D. M. *See* Twyman, F.

Spoehr, H. A. *See* Bailey, I. W.

Sprawozdanie Państwowego Muzeum Zoologicznego, 1929- . 8vo. Warsaw, 1931. (*Exchange.*)

Stewart, David (Earl of Buchan), and Walter Minto. An Account of the Life, Writings, and Inventions of John Napier of Merchiston. 4to. Perth, 1787. (*Presented by Mr John S. Ford.*)

Stheethman, H. A. The Geology of South-western Uganda, with Special Reference to the Stanniferous Deposits. 4to. The Hague, 1932. (*Exchange.*)

Sugimura, Kazue. *See* Ōtani, Ryōkichi.

Tables Annuelles des Constantes, etc. Tomes VIII, IX. 4to. Paris, 1931–32. (*Purchased.*)

Turner, A. Logan, and F. Esmond Reynolds. Intercranial Pyogenic Diseases : A Pathological and Clinical Study of the Pathways of Infection from the Face, the Nasal and Paranasal Air-Cavities. 8vo. Edinburgh, 1931. (*Presented by Dr A. Logan Turner.*)

Twyman, F., and D. M. Smith. Wave-length Tables for Spectrum Analysis. 2nd edition. 8vo. London, 1931. (*Presented by Messrs Adam Hilger, Ltd.*)

University Studies : Louisiana State University. No. 2- . 8vo. Baton Rouge. 1931. (*Exchange.*)

Van Es, Ir. L. J. C. The Age of Pithecanthropus. La. 8vo. The Hague, 1931. (*Exchange.*)

Vigoureux, P. Quartz Resonators and Oscillators. 8vo. London, 1931. (*Presented by Department of Scientific and Industrial Research.*)

Wood, Casey A. An Introduction to the Literature of Vertebrate Zoology. (McGill University Publications, Series XI [Zoology], No. 24.) 4to. London, 1931. (*Presented.*)

INDEX.

Index of Papers published in the "Transactions" during Session 1931–32.

PRINTED IN GREAT BRITAIN BY NEILL AND CO., LTD., EDINBURGH.